

Heating of ions by low-frequency Alfvén waves

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This paper reports a new physical mechanism that enables the heating of ions by a low-frequency parallel propagating Alfvén wave of finite amplitude in a low beta plasma. The heating does not rely on ion cyclotron resonance. The process has two stages: First, ions, whose initial average velocity is zero, are picked up in the transverse direction by the Alfvén wave and obtain an average transverse velocity. Second, at a given location the parallel thermal motions of ions produce phase differences (randomization) between ions leading to the heating of ions. The randomization (or heating) process saturates when phase differences are sufficiently large. The time scale over which ions are significantly heated is $\sim \pi/(kv_{th})$ (v_{th} is the initial ion thermal speed and k is the wave number). The heating is dominant in the perpendicular (to the background magnetic field) direction. Subsequently, a large ion temperature anisotropy is produced. During the heating process, ions are also accelerated in the parallel direction and obtain a bulk flow speed along the background magnetic field. © 2007 American Institute of Physics. [DOI: 10.1063/1.2715569]

I. INTRODUCTION

The heating of ions by Alfvén waves is of great importance in astrophysical and magnetically controlled laboratory plasmas.¹⁻⁶ Numerous papers, which are based on linear theory, have been published to investigate the resonant heating of ions by Alfvén waves.⁷⁻¹³ According to linear Vlasov theory, high-frequency Alfvén waves propagating along the background magnetic field heat charged particles only via cyclotron resonance: $\omega - kv_{\parallel} = \Omega_0$ (where ω and k are the frequency and wave number of the Alfvén waves, respectively. v_{\parallel} is the velocity component parallel to the background magnetic field and Ω_0 is the cyclotron frequency of the particle). Recently, Chen and his collaborators^{14,15} investigated the stochastic ion heating by obliquely propagating Alfvén waves with sufficiently large amplitude, and found that a significant perpendicular heating of particles can be achieved at a fraction of cyclotron frequency. Other ion heating mechanisms, for instance, low-frequency kinetic Alfvén waves and large amplitude MHD waves with strongly correlated wave phases,¹⁶⁻¹⁹ were also reported. In this paper, we demonstrate that a low-frequency Alfvén wave, which propagates along the background magnetic field $\mathbf{B}_0 = B_0 \mathbf{i}_z$, may heat ions even when the cyclotron resonant condition is not met if initially the average velocity of these ions is zero.

II. ANALYTIC THEORY AND TEST PARTICLE CALCULATIONS

We consider a monochromatic dispersionless Alfvén wave with frequency ω and wave number k , and the dispersion relation is $\omega = kv_A$ (v_A is the Alfvén speed). The wave magnetic field $\delta \mathbf{B}_w$ and electric field $\delta \mathbf{E}_w$ can be expressed as

$$\delta \mathbf{B}_w = B_k (\cos \phi_k \mathbf{i}_x - \sin \phi_k \mathbf{i}_y), \quad (1)$$

$$\delta \mathbf{E}_w = -\frac{v_A}{c} \mathbf{b} \times \delta \mathbf{B}_w, \quad \mathbf{b} = \frac{\mathbf{B}_0}{B_0},$$

where \mathbf{i}_x and \mathbf{i}_y are unit directional vectors, $\phi_k = k(v_A t - z)$ denotes the wave phase. The motion of a particle is described by

$$m_j \frac{d\mathbf{v}}{dt} = e_j \left(\delta \mathbf{E}_w + \frac{\mathbf{v}}{c} \times (B_0 \mathbf{i}_z + \delta \mathbf{B}_w) \right), \quad \frac{dz}{dt} = v_z. \quad (2)$$

Let $u_{\perp} = v_x + i v_y$ and $v_{\parallel} = v_z$; then we have

$$\frac{du_{\perp}}{dt} + i \Omega_0 u_{\perp} = i(v_{\parallel} - v_A) \Omega_k e^{-i\phi_k}, \quad (3)$$

$$\frac{dv_{\parallel}}{dt} = -\text{Im}(u_{\perp} \Omega_k e^{i\phi_k}), \quad \frac{dz}{dt} = v_{\parallel}, \quad (4)$$

where $\Omega_0 = e_j B_0 / (m_j c)$, $\Omega_k = e_j B_k / (m_j c)$, and the subscript j indicate physical quantities associated with ion species j . As a first-order approximation, we can assume $v_{\parallel} \approx v_{\parallel}(0)$, where $v_{\parallel}(0)$ is the initial parallel velocity of the particle. The approximation is valid when $\Omega_k / \Omega_0 = B_k / B_0$ is small enough and the frequency of the Alfvén wave is sufficiently low so $|\Omega_0| \gg |k(v_{\parallel} - v_A)|$. With the initial condition $u_{\perp} = u_{\perp}(0)$ and $z = z(0)$, the solution of Eq. (3) can be written as^{20,21}

$$u_{\perp} = u_{\perp}(0)e^{-i\Omega_0 t} - [v_A - v_{\parallel}(0)] \frac{\Omega_k}{\tilde{\Omega}_k} e^{-ik[v_A - v_{\parallel}(0)]t + ikz(0)} + [v_A - v_{\parallel}(0)] \frac{\Omega_k}{\tilde{\Omega}_k} e^{ikz(0)} e^{-i\Omega_0 t}, \tag{5}$$

where $\tilde{\Omega}_k = \Omega_0 - k[v_A - v_{\parallel}(0)] \approx \Omega_0$ and $z = z(0) + v_{\parallel}(0)t$. In a low beta plasma, $v_A - v_{\parallel}(0) \approx v_A$, then

$$u_{\perp} = u_{\perp}(0)e^{-i\Omega_0 t} - v_A \frac{B_k}{B_0} e^{-ik(v_A t - z)} + v_A \frac{B_k}{B_0} e^{ikz(0)} e^{-i\Omega_0 t}. \tag{6}$$

The transverse motion of the particle consists of three terms: the first term $u_{\perp}(0)e^{-i\Omega_0 t}$ describes the gyromotion of the particle in the background magnetic field; the second term $-v_A(B_k/B_0)e^{-ik(v_A t - z)}$ is the transverse motion due to the electric field of the Alfvén wave; and finally, $v_A(B_k/B_0)e^{ikz(0)}e^{-i\Omega_0 t}$ is the modification of the gyromotion due to the existence of the Alfvén wave.

Now we apply the above results to an ensemble of particles at a fixed position z . Initially, the average velocity of these particles is zero. The first term is irrespective of position z . The corresponding thermal speed of particles will be kept as their initial value, while the corresponding average transverse velocity is zero. From the second term, we can find that all particles at position z have the same velocity. This term describes the linear fluid velocity perturbations induced by an Alfvén wave and there is no thermal dispersion. The corresponding average transverse velocity is $-v_A(B_k/B_0)e^{-ik(v_A t - z)}$. Now let us consider the third term. If we assume the parallel velocity component of particles is constant, particles with initial position $z(0) = z - v_{\parallel}(0)t$ will arrive at position z after a finite time t . In a homogeneous plasma considered in this paper, whenever a particle arrives at position z , another particle with the same parallel velocity will leave position z simultaneously. Therefore, the particle velocity distribution at z in the parallel direction does not change. If we assume that particles initially have a Maxwellian velocity distribution function, the velocity distribution in the parallel direction will maintain Maxwellian. Therefore, the corresponding average transverse velocity is $(1/\sqrt{\pi}) \times (1/v_{th}) \int_{-\infty}^{\infty} v_A(B_k/B_0) e^{ik[z - v_{\parallel}(0)t]} e^{-i\Omega_0 t} e^{-[v_{\parallel}(0)/v_{th}]^2} dv_{\parallel}(0)$.

Summing up the three terms, the overall average transverse velocity at position z can be found as

$$U_{\perp} = -v_A \frac{B_k}{B_0} e^{-ik(v_A t - z)} + \frac{1}{v_{th}\sqrt{\pi}} \int_{-\infty}^{\infty} v_A \frac{B_k}{B_0} e^{ik[z - v_{\parallel}(0)t]} e^{-i\Omega_0 t} e^{-(v_{\parallel}(0)/v_{th})^2} dv_{\parallel}(0), \\ = -v_A \frac{B_k}{B_0} e^{-ik(v_A t - z)} + A_k v_A \frac{B_k}{B_0} e^{ikz} e^{-i\Omega_0 t}, \tag{7}$$

where $A_k = (1/\sqrt{\pi}) \int_{-\infty}^{\infty} \cos(kv_{th}tx) e^{-x^2} dx = e^{-k^2 v_{th}^2 t^2 / 4}$, $v_{th} = (2k_B T_{j0}/m_j)^{1/2}$ (where T_{j0} is the initial temperature of species j) is the particles' initial thermal speed. This average transverse velocity illustrates the pickup of particles by the Alfvén wave. The perpendicular temperature is

$$T_{\perp j} = \frac{m_j}{2k_B v_{th} \sqrt{\pi}} \int_{-\infty}^{\infty} |u_{\perp} - U_{\perp}|^2 e^{-(v_{\parallel}(0)/v_{th})^2} dv_{\parallel}(0), \\ = T_{j0} + \frac{m_j}{2k_B} \left[\frac{v_A^2 B_k^2}{B_0^2} (1 - A_k^2) \right] = T_{j0} \left[1 + \frac{m_j B_k^2}{m_p \beta_j B_0^2} (1 - A_k^2) \right], \tag{8}$$

where $\beta_j = 8\pi n_e k_B T_{j0} / B_0^2$ is the plasma beta of species j . Due to the initial random velocities of particles in the parallel direction, particles at position z will have different velocities after time t . According to the third term on the right-hand side of Eq. (6), $v_A(B_k/B_0)e^{ik[z - v_{\parallel}(0)t]}e^{-i\Omega_0 t}$, phase differences between particles will develop. As a result, a velocity dispersion will be produced and ions will be heated in the perpendicular direction. The heating process saturates when the phase difference due to particles with characteristic speed v_{th} reaches π and the time scale can be approximated as $\sim \pi/(kv_{th})$.

Substituting Eq. (6) into Eq. (4), and adopting $|\Omega_0| \gg |k[v_{\parallel}(0) - v_A]|$, $|v_{\parallel}(0)| \ll v_A$, $|u_{\perp}(0)| \ll v_A$, we can obtain

$$v_{\parallel} = v_{\parallel}(0) + v_A \frac{B_k}{B_0} \{1 - \cos[\Omega_0 t - kv_A t - kv_{\parallel}(0)t]\}, \tag{9}$$

It is obvious that v_{\parallel} can be considered as $v_{\parallel}(0)$ if the amplitude of the Alfvén wave is sufficiently small. Similar to the transverse motions, the average parallel velocity at z can be described as

$$U_{\parallel} = \frac{1}{v_{th}\sqrt{\pi}} \int_{-\infty}^{\infty} v_A \frac{B_k}{B_0} \{1 - \cos[\Omega_0 t - kv_A t - kv_{\parallel}(0)t]\} e^{-[v_{\parallel}(0)/v_{th}]^2} dv_{\parallel}(0), \\ = v_A \frac{B_k}{B_0} [1 - A_k \cos(\Omega_0 t - kv_A t)], \tag{10}$$

and the parallel temperature is

$$T_{\parallel j} = \frac{m_j}{k_B v_{th} \sqrt{\pi}} \int_{-\infty}^{\infty} |v_{\parallel} - U_{\parallel}|^2 e^{-[v_{\parallel}(0)/v_{th}]^2} dv_{\parallel}(0), \\ = T_{j0} \left\{ 1 + \frac{2m_j B_k^4}{m_p \beta_j B_0^4} [(C_k - A_k^2) \cos^2(\Omega_0 t - kv_A t) + D_k \sin^2(\Omega_0 t - kv_A t)] \right\}, \tag{11}$$

where $C_k = (1/\sqrt{\pi}) \int_{-\infty}^{\infty} \cos^2(kv_{th}tx) e^{-x^2} dx = 1/2 + 1/2 e^{-k^2 v_{th}^2 t^2}$ and $D_k = (1/\sqrt{\pi}) \int_{-\infty}^{\infty} \sin^2(kv_{th}tx) e^{-x^2} dx = 1/2 - 1/2 e^{-k^2 v_{th}^2 t^2}$, when $t \rightarrow \infty$, $A_k \rightarrow 0$, $C_k \rightarrow 0.5$, and $D_k \rightarrow 0.5$. Therefore, the asymptotic values of the average parallel and transverse velocities, the parallel and perpendicular temperatures, are

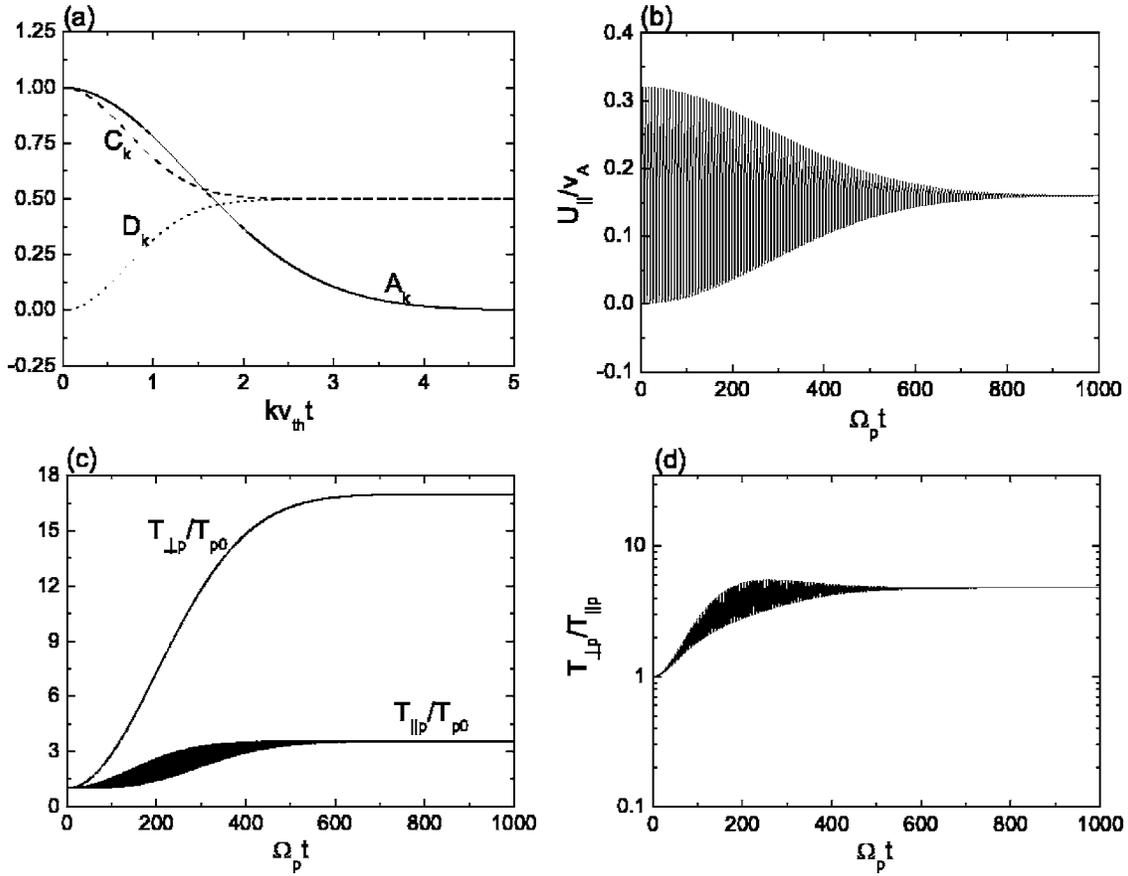


FIG. 1. The time evolution of (a) A_k , C_k , and D_k ; (b) the average parallel velocity $U_{||}/v_A$; (c) the perpendicular temperature $T_{\perp p}/T_{p0}$ and parallel temperature $T_{|| p}/T_{p0}$; and (d) the temperature anisotropy $T_{\perp p}/T_{|| p}$. The results are based on an analytical model.

$$U_{||} = v_A \frac{B_k^2}{B_0^2}, \quad U_{\perp} = -v_A \frac{B_k}{B_0} e^{-ik(v_A t - z)},$$

$$T_{|| j} = T_{j0} \left(1 + \frac{m_j B_k^4}{m_p \beta_j B_0^4} \right), \quad T_{\perp j} = T_{j0} \left(1 + \frac{m_j B_k^2}{m_p \beta_j B_0^2} \right), \quad (12)$$

Therefore, the temperature anisotropy is

$$\frac{T_{\perp j}}{T_{|| j}} = \frac{1 + \frac{m_j B_k^2}{m_p \beta_j B_0^2}}{1 + \frac{m_j B_k^4}{m_p \beta_j B_0^4}}. \quad (13)$$

In the above analysis, Eq. (8), (10) and (11) describe the time evolution of the perpendicular temperature, and the average parallel velocity and parallel temperature, which depend on A_k , C_k , and D_k . As an example, Fig. 1 shows the time evolution of (a) A_k , C_k , and D_k ; (b) the average parallel velocity $U_{||}/v_A$; (c) the perpendicular temperature $T_{\perp p}/T_{p0}$ and parallel temperature $T_{|| p}/T_{p0}$; and (d) the temperature anisotropy $T_{\perp p}/T_{|| p}$. Here protons are chosen as the test particles, and relevant parameters are $kv_A/\Omega_p = 0.05$, $\beta_p = 0.01$, $B_k^2/B_0^2 = 0.16$. For A_k , C_k , and D_k , they tend to be 0, 0.5, and 0.5 when $kv_{th}t > \pi$. From Figs. 1(b)–1(d) we can find that, when $\Omega_p t \geq 628$, an asymptotic state is attained. Obviously, protons are significantly heated, and the heating is more efficient in the perpendicular than in the parallel direction. Pro-

tons obtain a bulk flow speed along the background magnetic field. At the asymptotic stage, $U_{||}/v_A$, $T_{\perp p}/T_{p0}$, $T_{|| p}/T_{p0}$, and $T_{\perp p}/T_{|| p}$ are about 0.16, 17.0, 3.6, and 4.8, respectively.

To demonstrate the validity of the above analysis, test particle calculations using the same parameters in Fig. 1 are performed. In the test particle calculations, the motions of particles are described by Eq. (2), and the approximation that the parallel velocities of the particles are constant is relaxed. The equations are solved with the Boris algorithm,²² and the time step is $\Delta t = 0.02\Omega_0^{-1}$. Initially, particles with Maxwellian velocity distribution are evenly distributed in a region with length $1024v_A\Omega_0^{-1}$, and their average velocity (both parallel and transverse velocities) is zero. The total number of particles is 153 600. In the test particle calculations, the average parallel velocity, and the parallel and perpendicular temperatures were obtained by using the following procedure: We firstly divide the domain into 512 grid cells with size $2v_A\Omega_0^{-1}$, and then calculate $U_{||} = \langle v_z \rangle$, $T_{||} = (m_j/k_B) \langle (v_z - \langle v_z \rangle)^2 \rangle$, and $T_{\perp} = m_j/2k_B \langle (v_x - \langle v_x \rangle)^2 + (v_y - \langle v_y \rangle)^2 \rangle$ in every grid cell (the brackets $\langle \cdot \rangle$ denote an average over a grid cell). Finally, these quantities are averaged over all grid cells. The procedure eliminates the possible contribution of the perturbed wave velocity to the temperatures, as the analytical model has done. Figure 2 shows the time evolution of (a) the average parallel velocity $U_{||}/v_A$, (b) the perpendicular temperature $T_{\perp p}/T_{p0}$ and parallel temperature $T_{|| p}/T_{p0}$, and (c) the temperature anisotropy $T_{\perp p}/T_{|| p}$. The “shaded areas” are

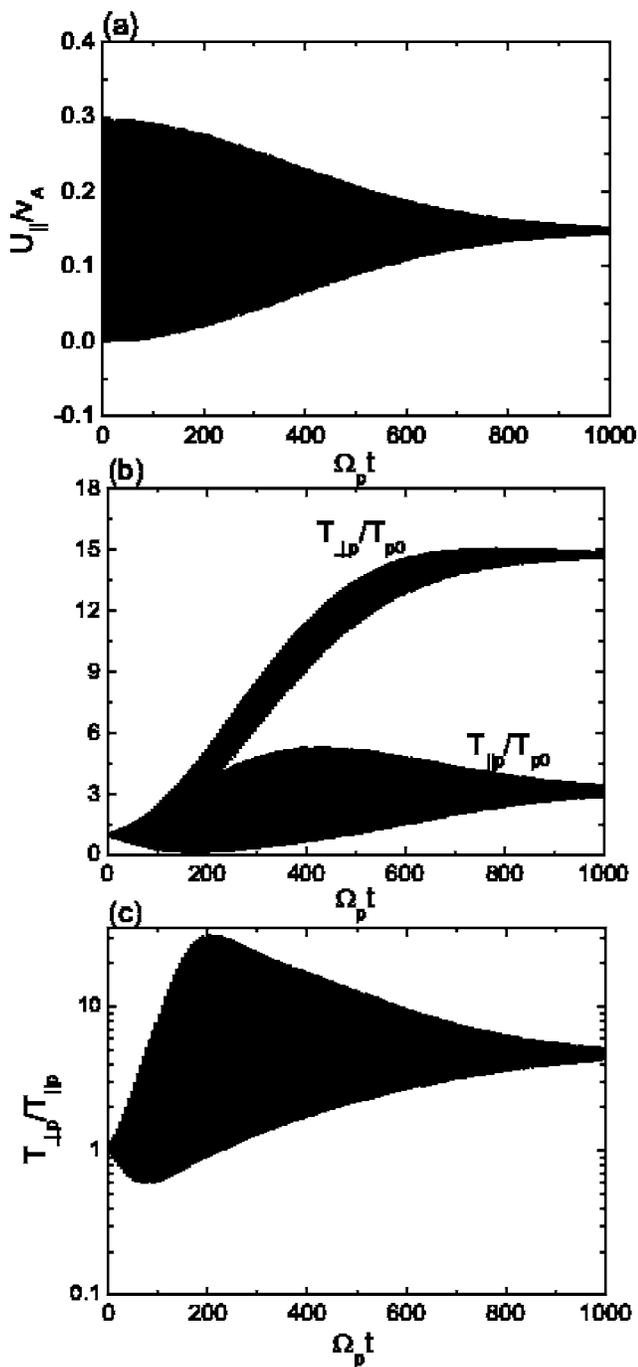


FIG. 2. The time evolution of (a) the average parallel velocity U_{\parallel}/v_A , (b) the perpendicular temperature $T_{\perp p}/T_{p0}$ and parallel temperature $T_{\parallel p}/T_{p0}$, and (c) the temperature anisotropy $T_{\perp p}/T_{\parallel p}$. The results are based on test particle calculations.

due to ion gyrations, and at this stage the randomization (or heating) is not fully developed and the temperatures oscillate with time. The results of the test particle calculations are strongly consistent with our analytical predictions. At about $\Omega_p t = 628$, the heating process saturates and an asymptotic stage is reached. At the asymptotic stage, the average parallel velocity $U_{\parallel}/v_A \approx 0.15$, the perpendicular temperature $T_{\perp p}/T_{p0} \approx 15$, and the parallel temperature $T_{\parallel p}/T_{p0} \approx 3.2$ with the temperature anisotropy $T_{\perp p}/T_{\parallel p} \approx 4.7$. The oscillation of temperatures almost disappears at this stage. The pro-

ton heating and a bulk acceleration by the Alfvén wave can also be found in Fig. 3, which shows the scatter plots of protons between 300 and $540v_A\Omega_p^{-1}$ at different times, $\Omega_p t = 0, 40$, and 800. Figures 3(a) and 3(b) depict the velocity component in the parallel and y directions, respectively. At $t=0$, protons satisfy Maxwellian distribution with the thermal speed $v_{th} = 0.1v_A$. At $\Omega_p t = 40$, protons are trapped by the electric field of the Alfvén wave and obtain average velocities in both the parallel and perpendicular directions. However, there is no obvious heating at this time. At $\Omega_p t = 800$, the heating of protons is obvious. The heating is more efficient in the perpendicular direction, and protons have an average parallel velocity of about $0.15v_A$.

III. CONCLUSIONS AND DISCUSSIONS

In summary, we demonstrated, both analytically and numerically, the heating of ions in a low beta plasma by a low-frequency Alfvén wave of finite amplitude. This is contrary to the linear theory, according to which ions can only be heated through resonant interactions with Alfvén waves. In our model, the frequency of the Alfvén wave is much lower than the ion cyclotron frequency so the cyclotron resonant condition is not met. The results show that ions can be significantly heated and accelerated by a finite-amplitude Alfvén wave through nonresonant interactions. The characteristic time scale over which ions are significantly heated and attain an asymptotic stage is $\sim \pi/(kv_{th}) = (\pi/\omega)\sqrt{m_j/(m_p\beta_j)}$. The heating mechanism preferentially energizes heavier ions, and ions can obtain a large temperature anisotropy with a much higher perpendicular than parallel temperature. These features are consistent with remote sensing observations in the lower solar corona above coronal holes,^{23,24} and our results may provide an interesting new mechanism for the heating of the solar corona by Alfvén waves. In Fig. 4, we describe the contour plots of (a) the proton temperature anisotropy $T_{\perp p}/T_{\parallel p}$ for different β_p and B_k^2/B_0^2 , and (b) the temperature anisotropy of O^{5+} $T_{\perp O}/T_{\parallel O}$ for different β_O and B_k^2/B_0^2 , based on Eq. (13). In general, ions obtain a large temperature anisotropy when the plasma beta is small, and O^{5+} have a larger temperature anisotropy than protons. For typical plasma beta ~ 0.01 in the lower corona, we find that, if B_k^2/B_0^2 is assumed to be around 0.1 the proton temperature anisotropy $T_{\perp p}/T_{\parallel p}$ and the temperature anisotropy of O^{5+} $T_{\perp O}/T_{\parallel O}$ are about 5–6 and 10–18, respectively. The results are consistent with the observations in the lower corona.²³ Meanwhile, in the corona, for an Alfvén wave with a period of 2 min and a plasma beta ~ 0.01 , the heating time for protons and O^{5+} ions due to the wave is 10 and 40 min, respectively. This is comparable to the solar wind expansion time.

The physical mechanism of this heating may be described as follows: Initially, ions, whose average velocity is zero, are not in phase with the perturbed fluid velocity induced by an Alfvén wave. Therefore, the ions are rapidly picked up by the Alfvén wave. This occurs mainly in the transverse direction. Ions firstly obtain a larger average transverse velocity [which is described by Eq. (7)] than what the wave field allows [the first term on right-hand side of Eq. (7), which corresponds to the linear fluid velocity perturbations

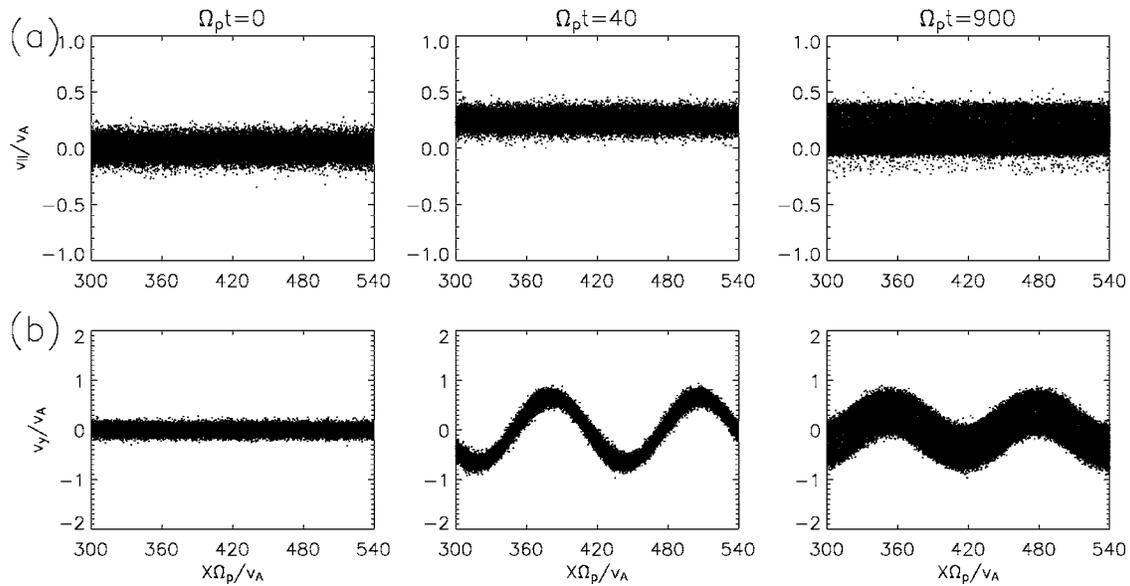


FIG. 3. Scatter plots of protons between 300 and $540v_A\Omega_p^{-1}$ at different times, $\Omega_p t=0, 40$, and 800 . Panels (a) and (b) display the scatter plot of the velocity component in the parallel and y directions, respectively.

induced by the Alfvén wave], and then that part of the energy is randomized into thermal energy. The randomization (heating) is roughly complete when ions (at the same location) with characteristic speed v_{th} produce a phase shift of π . Therefore the time scale of the heating can be approximated as $\sim \pi/(kv_{th})$. The phase difference is due to the parallel thermal motions of ions. However, a particle’s kinetic energy is conserved in the frame of the Alfvén phase speed. Hence

particles are accelerated in the parallel direction and the average parallel speed may reach a substantial fraction of the Alfvén speed. On the other hand, if particles initially are already riding on an Alfvén wave [their initial average transverse velocity can be described as $U_{\perp}(0)=-v_A(B_k/B_0)e^{ikz(0)}$, or their initial average parallel velocity is v_A], they are already settled in the wave field and cannot be heated. Our test particle calculations have shown this is indeed the case, which also validates our code. In our model, eventually particles will obtain a bulk parallel speed; this will also change the dispersion relation used in this paper. To consider this effect, self-consistent particle simulations are necessary. Our test particle calculations also ignore the influences of particles on the wave. To investigate the dissipation of the Alfvén wave also requires self-consistent particle simulations. This will be conducted in a future work.

This study is different from the work of Chen *et al.*,¹⁴ who studied the stochastic heating of ions by an Alfvén wave with a finite wave vector at a fraction of the cyclotron frequency. In our model, we have demonstrated the pickup and the ion heating by a parallel propagating Alfvén wave. The pickup of ions by a spectrum of Alfvén waves has been investigated by Wang *et al.*,²⁵ who calculated the amount of kinetic energy that the particles obtained from the waves. Consistent with our results, the pickup process is very fast and lasts only several ion cyclotron periods. However, we further discover a new heating process after the initial pickup, and the time scale of the heating process is $\sim \pi/(kv_{th})$. Ions are heated no matter what frequency or polarization (left or right handed) the Alfvén wave has. The frequency of the Alfvén wave only influences the time scale, over which the ion heating process saturates. Both the parallel and perpendicular temperatures at this (saturation) stage are independent of the frequency of the Alfvén wave. In this paper, we focused on the ion heating by a low-frequency

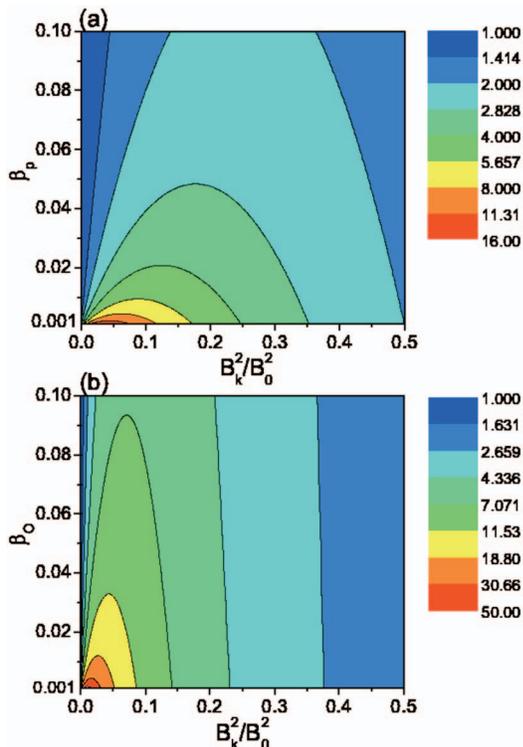


FIG. 4. (Color) Contour plots of (a) the proton temperature anisotropy $T_{\perp p}/T_{\parallel p}$ at different β_p and B_k^2/B_0^2 ; (b) the temperature anisotropy of O^{5+} $T_{\perp O}/T_{\parallel O}$ for different β_O and B_k^2/B_0^2 .

Alfvén wave; however, heating mechanisms like this may also apply to other types of waves.

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