# On the Irreducible Representations of the Specializations in Characteristics 2 and 3 of the Generic Hecke Algebra of type $F_4$*

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**Abstract**

A complete determination of the irreducible modules of specialized Hecke algebras of type $F_4$, with respect to specializations with equal parameters, has been obtained by M. Geck and K. Lux (1991, *Manuscripta Math.* **70**, 285-306) for all characteristics. A similar determination for specializations with $v = u^2$ and $v = u^4$ has been obtained by K. Bremke (1994, *Manuscripta Math.* **83**, 331-346). In an earlier paper (1999, *J.Algebra* **218**, 654-671), the authors determined the irreducible modules for all remaining specializations other than those into fields of characteristic 2 or 3, obtaining *en route* decompositions of the generic irreducible modules under such specializations. In this paper, the corresponding results for characteristic 2 or 3 are obtained. Again, it is found that the decomposition matrices may be expressed in lower uni-triangular form in all these cases and that the splitting fields are those generated by the images of the parameters.

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# 1 Introduction

In this paper, we continue the study of the irreducible representations of the specializations of the Hecke algebra of type $F_4$. In our previous paper [8], we determined all such representations where the characteristic of the underlying field was different from 2 and 3, apart from those already determined by Geck and Lux in [6] and Bremke in [1]. The work of Geck, Lux, and Bremke also deals with fields of characteristic 2 and 3. We now determine the irreducible representations of all the remaining specializations.

We continue with the notation of [8]. Let $A$ be the ring $\mathbf{Z}[u, u^{-1}, v, v^{-1}]$, where $u$ and $v$ are algebraically independent indeterminates, and let $H$ be the generic Hecke algebra over $A$ of type $F_4$. $H = H(u, v)$ is defined to be the associative $A$-algebra with generators $T_i$, $i = 1, 2, 3, 4$, and defining relations $T_i^2 = u1 + (u-1)T_i$ for $i = 1, 2$, $T_i^2 = v1 + (v-1)T_i$ for $i = 3, 4$, $T_iT_j = T_jT_i$ if $|i-j| > 1$, $T_iT_{i+1}T_i = T_{i+1}T_iT_{i+1}$ for $i = 1, 3$, and $T_2T_3T_2T_3 = T_3T_2T_3T_2$.

With each field $F$ and each homomorphism $f : A \to F$, there is an associated algebra $H_F$ known as the specialized algebra with respect to $f$ obtained by specializing $u$ and $v$ to $f(u)$ and $f(v)$, respectively, in the field $F$. The specialized algebra with respect to the homomorphism $f$ is the $F$-algebra defined by a similar set of generators and relations which are obtained from those above by replacing $u$ and $v$ throughout by $\bar{u} = f(u)$ and $\bar{v} = f(v)$, respectively.

In [5, Theorem 3.3], Geck has shown that the decomposition matrix of any specialization of a generic one-parameter Hecke algebra can be arranged into a lower uni-triangular shape if the characteristic is either 0 or a good prime and concludes that all irreducible representations of the specialization can be realized over the field generated by the image of the parameter. See also [7, Theorem 4.2] for an alternative derivation of this result. For type $F_4$, all primes other than 2 and 3 are good.

In [9], complete decompositions have been found of all generic irreducible representations of degrees $\leq 4$ under all specializations other than $\bar{u} = \bar{v} = -1$.

Combining the decomposition results of [1, 6, 9, 8] with the results below, we get the following theorem.

**Theorem** *Let $L$ be any algebraically closed field and let $L_0$ be the field of fractions of the image of $A$ under the specialization $f : A \to L$. Then*

(a) *There is a well-defined decomposition map from the Grothendieck group of $H_K$ to the Grothendieck group of $H_L$.*

(b) *The irreducible $H_K$-modules and the irreducible $H_L$-modules can be arranged so that the corresponding decomposition matrix has a lower uni-triangular shape. In particular, its elementary divisors are all 1.*

(c) *The field $L_0$ is a splitting field for $H_{L_0}$.*

This theorem has already been established for the cases considered in [1, 6] in these papers. For all remaining cases with different parameters, but characteristic different from 2 and 3, the theorem was established in [8]. For the cases with different parameters, but characteristic equal to 2 or 3, we proceed as in [8] to obtain the absolutely irreducible representations of the specializations and the decompositions of the specialized generic irreducible representations, from which parts (a) and (b) follow. Morever, since we are dealing with finite characteristic, the arguments of [3, Theorem (74.9)] show that the Schur indices of all these absolutely irreducible representations are 1, from which part (c) follows.

As in [8], we find that the specializations giving rise to non-semisimple algebras $H_F(\bar{u}, \bar{v})$ are those for which one or more of the following polynomials vanishes under specialization.

$$\begin{aligned} &2,\ 3,\ u+1,\ v+1,\ u+v,\ u^2+u+1,\ u^2-u+1,\ v^2+v+1, \\ &v^2-v+1,\ u^2+v^2,\ u+v^2,\ u^2+v,\ uv+1,\ u^2v+1,\ uv^2+1, \\ &u^2v^2+1,\ u^2v^2-uv+1,\ \text{and } u^2-uv+v^2. \end{aligned} \tag{1}$$

We also recall that, for a given choice of $\bar{u}$ and $\bar{v}$, the specializations $(\bar{u}, \bar{v})$, $(\bar{u}^{-1}, \bar{v})$, $(\bar{u}, \bar{v}^{-1})$, $(\bar{u}^{-1}, \bar{v}^{-1})$, $(\bar{v}, \bar{u})$, $(\bar{v}^{-1}, \bar{u})$, $(\bar{v}, \bar{u}^{-1})$, and $(\bar{v}^{-1}, \bar{u}^{-1})$ are equivalent to one another in the sense that the resulting algebras are all isomorphic to one another. So, we need only deal with one such parameter pair from each equivalence class.

Moreover, there is no need to consider cases of the form $\bar{v} = \bar{u}$, already dealt with in [6], or cases of the forms $\bar{v} = \bar{u}^2$ and $\bar{v} = \bar{u}^4$, already dealt with in [1].

Of the remaining cases to be considered, five are in characteristic 2 and eight are in characteristic 3.

In characteristic 2, the cases are

Case 2.1: $\bar{u} = u,\ \bar{v} = v.$ Case 2.2: $\bar{u} = u,\ \bar{v} = 1.$ Case 2.3: $\bar{u} = u,\ \bar{v} = \varepsilon.$
Case 2.4: $\bar{u} = u,\ \bar{v} = \varepsilon u.$ Case 2.5: $\bar{u} = \varepsilon,\ \bar{v} = 1.$

where $\omega$ is a primitive cube root of 1. We use $\varepsilon = \omega^2$ rather than $\omega$ so that the cases arising may be more easily related to the corresponding cases in [8]. Thus, Case 2.4 is a specialization of Case 7 in [8], while Case 2.5 is a specialization of Cases 8 and 12. Case 2.5 is also related to Case 13.

In characteristic 3, the cases are

Case 3.1: $\bar{u} = u,\ \bar{v} = v.$ Case 3.2: $\bar{u} = u,\ \bar{v} = -1.$ Case 3.3: $\bar{u} = u,\ \bar{v} = 1.$
Case 3.4: $\bar{u} = u,\ \bar{v} = -u.$ Case 3.5: $\bar{u} = u,\ \bar{v} = -u^2.$ Case 3.6: $\bar{u} = u,\ \bar{v} = iu.$
Case 3.7: $\bar{u} = \delta,\ \bar{v} = \delta^3.$ Case 3.8: $\bar{u} = \zeta,\ \bar{v} = \zeta^3.$

where $i$, $\delta$, and $\zeta$ are primitive fourth, eighth and tenth roots of 1 and $\delta^2 = i$.

From [9], we have complete decompositions of all generic irreducible representations of degrees $\leq 4$ under all specializations other than $\bar{u} = \bar{v} = -1$. For the remaining generic irreducible representations, we used the GAP3 Meat-Axe package (see [11, 12]) to decompose all thirteen specializations, by initially specializing further the cases with an indeterminate parameter $u$ into a field with $p^4$ elements, where $p$ is the characteristic. It was straightforward to reconstruct a decomposition into irreducible representations of each of the original specializations, in which the indeterminate $u$ was not specialized further. Moreover, we found that the irreducible components could be realized over the field $L_0$ generated by the images, $\bar{u}$ and $\bar{v}$, of the parameters. Indeed, with two exceptions, all the irreducible representations which occurred were equivalent to specializations of some of the 47 representations described in [8]. For explicit realizations of these representations, see [9, 10, 8].

We continue with the notation of [8], referring to the representations $M_1, \ldots, M_{47}$, of which the first 25 correspond to the 25 irreducible representations of the generic Hecke algebra, in the order used by Kondo (see [2, p.413]) and in GAP3 (see [12]).

The remaining two representations were constructed from two of the 47 using the constructions described in [8, Sect. 3]. We give the details of the constructions in Case 3.7 below and we will refer to these representations as $M_{48}$ and $M_{49}$.

We used Norton's Irreducibility Criterion (see [11, 6]) to check for absolute irreducibility, using the same procedure as described in [8]. In many cases, an appropriate matrix $B$ could be found by specializing one found in our previous work. For these calculations, we used GAP4 (see [4]) on account of the improved polynomial arithmetic.

We remind the reader that we use a compact notation to describe composition factors. For example, $M = X : Y : Z$ will denote the fact that the module $M$ has a series of submodules $0 \subseteq V_1 \subseteq V_2 \subseteq V_3 = M$ with $V_1 \cong X$, $V_2/V_1 \cong Y$ and $V_3/V_2 \cong Z$. If $Y$ is also a direct sum $U + V$, we may write $M = X : U + V : Z$.

# 2 Decomposition Details

In the case of those generic irreducible representations which decompose non-trivially, we record the decomposition giving, in each case, vectors generating the submodules which arise. As in [8], we need only record this information for one of each pair of associated representations. Recall that the associate $\rho'$ of a representation $\rho$ of $H_F$ is defined by $\rho'(T_i) = -\bar{u}\rho(T_i^{-1})$ for $i = 1, 2$ and $\rho'(T_i) = -\bar{v}\rho(T_i^{-1})$ for $i = 3, 4$. The associate pairs among $M_1, \ldots, M_{49}$ are: $M_1$, $M_4$; $M_{10}$, $M_{13}$; $M_{17}$, $M_{20}$; $M_n$, $M_{n+1}$ for $n \in \{2, 5, 7, 11, 18, 21, 23\}$, and $M_{2n}$, $M_{2n+1}$ for $13 \leq n \leq 24$. The remaining five modules are self-associates.

We record also any isomorphisms between generic irreducible representations which remain irreducible under the specialization being considered. Thus, in each case, the modules among $M_1, \ldots, M_{25}$ which are not mentioned remain irreducible, inequivalent to one another and inequivalent to all other irreducibles mentioned explicitly in that case. Note, in particular, that $M_{25}$ decomposes in all characteristic 2 cases and $M_{16}$ decomposes in all characteristic 3 cases.

*Characteristic 2.*

*Case 2.1*: $\bar{u} = u$, $\bar{v} = v$.

We have the decomposition $M_{25} = M_9 : M_{16}$.

The proper submodule of $M_{25}$ is generated by $[1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0]$.

*Case 2.2*: $\bar{u} = u$, $\bar{v} = 1$.

We get the isomorphisms $M_1 \cong M_2$, $M_3 \cong M_4$, $M_7 \cong M_8$, $M_{10} \cong M_{11}$, $M_{12} \cong M_{13}$, $M_{14} \cong M_{15}$, $M_{17} \cong M_{18}$, and $M_{19} \cong M_{20}$, and the decompositions $M_{23} = M_{14} : M_7$,

$M_{24} = M_{14} : M_7$, and $M_{25} = M_9 : M_{16}$.

The proper submodules of $M_{23}$ amd $M_{25}$ are generated by $[1,1,1,0,0,0,0,0]$ and $[1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0]$, respectively.

*Case 2.3*: $\bar{u} = u$, $\bar{v} = \varepsilon = \omega^2$.

We get the decompositions $M_5 = M_2 : M_1$, $M_6 = M_4 : M_3$, $M_9 = M_8 : M_7$, $M_{16} = M_{26} : M_{27}$, $M_{21} = M_{17} : M_{18}$, $M_{22} = M_{20} : M_{19}$, $M_{23} = M_{27} : M_7$, $M_{24} = M_{26} : M_8$, and $M_{25} = M_7 : M_8 : M_{27} : M_{26}$.

The proper submodules of $M_5$, $M_9$, $M_{16}$, $M_{21}$, and $M_{23}$ are generated by $[1,\varepsilon^2]$, $[1,0,\varepsilon^2,0]$, $[1,0,0,1+u,\varepsilon^2+\varepsilon u^2,\varepsilon+\varepsilon^2 u,\varepsilon^2+u^2,0,0,\varepsilon+\varepsilon^2 u+\varepsilon^2 u^3,\varepsilon u^2,\varepsilon u+\varepsilon u^2]$, $[0,u\varepsilon+u,0,u\varepsilon,\varepsilon,0,0,1]$, and $[1,1,1,0,0,0,0,0]$, respectively. Those of $M_{25}$ are generated by $[1,0,0,0,0,0,0,0,\varepsilon,0,0,0,0,0,0,0]$, $[0,0,0,0,0,0,0,0,1,0,0,0,0,0,0,0]$, and $[0,0,\varepsilon^2+\varepsilon u,0,0,u,u,\varepsilon^2 u+\varepsilon^2 u^3,\varepsilon u,0,0,0,0,0,0,0]$.

*Case 2.4*: $\bar{u} = u$, $\bar{v} = \varepsilon u$.

We get the decompositions $M_{14} = M_{33} : M_{32}$, $M_{18} = M_{32} : M_2$, $M_{19} = M_{33} : M_3$, and $M_{25} = M_9 : M_{16}$.

The proper submodules of $M_{14}$, $M_{18}$, and $M_{25}$ are generated by $[0,u,1,u,1,0]$, $[1,1,0,0]$, and $[1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0]$, respectively.

*Case 2.5*: $\bar{u} = \varepsilon$, $\bar{v} = 1$.

We get the isomorphisms $M_1 \cong M_2$, $M_3 \cong M_4$, $M_{10} \cong M_{11}$, and $M_{12} \cong M_{13}$, and the decompositions $M_7 = M_4 : M_1$, $M_8 = M_4 : M_1$, $M_9 = M_6 : M_5$, $M_{14} = M_{42} : M_{43}$, $M_{15} = M_{43} : M_{42}$, $M_{16} = M_{46} : M_{47}$, $M_{17} = M_1 : M_{43}$, $M_{18} = M_{43} : M_2$, $M_{19} = M_{42} : M_3$, $M_{20} = M_4 : M_{42}$, $M_{21} = M_{47} : M_5$, $M_{22} = M_{46} : M_6$, $M_{23} = M_{43} : M_1 : M_{42} : M_4$, $M_{24} = M_{42} : M_4 : M_{43} : M_1$, and $M_{25} = M_5 : M_{47} : M_6 : M_{46}$.

The proper submodules of $M_7$, $M_9$, $M_{14}$, $M_{15}$, $M_{16}$, $M_{17}$, $M_{18}$, and $M_{21}$ are generated by $[1,\omega^2]$, $[1,\omega^2,0,0]$, $[1,0,0,0,\omega,1]$, $[0,0,\omega,1,1,1]$, $[1,0,0,\omega^2,0,0,0,0,0,0,0,0]$, $[\omega,1,0,1]$, $[1,1,0,0]$, and $[0,0,0,0,0,0,1,1]$, respectively. Those of $M_{23}$ are generated by $[1,1,0,0,1,0,\omega^2,\omega^2]$, $[0,1,0,0,0,\omega,\omega,\omega]$, and $[0,0,0,0,0,0,1,1]$. Those of $M_{25}$ are generated by $[1,\omega,0,0,0,0,0,0,0,0,0,0,0,0,0,0]$, $[0,0,1,0,0,\omega^2,\omega^2,\omega,0,0,0,0,0,0,0,0]$, and $[0,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0]$.

*Characteristic 3.*

*Case 3.1*: $\bar{u} = u$, $\bar{v} = v$.

We have the decomposition $M_{16} = M_{15} : M_{14}$. The proper submodule of $M_{16}$ is generated by $[1,0,0,0,0,0,0,0,0,0,0,0]$.

*Case 3.2*: $\bar{u} = u$, $\bar{v} = -1$.

We get the isomorphisms $M_1 \cong M_2$, $M_3 \cong M_4$, $M_7 \cong M_8$, $M_{10} \cong M_{11}$, $M_{12} \cong M_{13}$, $M_{14} \cong M_{15}$, $M_{17} \cong M_{18}$, and $M_{19} \cong M_{20}$, and the decompositions $M_{16} = M_{14} : M_{14}$, $M_{23} = M_{14} : M_7$, and $M_{24} = M_{14} : M_7$.

The proper submodules of $M_{16}$ and $M_{23}$ are generated by $[1,0,0,0,0,0,0,0,0,0,0,0]$ and $[0,0,0,0,0,0,1,1]$, respectively.

*Case 3.3*: $\bar{u} = u$, $\bar{v} = 1$.

We get the decompositions $M_5 = M_2 : M_1$, $M_6 = M_4 : M_3$, $M_9 = M_8 : M_7$, $M_{16} = M_{15} : M_{14}$, $M_{21} = M_{17} : M_{18}$, $M_{22} = M_{20} : M_{19}$, and $M_{25} = M_{23} : M_{24}$.

The proper submodules of $M_5$, $M_9$, $M_{16}$, $M_{21}$, and $M_{25}$ are generated by $[1, -1]$, $[1, 0, -1, 0]$, $[1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0]$, $[0, 0, 0, 1, 1, -1, -1, 0]$, and $[0, 0, 0, 0, 0, -u, u, u, 0, 0, 0, 0, 0, 1, 0, 0]$, respectively.

*Case 3.4*: $\bar{u} = u$, $\bar{v} = -u$.

We get the decompositions $M_{11} = M_9 : M_2 : M_{32} : M_2$, $M_{12} = M_9 : M_3 : M_{33} : M_3$, $M_{14} = M_{33} : M_{32}$, $M_{15} = M_2 + M_3 : M_9$, $M_{16} = M_2 : M_{32} : M_9 : M_3 : M_{33}$, $M_{17} = M_5 : M_7$, $M_{18} = M_{32} : M_2$, $M_{19} = M_{33} : M_3$, and $M_{20} = M_6 : M_8$.

The proper submodules of $M_{14}$, $M_{17}$, and $M_{18}$ are generated by $[0, -u, 1, -u, 1, 0]$, $[0, 0, 0, 1]$, and $[1, 1, 0, 0]$, respectively. Those of $M_{11}$ are generated by $[0, 0, 0, 0, 0, u, 1, u, 1]$, $[-u^2, -u, 0, 0, 1, 0, 0, 1, 0]$ and $[1, 1, 0, 0, 0, 0, 0, 0, 0]$. Those of $M_{15}$ are generated by $[-u, 0, 1, 0, 0, 0]$ and $[0, u^2, 0, 1, -u, -u]$. Those of $M_{16}$ are generated by $[0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0]$, $[0, 0, 0, 0, 0, 0, 0, 0, 1, 0, -1, -u]$, $[1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0]$, and $[0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0]$.

*Case 3.5*: $\bar{u} = u$, $\bar{v} = -u^2$.

We get the decompositions $M_{10} = M_5 : M_{29}$, $M_{13} = M_6 : M_{28}$, $M_{16} = M_{15} : M_{14}$, $M_{23} = M_{29} : M_3$, and $M_{24} = M_{28} : M_2$.

The proper submodules of $M_{10}$, $M_{16}$ and $M_{23}$ are generated by $[u^2, -u, 1, 0, -u^2, 0, u, 0, -1]$, $[1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0]$, and $[0, 0, 0, 0, 0, 0, 1, 1]$, respectively.

*Case 3.6*: $\bar{u} = u$, $\bar{v} = iu$.

We get the decompositions $M_{11} = M_{31} : M_2$, $M_{12} = M_{30} : M_3$, $M_{16} = M_{15} : M_{14}$, and $M_{25} = M_{30} : M_{31}$.

The proper submodules of $M_{11}$, $M_{16}$ and $M_{25}$ are generated by $[1, 1, 0, 0, 0, 0, 0, 0, 0]$, $[1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0]$ and $[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0]$, respectively.

*Case 3.7*: $\bar{u} = \delta$, $\bar{v} = \delta^3$.

We get the decompositions $M_{10} = M_9 : M_1 : M_{48} : M_1$, $M_{11} = M_{31} : M_2$, $M_{12} = M_{30} : M_3$, $M_{13} = M_9 : M_4 : M_{49} : M_4$, $M_{14} = M_1 + M_4 : M_9$, $M_{15} = M_{48} : M_{49}$, $M_{16} = M_{49} : M_4 : M_9 : M_{48} : M_1$, $M_{17} = M_1 : M_{48}$, $M_{18} = M_8 : M_5$, $M_{19} = M_7 : M_6$, $M_{20} = M_4 : M_{49}$, and $M_{25} = M_{30} : M_{31}$.

The proper submodules of $M_{11}$, $M_{15}$, $M_{17}$, $M_{18}$, and $M_{25}$ are generated by $[0, 0, 0, 0, 0, 0, 0, 1, \delta]$, $[0, 0, \delta, 1, 1, -\delta]$, $[\delta^2, 1, 1, \delta]$, $[0, 0, 1, \delta^3]$, and $[0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0]$, respectively. Those of $M_{10}$ are generated by $[0, 0, 0, 0, 0, \delta, 1, \delta, 1]$, $[0, \delta^3, 0, 1, 0, -1, 0, -\delta^2, \delta^3]$ and $[0, 0, 0, 0, 0, 1, 0, 1, 0]$. Those of $M_{14}$ are generated by $[1, 1, 1, 0, 0, 0]$ and $[0, 1, 0, \delta, 0, \delta^2]$. Those of $M_{16}$ are generated by $[\delta^2, 0, 1, 0, 0, 0, 1, 0, 0, 0, 0, 0]$, $[-\delta^3, -\delta^2, -1, 1, -\delta^3, \delta^2, 0, 0, 0, 0, 0, 0]$, $[-\delta, \delta^2, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0]$, and $[1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0]$.

The representation $M_{48}$ is obtained from $M_{32}$ (see [9, Sect. 1.2.3]) by multiplying the third and fourth matrices by $-\delta^3$, and observing that $\varepsilon = -1$ in characteristic 3. This is a combination of constructions (C2) and (C3) of [8, Sect. 3].

*Case 3.8*: $\bar{u} = \zeta$, $\bar{v} = \zeta^3$.

We get the decompositions $M_{10} = M_{37} : M_7$, $M_{11} = M_5 : M_{38}$, $M_{12} = M_6 : M_{39}$, $M_{13} = M_{36} : M_8$, $M_{16} = M_{15} : M_{14}$, $M_{21} = M_{37} : M_2$, $M_{22} = M_{36} : M_3$, $M_{23} = M_1 : M_{39}$, and $M_{24} = M_4 : M_{38}$.

The proper submodules of $M_{10}$, $M_{11}$, $M_{16}$, $M_{21}$, and $M_{23}$ are generated by $[1, 0, 0, 0, 0, 0, 0, 0, -\zeta^3]$, $[\zeta^2, -\zeta, 1, 0, -\zeta^2, 0, \zeta, 0, -1]$, $[1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0]$, $[0, 0, 0, 0, 0, 0, 1, 1]$, and $[-1-\zeta^2, 1, -\zeta^2, 1-\zeta+\zeta^2-\zeta^3, \zeta, 1+\zeta^2, \zeta^3+\zeta-1, \zeta^3-\zeta^2-1]$, respectively. This concludes the proof of the theorem.

# References


[1] K. Bremke, The decomposition numbers of Hecke algebras of type $F_4$ with unequal parameters, *Manuscripta Math.* **83** (1994), 331–346.

[2] R. W. Carter, "Finite Groups of Lie Type: Conjugacy Classes and Complex Characters", Wiley, New York, 1985.

[3] C. W. Curtis and I. Reiner, "Methods of Representation Theory, I & II", Wiley, New York, 1981/1987.

[4] The GAP Group, Lehrstuhl D für Mathematik, RWTH Aachen, Germany and School of Mathematical and Computational Sciences, University of St. Andrews, Scotland. "GAP – Groups, Algorithms, and Programming, Version 4", 1997. `http://www-gap.dcs.st-and.ac.uk/~gap/`

[5] M. Geck, Kazhdan-Lusztig cells and decomposition numbers, *Representation Theory* 2 **68** (1998), 264–277.

[6] M. Geck and K. Lux, The decomposition numbers of the Hecke algebra of type $F_4$, *Manuscripta Math.* **70** (1991), 285–306.

[7] M. Geck and R. Rouquier, Filtrations on projective modules for Iwahori-Hecke algebras, preprint.

[8] T. P. McDonough and C. A. Pallikaros, On the irreducible representations of the specializations of the generic Hecke algebra of type $F_4$, *J. Algebra* **218** (1999), 654–671.

[9] C. A. Pallikaros, Some decomposition numbers of Hecke algebras, *J. Algebra* **187** (1997), 493–509.

[10] C. A. Pallikaros, A note on the representation theory of the Hecke algebra of type $F_4$, *Glasgow Math. Journal*, **39** (1997) 43–50.

[11] R. A. Parker, The computer calculation of modular characters (the Meat-Axe) 267–274, *in* "Computational Group Theory", (M. D. Atkinson Ed.), Academic Press, London, 1984.

[12] M. Schönert et al. “GAP – Groups, Algorithms, and Programming,” 5th ed., Lehrstuhl D für Mathematik, Rheinisch Westfälische Technische Hochschule, Aachen, Germany, 1995.