Some Tight Bounds for Genetic Programming on MAJORITY and ORDER

Timo Kötzing

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Known Results

What can we do so far?

- 1+1 EA and RLS on OneMax.
- 1+1 EA and RLS on linear functions.
- ACO on OneMax.
- 1+1 EA and RLS on MST and SSSP.
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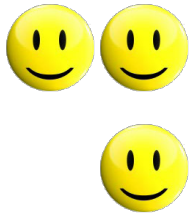

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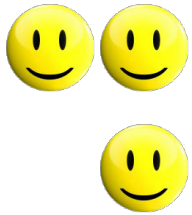

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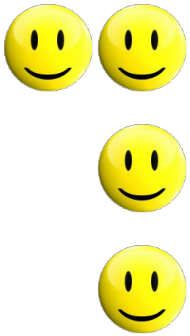

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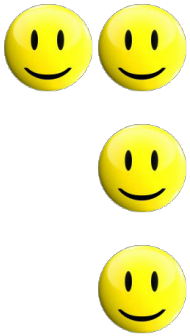

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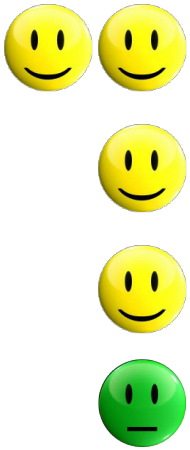

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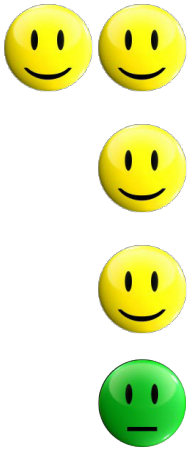

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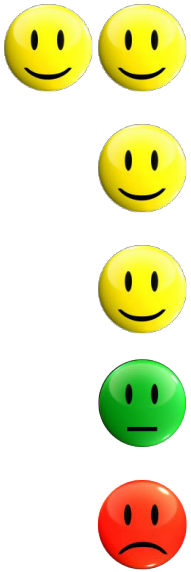

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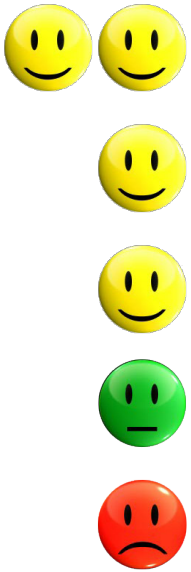

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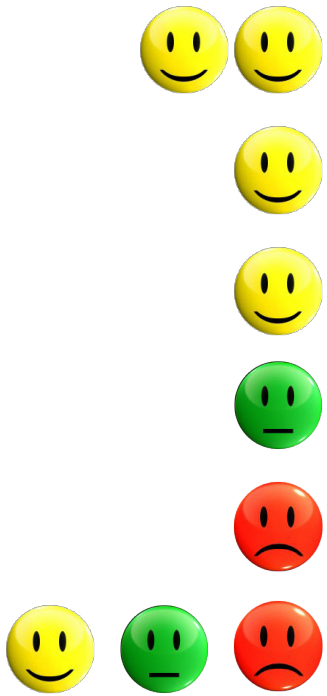

Majority and Order

- Imagine a list of variable length with entires from $1, \ldots, n$ and $\overline{1}, \ldots, \overline{n}$.

3	$\overline{6}$	1	3	$\overline{3}$	...	2	$\overline{2}$	5

- MAJORITY and ORDER assign a fitness to such lists.

MAJORITY

One fitness point per $i \leq n$ such that at least as many i are listed as $\overline{i}$ (and at least one i is listed).

ORDER

One fitness point per $i \leq n$ such there is an i listed before any $\overline{i}$.

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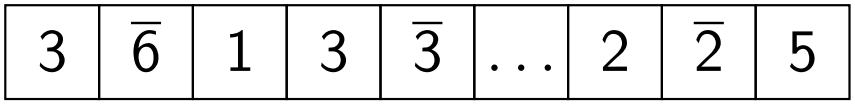

- GP algorithms work on these lists with the following randomized operators.
  - Relabel.
  - Insert.
  - Delete.
- Either $k = 1$ uses of operators per offspring or $k = 1+\text{Pois}(1)$.
- Bloat control: In case of equal fitness, favor shorter lists.

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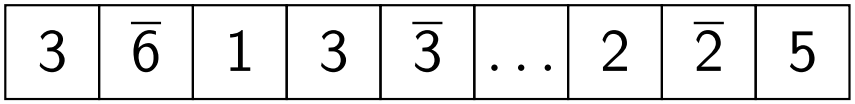

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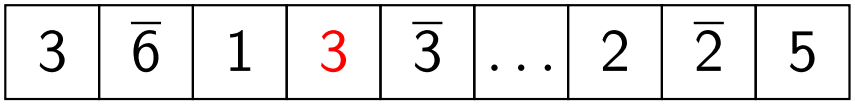

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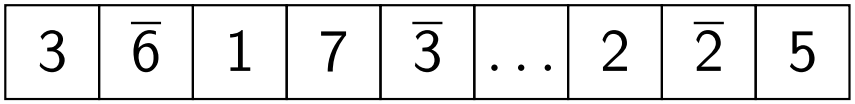

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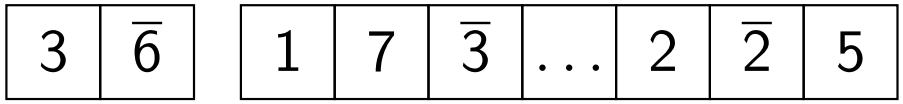

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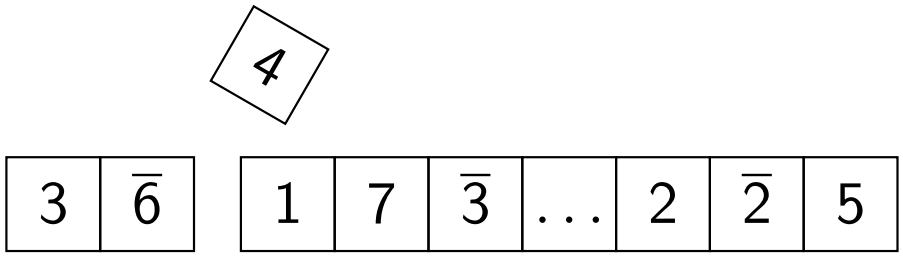

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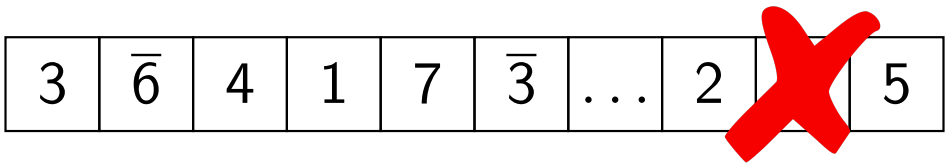

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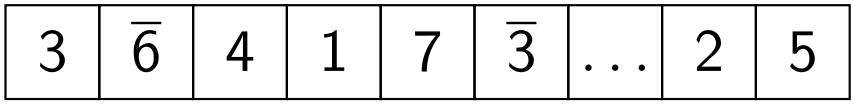

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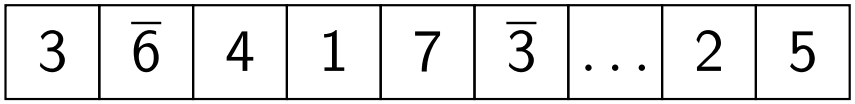

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	k=1	k=1+Pois(1)
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WORDER	$O(nT_{max}) \star$ $O(T_{init} + n \log n)$ Urli et al. (2012)	$O(nT_{max}(\log n + \log w_{max})) \star$ $O(T_{init} + n \log n)$ Urli et al. (2012)
MAJORITY	$O(n^2 T_{max} \log n)$ Durrett et al. (2011) $O(T_{init} + n \log n)$ Urli et al. (2012)	? $O(T_{init} + n \log n)$ Urli et al. (2012)
WMAJORITY	? $O(T_{init} + n \log n)$ Urli et al. (2012)	? $O(T_{init} + n \log n)$ Urli et al. (2012)

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- (Source: Nguyen et al. 2013)

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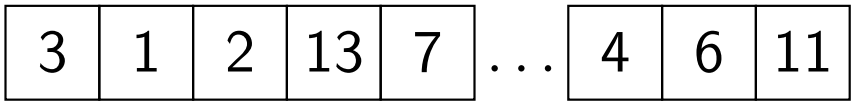

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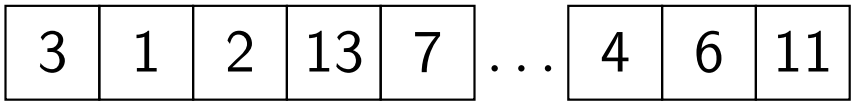

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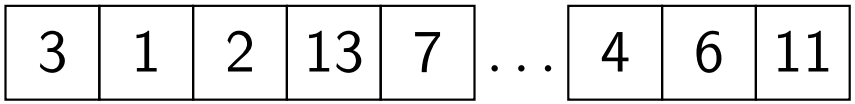

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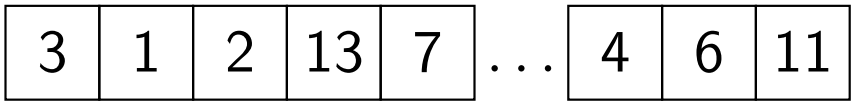

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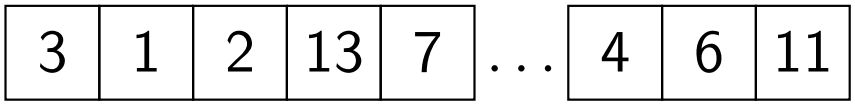

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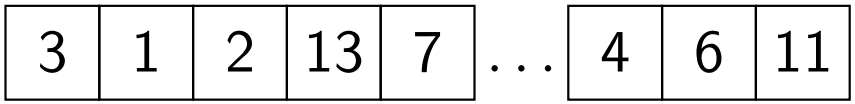

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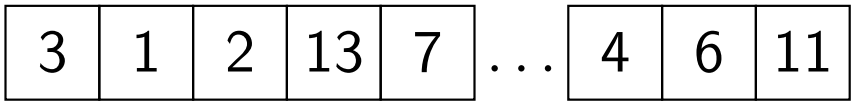

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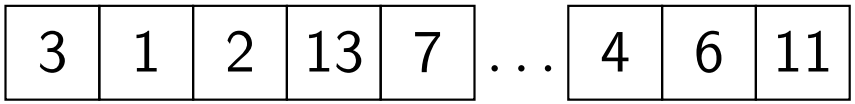

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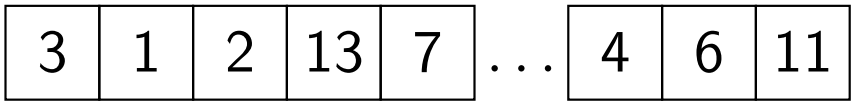

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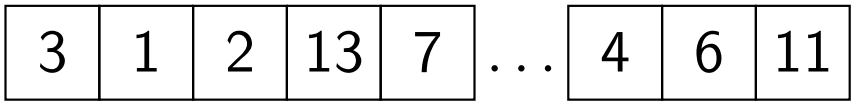

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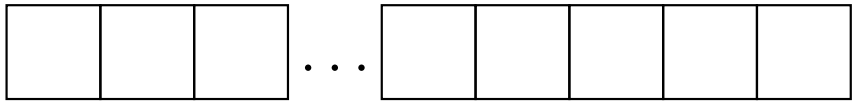

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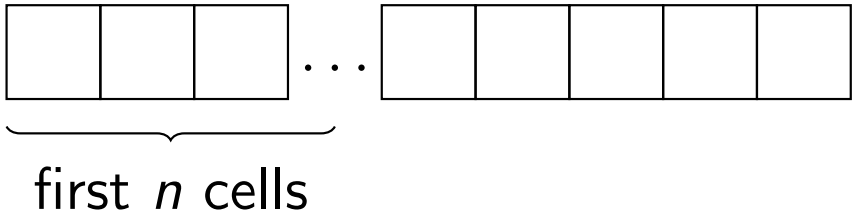

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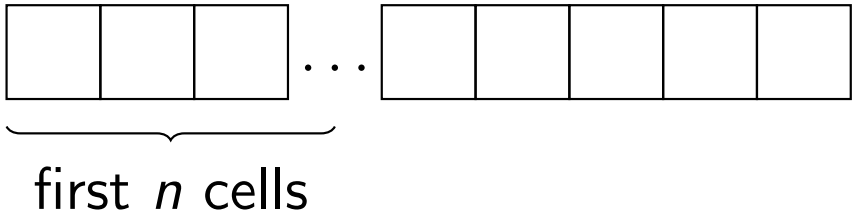

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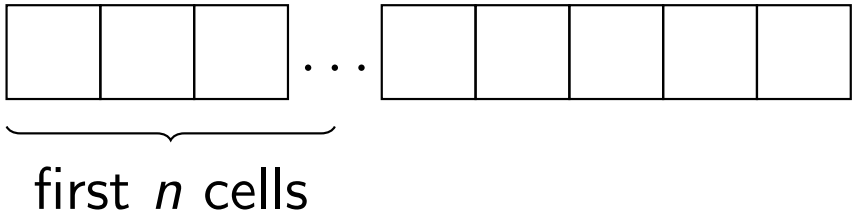

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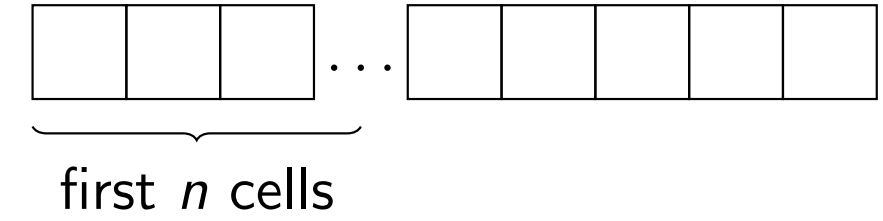

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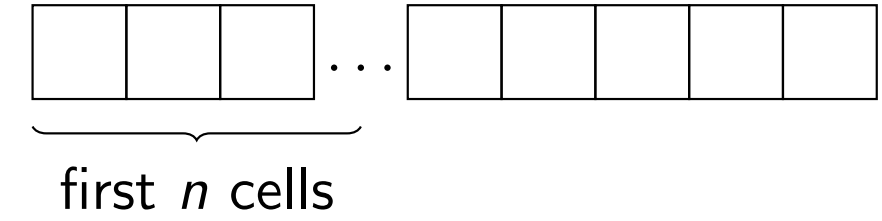

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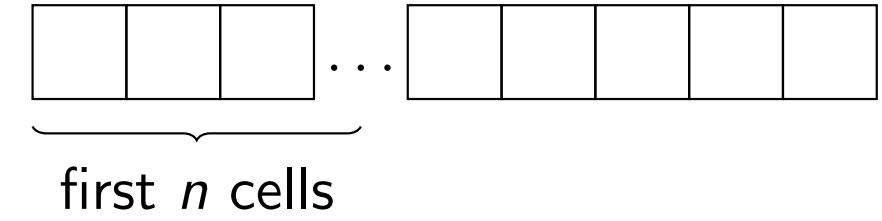

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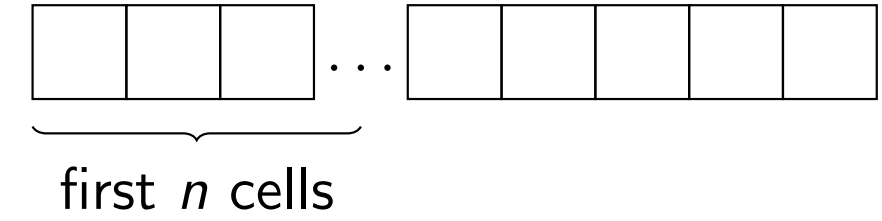

- Bound on maximal list length again?
- $\Rightarrow$ mixing time for first n cells: $O(T_{\text{init}} \log n + n \log n)$.
- Sounds like a good bound. . .
- Best known: $O(T_{\max} n)$.
- A semi-decent bound on ORDER is much easier than on MAJORITY . . .
- . . . for a good bound, it seems to be the reverse.

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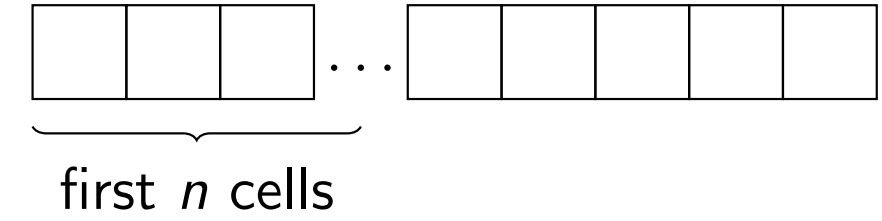

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More Work?

- What else is there?
- Random number of mutations with weighted case.
- Some Multi-Objective variants are also still open.
- What can we learn from abstract results?
- $\Rightarrow$ implicit bloat control: Make delete more likely than insert.
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Thank You!