The dynamics of a topological change in a system of soap films

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Abstract

The study of soap films spanning wire frames continues to provide insight into both static and dynamic properties of foams. Experiments show that sufficiently small triangular faces shrink spontaneously, with their area varying with time as $t^{0.8}$. The rate of growth of the emerging Plateau border after a topological change depends on the bulk viscosity and features three distinct regimes, including one of damped oscillations. We use computer simulations to examine the stability of soap film configurations in frames with more than three sides.

Key words:
PACS: 8270.Rr - Foams
PACS: 8380.yz - Emulsions and foams
PACS: 4720.Dr - Surface-tension-driven instability

1. Introduction

Plateau's celebrated rules describe the topology and geometry of a system of soap films [1]. In particular they state that three films meet symmetrically in a line (called a Plateau border) at angles of 120 degrees, and that four Plateau borders meet symmetrically in a vertex at the tetrahedral angle of $\cos^{-1}(-1/3) \simeq 109.47$ degrees [2]. No other configurations are allowed in a dry foam. However, it was recently demonstrated, both experimentally and computationally, that these rules, albeit necessary, are not sufficient for stability [3]. In particular it was shown that small triangular faces are not stable, with the result that the soap films will undergo spontaneous topological transitions that remove such faces, once the face area has decreased below a critical value.

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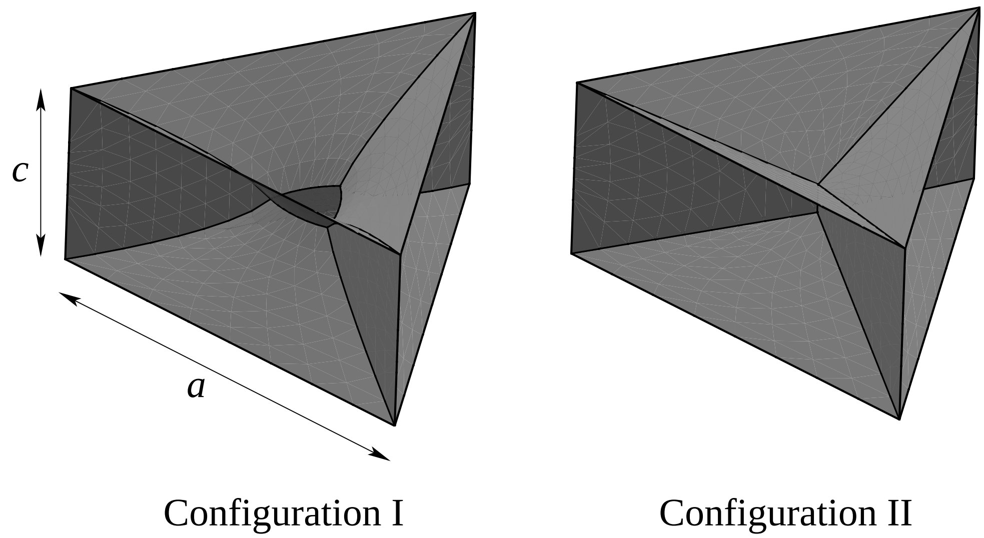


Fig. 1. Two possible configurations of soap films spanning a triangular frame. Configuration I: aspect ratio $c/a < 0.487$. Configuration II: aspect ratio $c/a > 0.413$.

In this paper we extend our previous work [3] in several directions. We report new experiments that examine the dynamics of the vanishing triangular faces in greater detail. We find that the bulk viscosity of the soap solutions begins to affect the speed of these transitions only if the viscosity is increased by at least a factor of 10. We also present data for the growth of the new Plateau border formed in this transition. We find that in this case bulk viscosity plays a prominent role and we compare this with a related 2D experiment [4]. Finally we also report new computations with regard to equilibrium configurations in frames with six, seven, eight and nine-sided frames. The marginal case $n = 6$ is still not well understood.

2. Soap films in a triangular prism: Pre-emptive instability

Plateau's rules are of direct relevance to the configuration of soap films in foams with a low liquid content (less than about five percent by volume). They were derived from experiments where wire frames were dipped into soap solutions, and then carefully withdrawn. This results in soap films spanning the frame, in configurations that depend on both frame geometry and the way that the frames have been withdrawn.

For the triangular prism of Fig. 1, and an aspect ratio of the two side lengths $c/a < 0.487$ the resulting film configuration I is shown in Fig. 1, featuring a plane triangular film parallel to the triangular base of the frame. For $c/a > 0.413$, the equilibrium configuration II features a Plateau border in the centre of the frame, parallel to the $c-$axis. For $0.413 < c/a < 0.487$ both configurations are possible and which configuration is actually realised depends on the experimental details/history.

Fig. 2 displays the variation of total energy (equivalent to the total surface area) as a function of the ratio c/a. Configuration I has a higher energy than configuration II for $c/a > 0.472$, but remains stable until $c/a = 0.487$. One would intuitively think that the

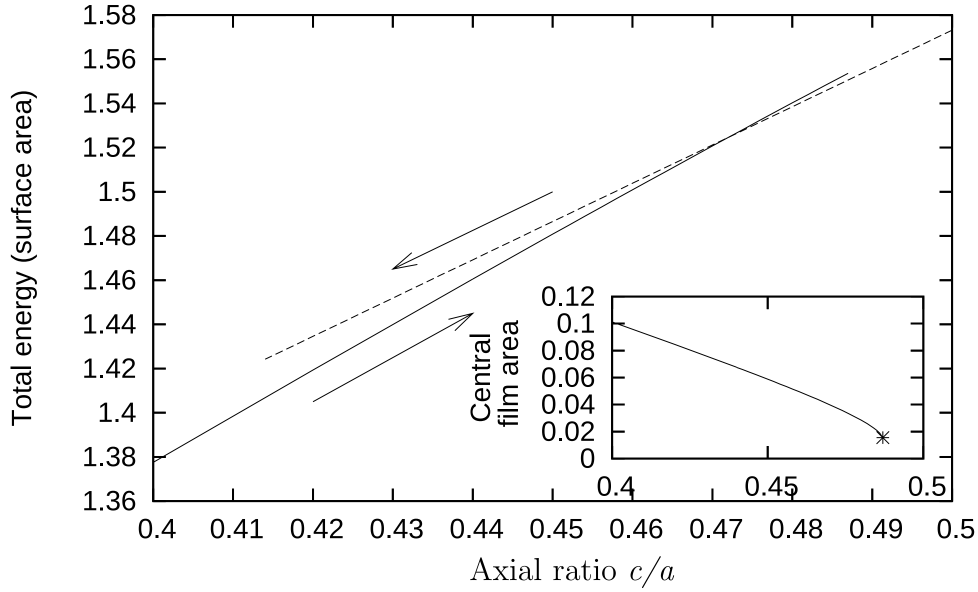


Fig. 2. A Surface Evolver calculation allows us to calculate the total film area of configurations I (solid line) and II (dashed line) as a function of c/a. At $c/a = 0.487$ configuration I is unstable, despite the finite area A of the triangular face (see inset).

loss of stability of the two configurations coincides with the value of c/a at which the triangular face has shrunk to zero area, i.e. where six Plateau borders would meet in a single point, in violation of Plateau's rules. However, the inset of Fig. 2 shows that at the actual point of instability, the area of the triangular face is finite. In other words, the instability that is demanded by Plateau's rules is *pre-empted*.

Figs. 1 and 2 were obtained from numerical computations using the Surface Evolver, written by Ken Brakke [5]. The computations involve the minimisation of surface energy (area) for a given topology of the connecting surfaces. The stability of a configuration is determined by computing the eigenvalues of the Hessian matrix: when the smallest of these becomes negative, the configuration becomes unstable, implying that a transition to another configuration should occur. These calculations concern equilibrium configurations only; the *dynamics* of the transition at the point of instability cannot be resolved using this method. This would require knowledge of the various dissipation mechanisms involved in the transition. Some insight can be gained from our experiments described in the next section.

3. Experiments on soap films in a triangular prism

3.1. *Experimental set-up*

All of our experiments were made with a 1% volume aqueous solution of the commercial detergent Fairy Liquid. We used three different sizes for the triangular prisms made of thin wire (diameter 0.70mm) with values for (c, a) of $(1.6\text{cm}, 3.2\text{cm})$, $(3.2\text{cm}, 6.4\text{cm})$ and

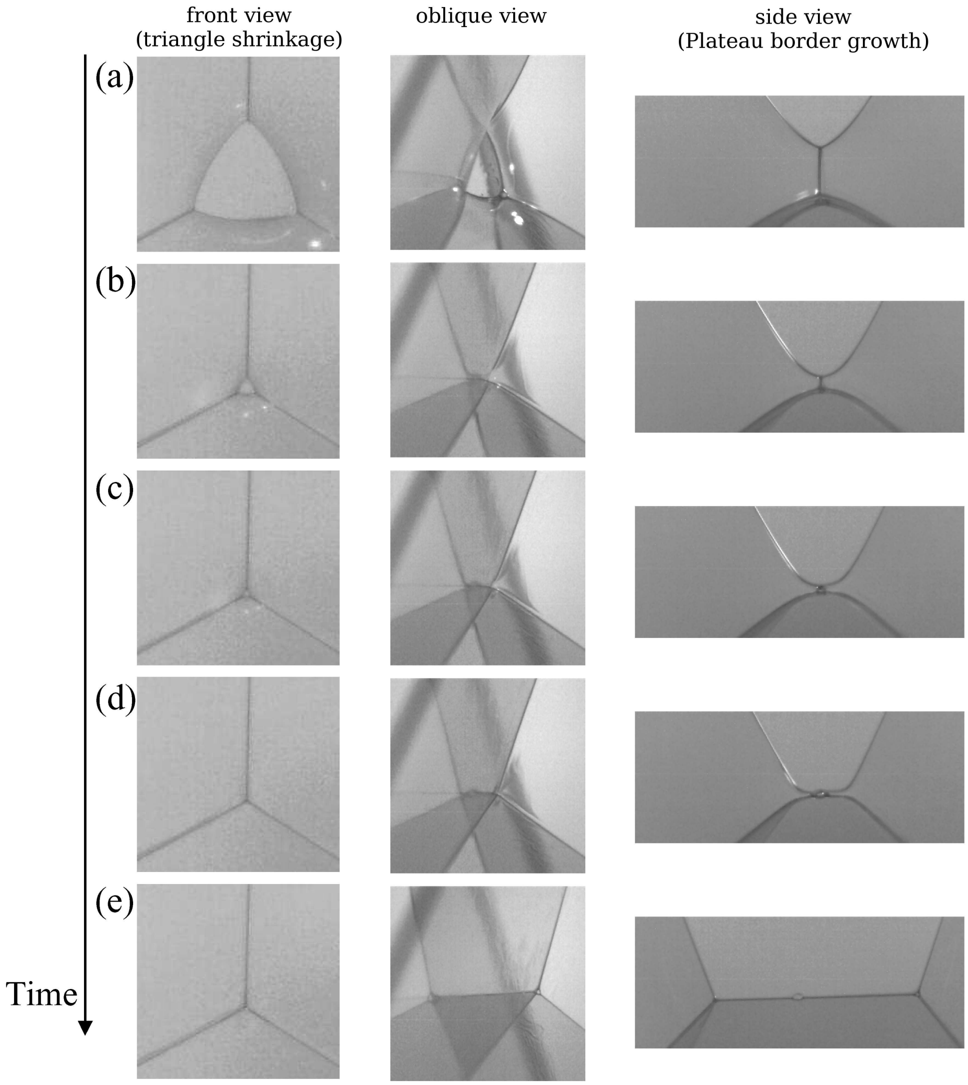


Fig. 3. Photographs of the transition from configuration I to II (see Fig. 1). The sequences were obtained using a high speed camera. They show the shrinkage of the triangular face and the growth of the Plateau border. The disappearance of the face is accompanied by entrainment of air, resulting in the trapping of a small bubble in the Plateau border (c-e).

(6.4cm, 12.8cm).

The dynamics of the transition was studied by inducing the instability for a fixed axial ratio c/a slightly above 0.487 by performing the following experiment. Dipping such a frame into soap solution we obtain configuration II. By blowing carefully against one of the central Plateau border junctions, in the direction of the central Plateau border (see Fig. 1), we can force a transition to configuration I. This configuration is not stable and quickly returns to II. An analysis of the return transition was performed based on video images.

Focusing on the triangular face, we can monitor the shrinking of its area in time, during the relaxation to configuration II. Similarly we can observe the formation and subsequent growth of the emerging Plateau border which is perpendicular to the plane of the triangular face. We have recorded this transition using a high speed camera (PCO 1200hs) with up to 5000 frames per second. Some still frames are shown in Fig. 3: the left column shows the disappearance of the triangular face, the right column shows the appearance of the Plateau border, and the middle column shows an oblique view in which both the disappearance of the triangular face and the appearance of the Plateau border are seen.

The surface area of the triangular face in successive images was analysed using the ImageJ software [1]. Fig. 4 shows data for the area of the vanishing triangular face as a function of time for our three different frame sizes. To measure the length of the emerging Plateau border, the Particle Image Velocimetry (PIV) algorithm combined with particle tracking has been applied to the image sequences. The high liquid content in the vertices (the two ends of the emerging Plateau border seen in Fig. 3(e)), allows the particle tracking algorithm to follow the end points of the Plateau border. In this way we can measure its length in each frame.

3.2. *The vanishing of a triangular face*

The oscillations in the initial shrinking of the triangular face, shown in 4(a), are incidental consequences of the initial disturbance by blowing. For our analysis we consider only the last 0.1s of the existence of the triangular face, which does not depend on the history of its formation (Fig. 4(b)).

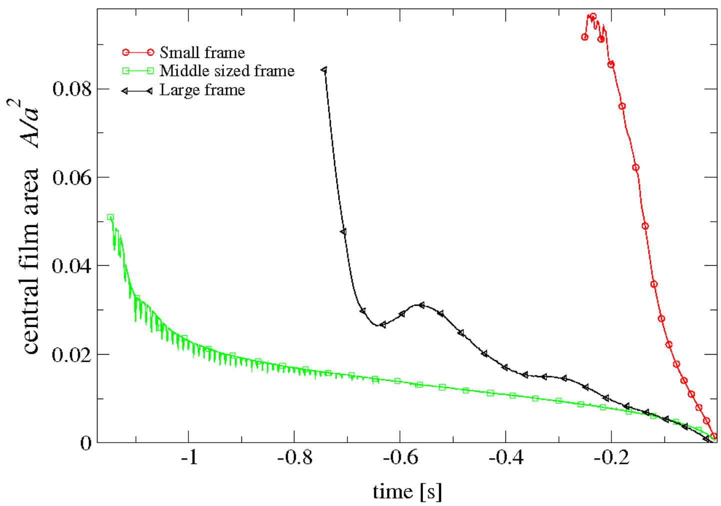
For a plastic frame with struts 4.30mm thick, the area of the triangular face was previously found to decrease approximately quadratically in time [3]. We have repeated this experiment using three different wire frames, including the frame size used in [3] ($(c, a) = (3.2\text{cm}, 6.4\text{cm})$), but now with thinner struts. The time resolution in the new experiment was 100 times better. Performing a total of 46 experiments, we find that close to the vanishing of the triangular face at time t_c, its area A scales with time t as $A(t)/a^2 = \alpha(t_c - t)^\beta$, where $\beta \simeq 0.80 \pm 0.02$, independent of the frame size. The prefactor α increases roughly linear with frame size. The results of our analysis are summarised in Table 1.

To study the role of viscosity we repeated the experiment with the frame dimension of $(c, a) = (3.2\text{cm}, 6.4\text{cm})$ using solutions with different bulk viscosity. These were made by adding glycerol to the detergent to obtain concentrations of 5, 10, 20, 60 and 80 volume percentage. The resulting values of bulk viscosity of the solutions were taken from ref. [6].

Based on 77 experiments we find that an increase of viscosity of up to a factor 10 does not result in a change of the power law. Only higher values of the viscosity (> 60 mPa s) lead to a gradual decrease of β to 0.70. The prefactor only decreases if the viscosity is increased by more that a factor of 10. That viscosity has such a small role in the disappearance of the triangular face is surprising, since the topological change involves

[1] A freeware available from: http://rsb.info.nih.gov/ij/

(a)



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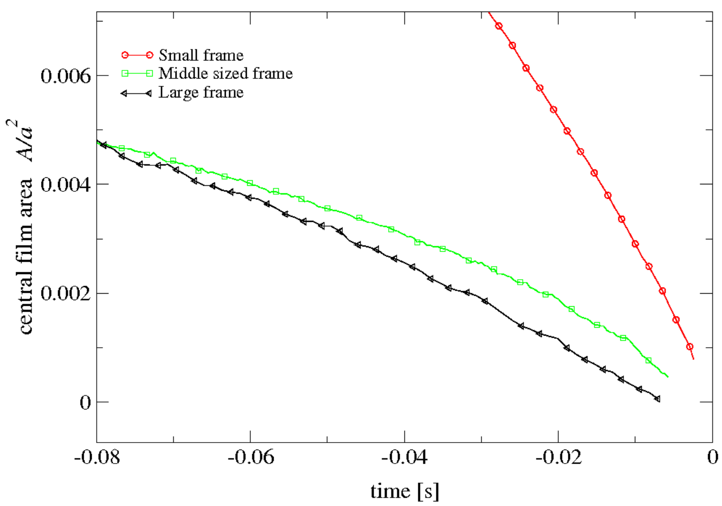


Fig. 4. (a) Shrinkage of the triangular face as a function of time for all three frame sizes used in our experiment. (b) Expanded region close to the transition point from configuration I to II, where we fit a power law function to the data.

a flow of liquid and viscosity plays a major role in the drainage of Plateau borders and films. However, recent experiments by Durand and Stone [4] find that topological changes in quasi-two-dimensional foams are also largely unaffected by the bulk viscosity of the solution.

The implications of these results for the 3D case are as yet not clear; in particular we cannot give an explanation for either the value of the exponent β or the lack of dependence on viscosity, other than to point to the the possibility of dominant dissipative mechanism associated with surface viscosity.

a [cm]	Glycerol conc. [volume %]	Viscosity [mPa s] (from [6])	α $[T^{-\beta}]$	β	No. of measurements
3.2	0	1.00	1.32 ± 0.19	0.820 ± 0.03	18
6.4	0	1.00	3.07 ± 0.70	0.805 ± 0.11	18
12.8	0	1.00	6.69 ± 0.60	0.790 ± 0.08	10
6.2	5	1.14	3.12 ± 0.14	0.823 ± 0.04	10
6.2	10	1.31	3.06 ± 0.07	0.834 ± 0.08	15
6.2	20	1.77	3.05 ± 0.11	0.826 ± 0.12	15
6.2	60	10.97	2.16 ± 0.09	0.773 ± 0.07	18
6.2	80	62.00	1.33 ± 0.04	0.704 ± 0.05	19

Table 1
Summary of the experimental results for the shrinkage of the area A of the triangular face with time t $(A(t)/a^2 = \alpha(t_c - t)^\beta)$.Here a is the longest edge of the frame.

3.3. *The growth of the Plateau border*

Fig. 5(a) shows the growth of the Plateau border with time for the frame of size $(c, a) = (3.2\text{cm}, 6.4\text{cm})$. Three distinct regimes characterise its growth, shown in Fig. 5(a). A steep linear increase lasting less than 5ms is followed by a roughly exponential decay over a period of 10ms. Finally there is a relatively long damped oscillatory phase with a well-defined wave length of $65 - 100$ Hz, depending on the thickness of the wire struts constituting the frame.

The inset of 5(a) shows a linear fit to the three different experiments for $(c, a) = (3.2\text{cm}, 6.4\text{cm})$. A qualitative comparison of the time scale over which the triangular face disappears and the Plateau border appears reveals that the latter takes place much more quickly (by two orders of magnitude).

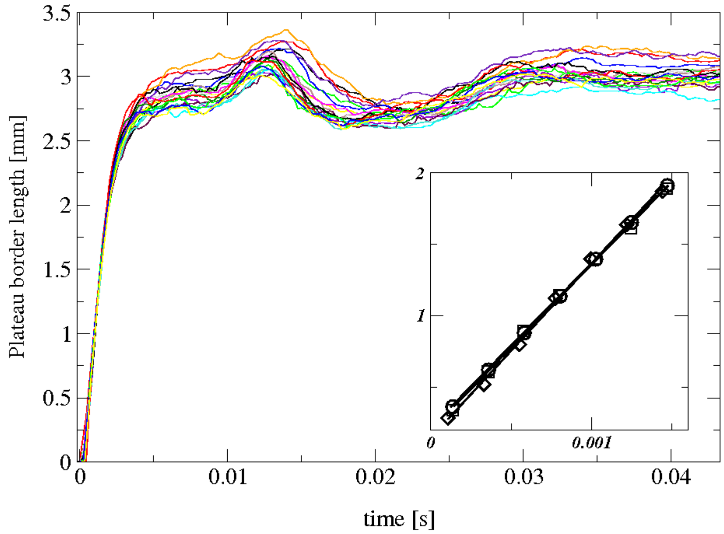
Note that the Plateau border has a finite length once we are able to measure it ($t = 0$). This is due to the surprising fact that before the triangular face disappears the surrounding films form an elongated *tunnel* which, as the triangular face finally disappears, collapses into a Plateau border which already has a finite length. We observe that gas is trapped during the collapse of this tunnel, which results in the creation of small bubbles (see Fig. 3(e)).

The observation of damped oscillations is also surprising, but they are presumably due to vibrations about equilibrium. We found that the oscillations are more pronounced when using plastic frames (Fig. 6). These have a larger strut thickness than the wire frames, which result in thicker Plateau borders and films.

As is evident from Fig. 6(a), the oscillatory phase of the growth of Plateau border does not seem to be affected by the aspect ratio of the triangular prism.

In order to establish whether the frequency is related to the normal mode of vibration of the film configuration, we subjected configuration II to a sound wave produced by a square wave sound generator. We find that the central Plateau border oscillates at a frequency of about 67Hz (the solid line in Fig. 6(a)), consistent with the frequency of oscillation of the emerging Plateau border in the same frame. An estimate of the vibration frequency, based on the mass of the Plateau border and surface tension, was found to

(a)



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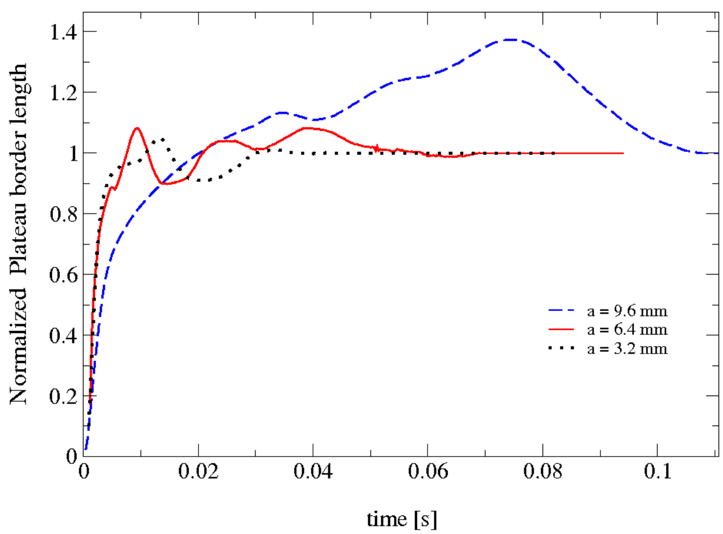
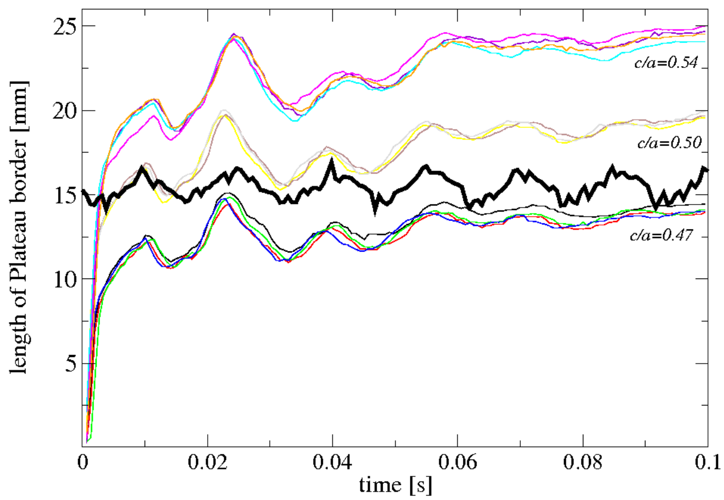


Fig. 5. (a) The increase in length of the Plateau border in a frame of size $c = 3.2$cm, $a = 6.4$cm consisting of wires with a thickness of 0.7mm. A sharp linear increase is followed by a long relaxation period, with some oscillations. The inset shows linear fits to three of the data sets in the regime of initial growth. (b) The growth of the Plateau border averaged over many experiments for all three frame sizes.(The Plateau border lengths for the three different frame sizes are normalised by their respective final length.)

grossly exceed the observed value. It would therefore appear to be necessary to include the mass of the displaced air to obtain a frequency of the order of that observed.

(a)



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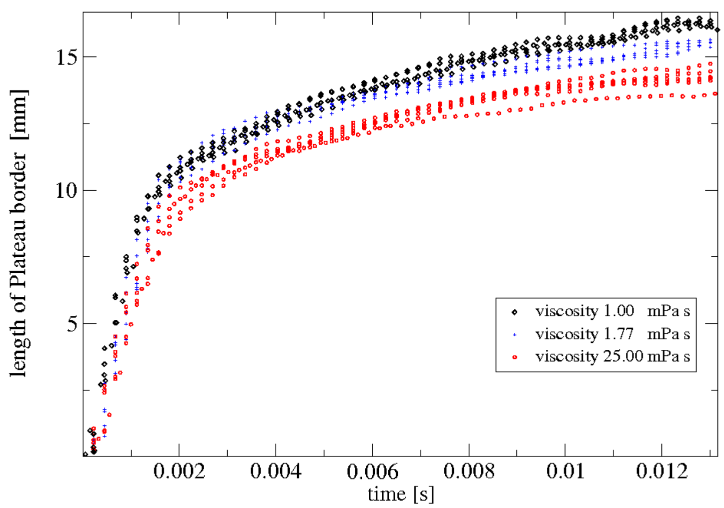

Fig. 6. (a) The gray lines show the oscillations of the emerging central Plateau border (PB). The frequency is approximately 66Hz, and is independent of the ratio c/a for the three values used in our experiments. The solid line shows the resonance of the PB when subject to a sound wave of 67 Hz. (b) The first 13ms after the formation of the PB for three different values of bulk viscosity indicates the reduction of growth-rate with an increase in viscosity. For all of the experiments in this figure we used plastic frames with a strut diameter of 4.30mm.

3.4. *Role of viscosity on the emerging Plateau border*

We investigated the effect of viscosity on the emerging Plateau border by varying the value of bulk viscosity of our soap solution. Fig. 6(b) shows the initial 13ms of the growth of the Plateau border. Clearly the growth is slowed down with increasing viscosity. In

Glycerol conc. [volume %]	Viscosity [mPa s] (from [6])	Growth rate [m/s]	No. of measurements
0	1.00	0.539 ± 0.08	7
20	1.77	0.379 ± 0.05	7
70	11.30	0.307 ± 0.01	7

Table 2
The initial growth rate of a Plateau border for different values of bulk viscosity.

table 2 we present the slope of the linear part of the growth (first 2ms) of the plots averaged over different measurements of the same viscosity.

Our experiment is the 3D equivalent to the work of Durand and Stone [4] (see their figure 3), who performed experiments on foams confined between two glass plates. In particular, they recorded the growth of a film in time, after a topological change, and report that the bulk viscosity has a minor role in comparison with surface viscoelasticity.

Note that while in the experiments of [4] the time scale of the growth of the Plateau border is of the order of seconds, in our case it is of the order of 10ms. This is another clear indication of the role of friction [7], imposed by the confining plates in [4].

4. The stability of small faces with more than three sides: further computations

The pre-emptive instability occurs for other frames with up to five sides, but the critical size of the polygonal face which vanishes becomes very small (Fig. 7), and hardly worth pursuing experimentally. Fig. 8 shows Surface Evolver simulations of the two con-

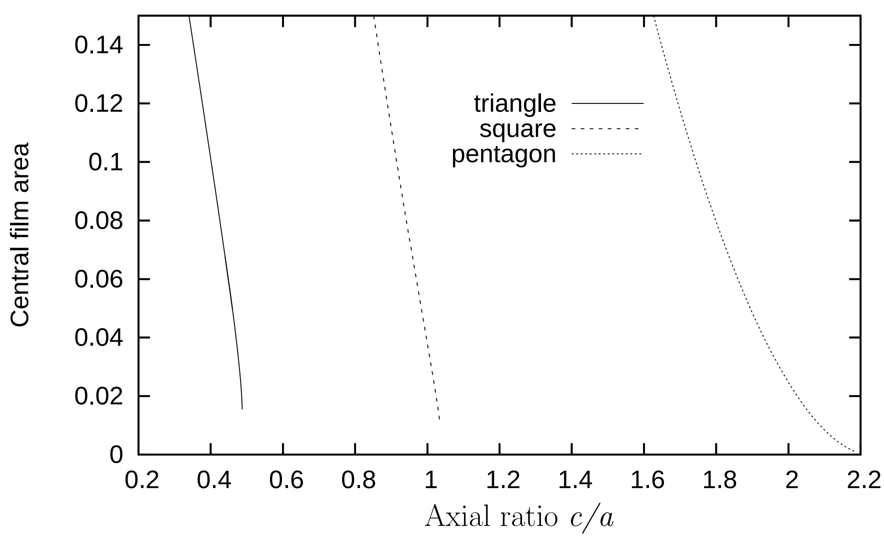

Fig. 7. As the aspect ratio of a four- or five-sided frame increases, the area of the central film shrinks in a manner analogous to the $n = 3$ case. Surface Evolver simulations show that the instability predicted by Plateau's Laws is triggered pre-emptively in these cases, while the central film retains finite area [3]. The critical area becomes smaller as the number of sides of the frame increases.

(a)

(b)

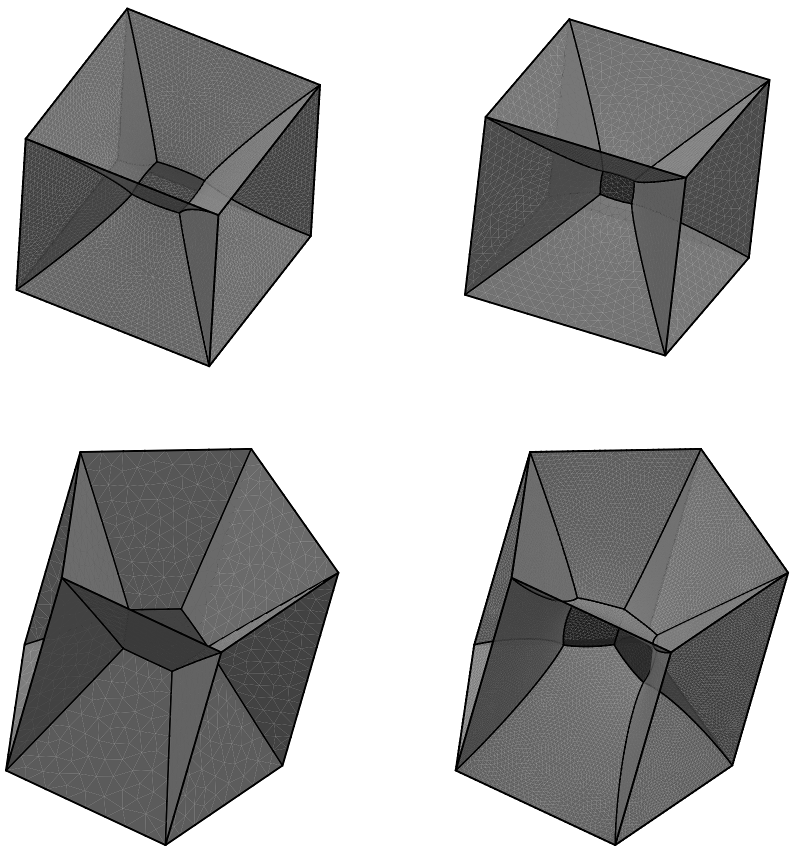

Fig. 8. Two possible configurations of soap films spanning square and pentagonal prisms, demonstrating the hysteresis present. (a) Square frame, with $a = 1$ and $c = 0.960$. The critical value is $c/a = 1.043$, when the area of the square face is close to $0.01a^2$. This is the topological change familiar from the cubic frame [8], with two possibilities for the new face by symmetry. (b) Pentagonal frame, with $a = 1$ and $c = 1.60$. The critical value is $c/a = 2.184$, when the area of the pentagonal face is close to $0.001a^2$. In the second configuration, there are two vertical faces.

figurations for square ($n = 4$) and pentagonal ($n = 5$) prisms, where the topological transition is more complex than in the case of the triangular prism. Here a is defined to be the length of each edge of the base of the frame, and c its height; in the simulations we vary c and keep a fixed and equal to one.

Beyond $n = 5$, the scenario changes. The case $n = 6$ is marginal and we will return to it below. For $n > 6$, the configuration with the single central film, which is analogous to that of Fig. 1 (configuration I), remains stable for an indefinite increase of the aspect ratio, and the area of the film tends to a constant as $c/a \to \infty$. This is because the adjoining angled Plateau borders connect to the sides rather than the corners of the frame (see Fig. 9). We have verified this behaviour experimentally in the case $n = 8$, for a frame made of thin copper wire (Fig. 9).

For the marginal case $n = 6$, the edges at the ends of the frame make an angle of $120°$.

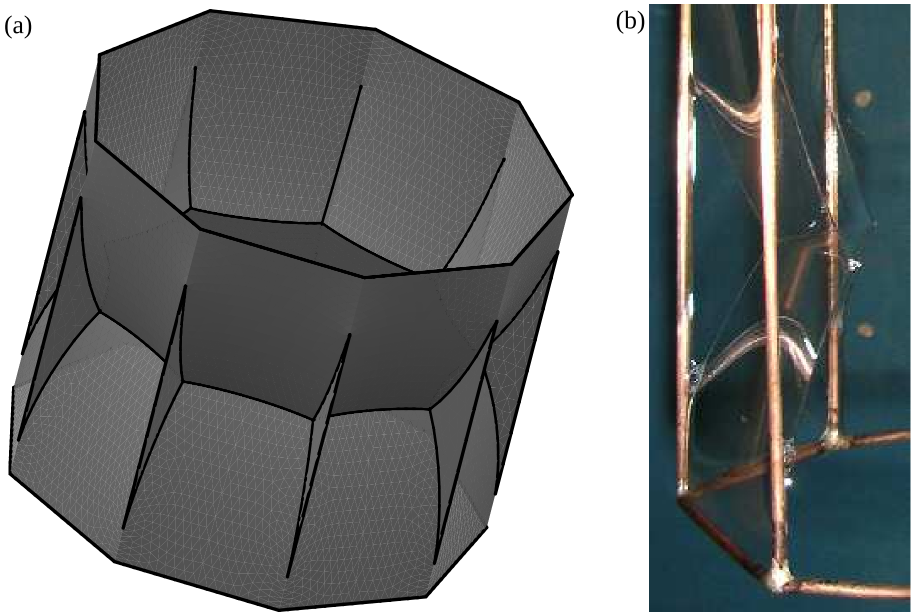

Fig. 9. (a) The (only) soap film configuration that spans a frame with nine sides, from simulation, with $c/a = 2.30$. This configuration remains stable when the aspect ratio is increased because of the way in which the Plateau borders join the sides of the frame rather than the corners. (b) Image from an experiment with an eight-sided frame (c = 120mm, a = 40mm), magnifying the region where the Plateau border meets the frame.

The Plateau borders now remain attached to the corners, but make an angle with the edges of the frame which tends to zero as $c \to \infty$. It is not obvious what form should be taken in that limit: the calculations presented in Fig. 10 suggest that the area of the central face goes to zero as c^{-3}. This result, based purely on statics, poses an unsolved problem.

5. Conclusion

Interest in the physics of foams has moved from static to dynamic properties [9], posing fresh challenges for both theory and experiment. In this work we have taken a case which, in addition to its intrinsic appeal, offers a good test for future theory. At present this is limited as much by our lack of understanding of the physical factors that are at work as by technical problems of simulation.

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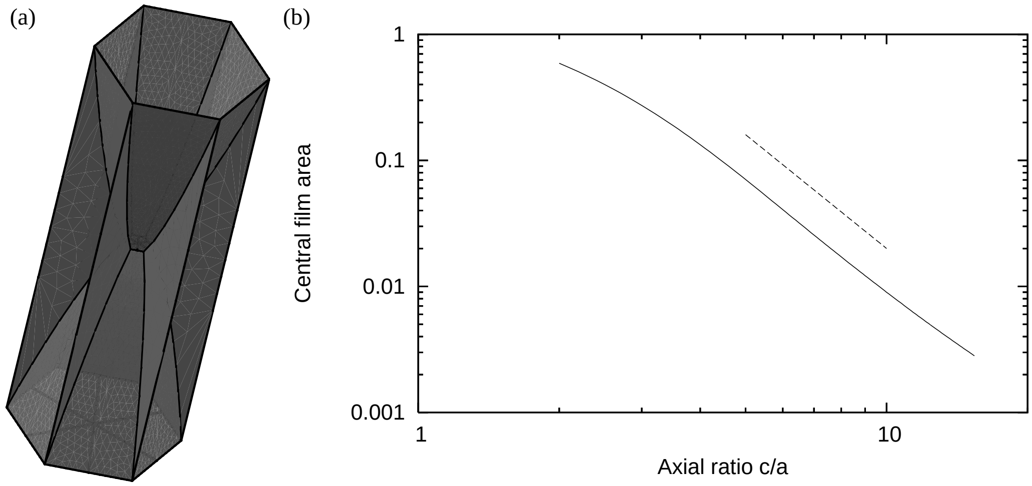

Fig. 10. (a) The (only) soap film configuration that spans a frame with six sides, from simulation, with $c/a = 5.70$. (b) The aspect ratio is increased by varying the area of the central face tends to zero. Note that the data is plotted on log axes, and the dashed line has a slope of -3.

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