

The dynamics of a topological change in a system of soap films *(revised manuscript)*

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Abstract

The study of soap films spanning wire frames continues to provide insight into both static and dynamic properties of foams. Experiments show that sufficiently small triangular faces shrink spontaneously, with their area varying with time as $t^{0.8}$. The growth of the Plateau border that emerges after the collapse of the triangular face is initially linear, followed by an oscillatory relaxation to the equilibrium length. Its initial growth rate decreases with the viscosity of the liquid. We also consider wire frames in the shape of n-sided prisms with regular polygonal ends, $n \geq 3$, and employ experiments and computer simulations to examine the stability of soap film configurations within them. Spontaneous shrinking of small faces occurs in 4 and 5-sided prismoidal frames, while this instability is suppressed for $n \geq 6$.

Key words:

PACS: 8270.Rr - Foams

PACS: 8380.yz - Emulsions and foams

PACS: 4720.Dr - Surface-tension-driven instability

1. Introduction

Plateau's celebrated rules describe the topology and geometry of a system of soap films [1]. In particular they state that three films meet symmetrically in a line (called a Plateau border) at angles of 120 degrees, and that four Plateau borders meet symmetrically in a vertex at the tetrahedral angle of $\cos^{-1}(-1/3) \simeq 109.47$ degrees [2]. No other configurations are allowed in a dry foam. The 120 degree rule reflects the equilibrium of the three surface tension vectors at the intersection of three soap films. The rule that

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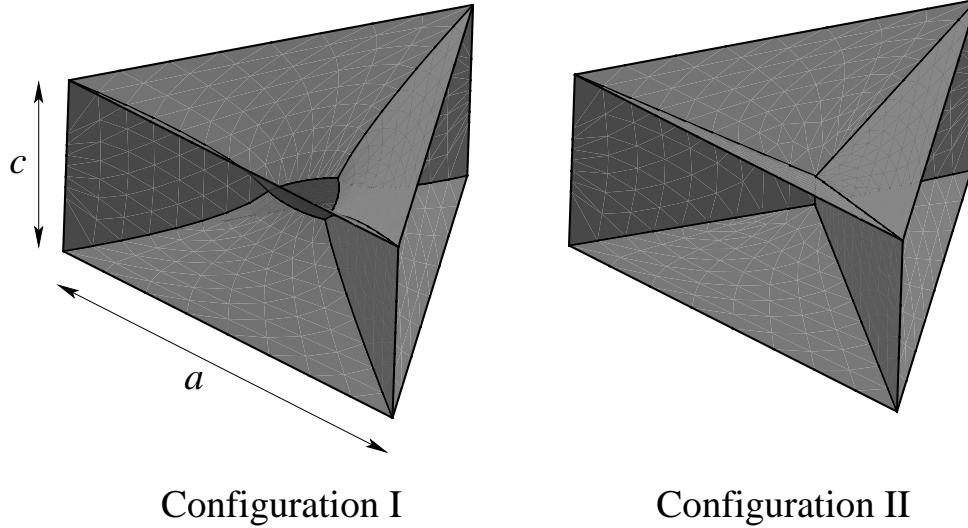


Fig. 1. Two possible configurations of soap films spanning a triangular frame. Configuration I: aspect ratio $c/a < 0.487$. Configuration II: aspect ratio $c/a > 0.413$.

not more than four of these intersection lines can meet in a vertex was only proven in 1976 with full mathematical rigour [3] (the origins of this proof date back to the 19th century [4]). The symmetry of the vertex then follows from the symmetry of the adjoining intersections of the films.

However, it was recently demonstrated, both experimentally and computationally, that Plateau's rules, albeit necessary, are not sufficient for stability [5]. In particular it was shown that small triangular faces are not stable, with the result that the soap films will undergo spontaneous topological transitions that remove such faces, once the face area has decreased below a critical value.

In this paper we extend our previous work [5] in several directions. We employ *experiments* to help us to understand the dynamics of this instability, which is probably important in the rheology of foams, and static (or quasi-static) *simulations* using the Surface Evolver to establish equilibrium film shapes in different frame shapes and sizes, providing a benchmark configuration for our experiments. Our experiments concern both the dynamics of the vanishing triangular face and the growth of a new Plateau border. We find that the bulk viscosity of the soap solution slows down the shrinking of the face only if the viscosity is increased (by adding glycerol) by at least a factor of 10. For the growth of the new Plateau border we find that the bulk viscosity plays a much more prominent role. Finally we report new computations with regard to equilibrium configurations in frames with six, seven, eight and nine sides.

2. Soap films in a triangular prism: Pre-emptive instability

Plateau's rules are of direct relevance to the configuration of soap films in foams with a low liquid content (less than about five percent by volume). They were derived from experiments where wire frames were dipped into soap solutions, and then carefully with-

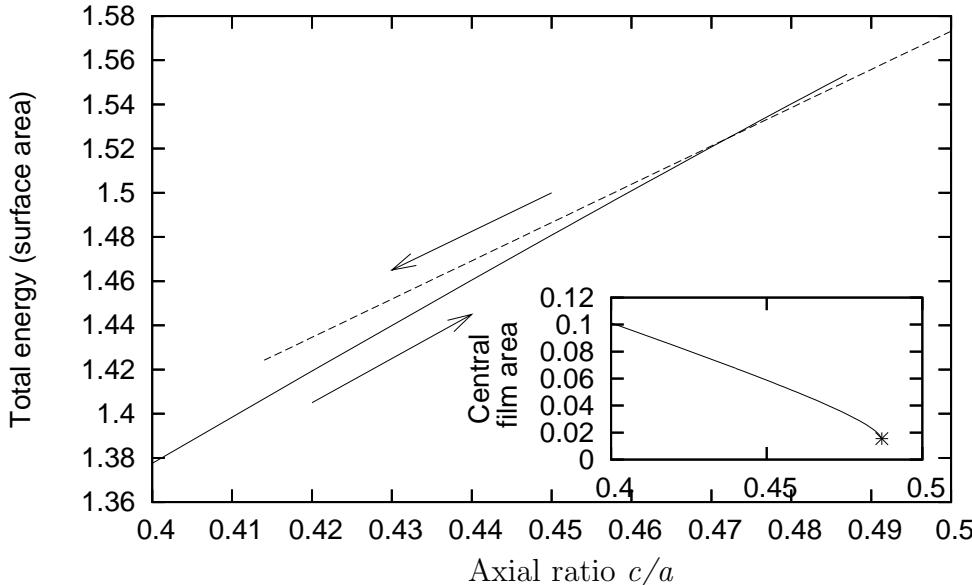


Fig. 2. A Surface Evolver calculation allows us to calculate the total film area of configurations I (solid line) and II (dashed line) as a function of c/a . As c/a is increased from a small value (lower arrow), configuration I remains stable until $c/a = 0.487$. Despite the finite area A of the triangular face at this point (see inset), the soap film configuration spontaneously evolves to configuration II (dashed line). If c/a is decreased from a large value (upper arrow), configuration II remains stable down to $c/a = 0.413$. At this point the central Plateau border shrinks to zero length and configuration I is regained.

drawn. This results in soap films spanning the frame, in configurations that depend on both frame geometry [6] and the way that the frames have been withdrawn.

For the triangular prism of Fig. 1, and an aspect ratio of the two side lengths $c/a < 0.487$ the resulting film configuration I is shown in Fig. 1, featuring a plane triangular film parallel to the triangular base of the frame. For $c/a > 0.413$, the equilibrium configuration II features a Plateau border in the centre of the frame, parallel to the c -axis. For $0.413 < c/a < 0.487$ both configurations are possible and which configuration is actually realised depends on the experimental details/history.

The critical values of c/a were obtained from numerical computations using the Surface Evolver, written by Ken Brakke [7]. The computations involve the minimisation of surface energy (area) for a given topology of the connecting surfaces. The stability of a configuration is determined by computing the eigenvalues of the Hessian matrix: when the smallest of these becomes negative, the configuration becomes unstable, implying that a transition to another configuration should occur.

Fig. 2 displays the variation of total energy (equivalent to the total surface area) as a function of the ratio c/a , computed using the Surface Evolver with $a = 1$. Configuration I has a higher energy than configuration II for $c/a > 0.472$, but remains stable until $c/a = 0.487$. One would intuitively think that the loss of stability of the two configurations coincides with the value of c/a at which the triangular face has shrunk to zero area, i.e. where six Plateau borders would meet in a single point, in violation of Plateau's rules. However, the inset of Fig. 2 shows that at the actual point of instability, the area of the

triangular face is finite. In other words, the instability that is demanded by Plateau's rules is *pre-empted*. For a discussion of the re-entrant form of the central film area as a function of c/a see Appendix A.

These Surface Evolver calculations concern equilibrium configurations only; the *dynamics* of the transition at the point of instability cannot be resolved using this method. This would require knowledge of the various dissipation mechanisms involved in the transition. Some insight can be gained from our experiments described in the next section.

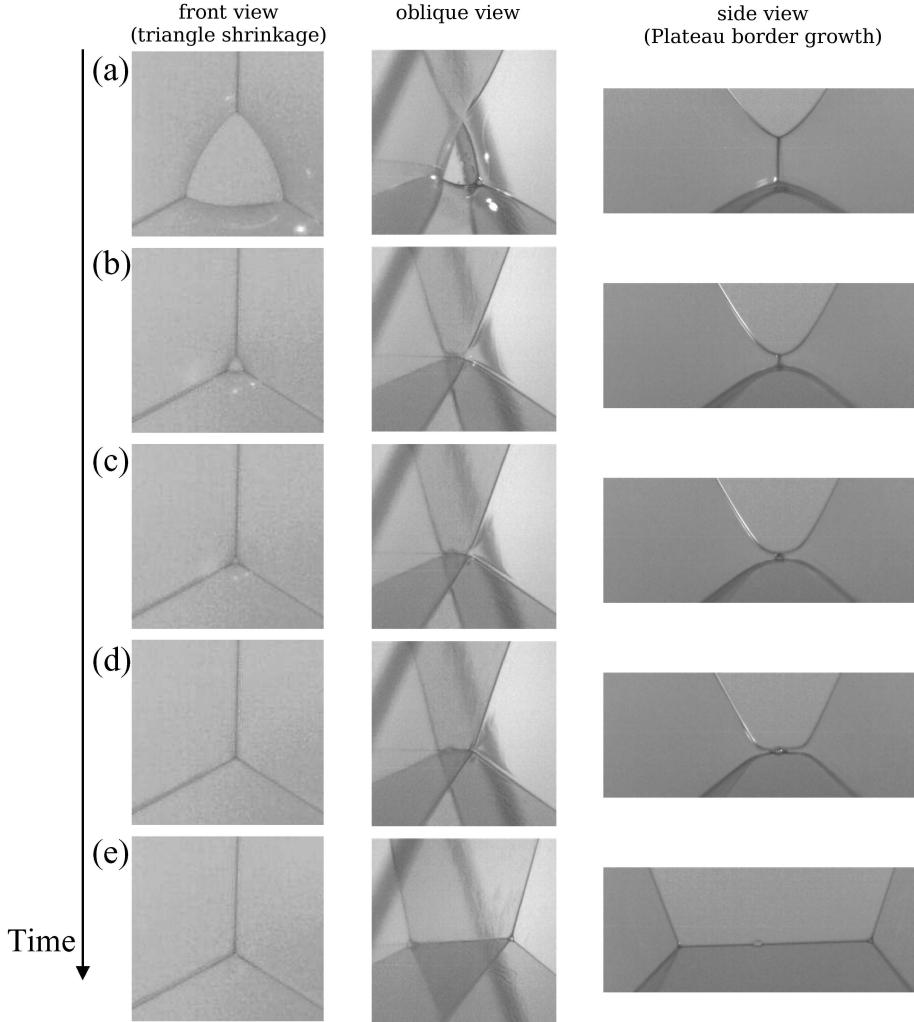


Fig. 3. Photographs of the transition from configuration I to II (see Fig. 1). The sequences were obtained using a high speed camera. They show the shrinkage of the triangular face and the growth of the Plateau border. The disappearance of the face is accompanied by entrainment of air, resulting in the trapping of a small bubble in the Plateau border (c-e). The entire sequence from (a) to (e) spans over 0.2 s but the time intervals are not equal.

3. Experiments on soap films in a triangular prism

3.1. Experimental set-up

All of our experiments were carried out with a 1% volume aqueous solution of the commercial detergent Fairy Liquid which is known to produce very stable foams. We used three different sizes for the triangular prisms made of thin wire (diameter 0.70 mm) with values for (c, a) of (1.6 cm, 3.2 cm), (3.2 cm, 6.4 cm) and (6.4 cm, 12.8 cm).

The dynamics of the transition was studied by inducing the instability for a fixed axial ratio c/a slightly above 0.487 by performing the following experiment. Dipping such a frame into soap solution we obtain configuration II. By blowing carefully against one of the central Plateau border junctions, in the direction of the central Plateau border (see Fig. 1), we force a transition to configuration I. This configuration is not stable and quickly returns to II. An analysis of the return transition was performed based on video images.

Focusing on the triangular face, we can monitor the shrinking of its area in time, during the relaxation to configuration II. Similarly we can observe the formation and subsequent growth of the emerging Plateau border which is perpendicular to the plane of the triangular face. We have recorded this transition using a high speed camera (PCO 1200hs) with up to 5000 frames per second. Some still frames are shown in Fig. 3: the left column shows the disappearance of the triangular face, the right column shows the appearance of the Plateau border, and the middle column shows an oblique view in which both the disappearance of the triangular face and the appearance of the Plateau border are seen.

The surface area of the triangular face in successive images was analysed using the ImageJ software¹. Fig. 4 shows data for the area of the vanishing triangular face as a function of time for our three different frame sizes. To measure the length of the emerging Plateau border, the Particle Image Velocimetry (PIV) algorithm combined with particle tracking has been applied to the image sequences. The high liquid content in the vertices (the two ends of the emerging Plateau border seen in Fig. 3(e)), allows the particle tracking algorithm to follow the end points of the Plateau border. In this way we can measure its length in each frame.

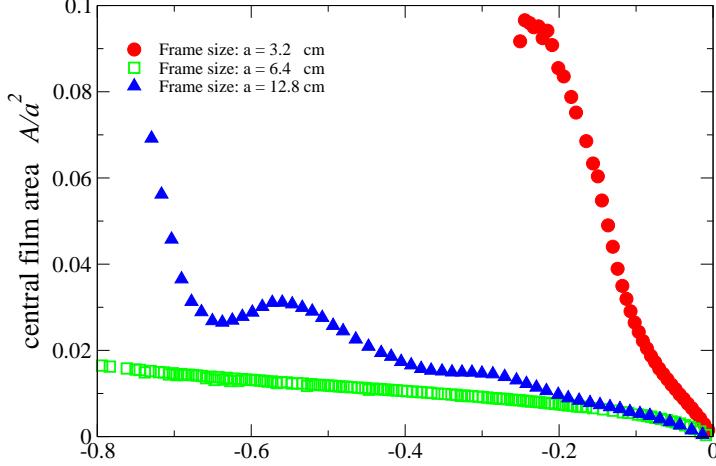
3.2. The vanishing of a triangular face

The oscillations in the initial shrinking of the triangular face, shown in 4(a), are incidental consequences of the initial disturbance by blowing. For our analysis we consider only the last 0.02 s of the existence of the triangular face, which does not depend on the history of its formation (Fig. 4(b)).

For a plastic frame with hollow struts of 4.30 mm thickness, the area of the triangular face was previously found to decrease approximately quadratically in time [5]. We have repeated this experiment using three different wire frames, including the frame size used in [5] ($(c, a) = (3.2 \text{ cm}, 6.4 \text{ cm})$), but now with thinner (solid) struts (thickness 0.7 mm), resulting in thinner Plateau borders. The time resolution in the new experiment was 100

¹ A freeware available from: <http://rsb.info.nih.gov/ij/>

(a)



(b)

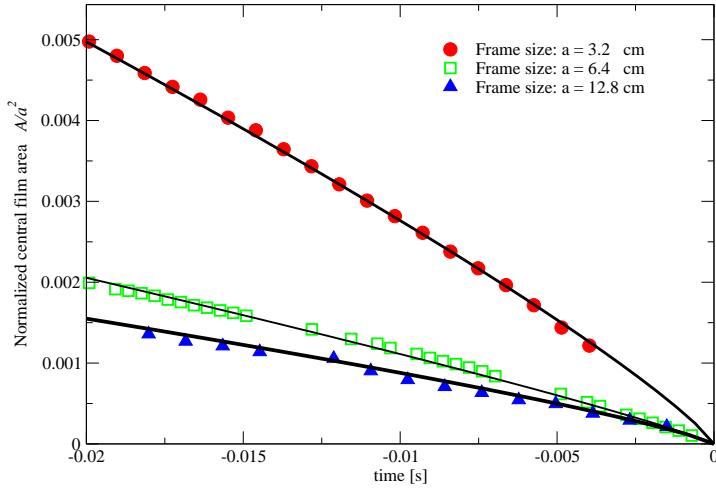


Fig. 4. (a) Three individual data sets showing the decrease in area of the triangular face as a function of time (thickness of the struts 0.7 mm). (b) Expanded region close to the transition point from configuration I to II, where we fit a power law (eqn. 1) to the data (solid line). The exponents β for the three data sets are 0.82 (edge length $a = 3.2$ cm), 0.89 ($a = 6.4$ cm) and 0.82 ($a = 12.8$ cm).

times better. Performing a total of 46 experiments, we find that close to the vanishing of the triangular face at time t_c , its normalized area A/a^2 scales with time t as

$$A(t)/a^2 = \alpha(t_c - t)^\beta, \quad (1)$$

where $\beta \simeq 0.80 \pm 0.07$, independent of the frame size (Fig. 4(b)). Such a power law behaviour is typical for critical phenomena close to a singularity. The results of our data analysis are summarised in Table 1, they also show that the prefactor α increases with

a [cm]	Glycerol conc. [volume %]	Viscosity [mPa s] (from [8])	α [$T^{-\beta}$]	β	No. of measurements
3.2	0	1.00	1.32 ± 0.19	0.820 ± 0.03	18
6.4	0	1.00	3.07 ± 0.70	0.805 ± 0.11	18
12.8	0	1.00	6.69 ± 0.60	0.790 ± 0.08	10
6.4	5	1.14	3.12 ± 0.14	0.823 ± 0.04	10
6.4	10	1.31	3.06 ± 0.07	0.834 ± 0.08	15
6.4	20	1.77	3.05 ± 0.11	0.826 ± 0.12	15
6.4	60	10.97	2.16 ± 0.09	0.773 ± 0.07	18
6.4	80	62.00	1.33 ± 0.04	0.704 ± 0.05	19

Table 1

Summary of the experimental results for the shrinkage of the area A of the triangular face with time t ($A(t)/a^2 = \alpha(t_c - t)^\beta$). Here a is the longest edge of the frame. In all our experiments the ratio c/a was 0.5

frame size.

To study the role of viscosity we repeated the experiment with the frame dimension of $(c, a) = (3.2 \text{ cm}, 6.4 \text{ cm})$ using solutions with different bulk viscosity. These were made by adding glycerol to obtain concentrations of 5, 10, 20, 60 and 80 volume percent. The resulting values of the bulk viscosity of the solutions were taken from [8].

Based on 77 experiments we find that an increase of viscosity of up to a factor 10 does not result in a change of the power law. Only higher values of the viscosity ($> 60 \text{ mPa s}$) lead to a gradual decrease of β to 0.70, see table 1. For a fixed frame size the prefactor α only decreases if the viscosity is increased by more than a factor of 10. That viscosity plays such a small role in the disappearance of the triangular face is surprising, since the topological change involves a flow of liquid and viscosity plays a major role in the drainage of Plateau borders and films. We will discuss this issue in the conclusions.

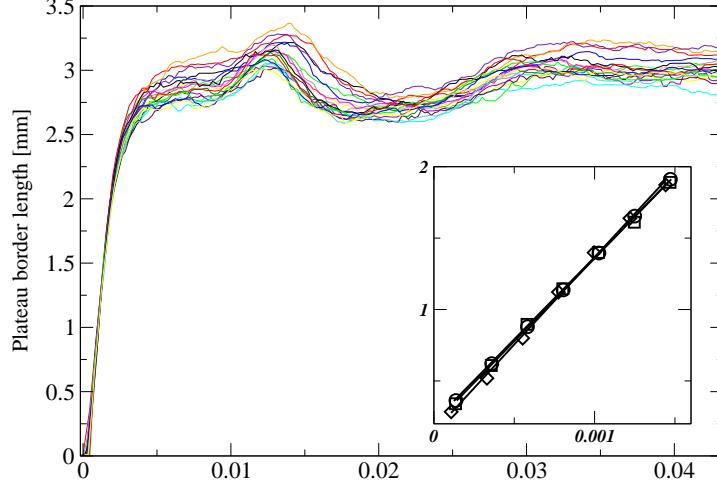
3.3. The growth of the Plateau border

Fig. 5(a) shows the growth of the Plateau border with time for the frame of size $(c, a) = (3.2 \text{ cm}, 6.4 \text{ cm})$. A linear increase of the Plateau border length, lasting 5×10^{-3} seconds, is followed by an oscillatory relaxation to its equilibrium length. We find such behaviour for all our frame sizes, see Figure 5(b). A qualitative comparison of the time scale over which the triangular face disappears and the Plateau border appears reveals that the latter takes place much more quickly (by two orders of magnitude).

Note that the Plateau border is created with a finite length (at $t = 0$ in our measurements). This is due to the surprising fact that before the triangular face disappears the surrounding films form an elongated *tunnel* which, as the triangular face finally disappears, collapses into a Plateau border which already has a finite length. We observe that gas is trapped during the collapse of this tunnel, which results in the creation of small bubbles (see Fig. 3(e)).

We found that the damped oscillations of the emerging Plateau border about its equilibrium length are more pronounced when using plastic frames with a larger strut thickness than the wire frames, resulting in thicker Plateau borders and films. Figure 6 shows

(a)



(b)

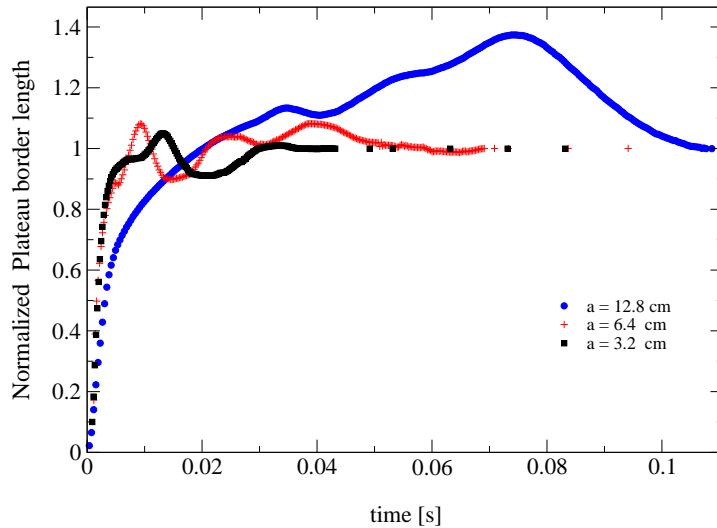


Fig. 5. (a) The increase in length of the Plateau border in a frame of size $c = 3.2$ cm, $a = 6.4$ cm consisting of wires with a thickness of 0.7 mm. A sharp linear increase is followed by a long relaxation period, with some oscillations. The inset shows linear fits to three of the data sets in the regime of initial growth. (b) The growth of the Plateau border averaged over many experiments for all three frame sizes.(The Plateau border lengths for the three different frame sizes are normalised by their respective final length.)

that the oscillations have a frequency of about 66 Hz, independent of the aspect ratio of the frame.

In order to establish whether the frequency is related to the normal mode of vibration of

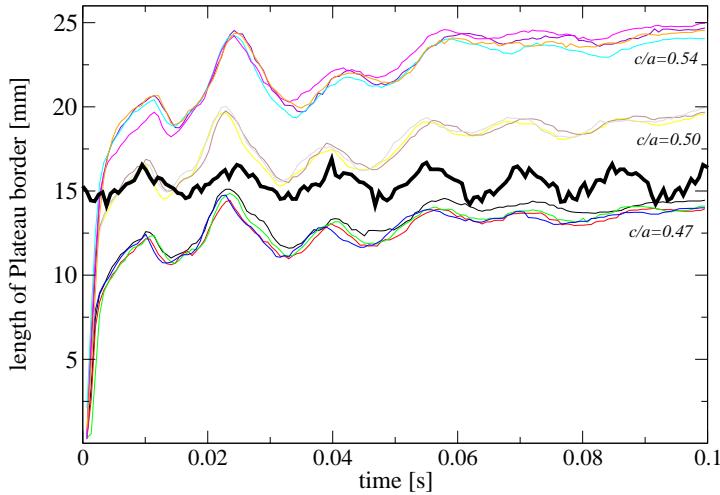


Fig. 6. The thin gray lines show the oscillations of the emerging central Plateau border (PB). The frequency is approximately 66 Hz, and is independent of the ratio c/a for the three values used in our experiments. The thick black line shows the resonance of the PB when subject to a sound wave of 67 Hz. The data was taken using plastic frames with strut thickness of 4.3 mm.

the film configuration, we placed a loudspeaker in front of the triangular prism, containing the soap films in configuration II. Performing a sweep of frequencies using a square wave sound generator we found a resonance of the central Plateau border at a frequency of about 67 Hz, i.e. consistent with the frequency of oscillation of the emerging Plateau border in the same frame. Our measurements are shown as thick solid line in Fig. 6). Since we did not find any movement of the frame when subjecting it to sound at 67 Hz *without* the soap films, we believe this frequency is an inherent property of the soap film configuration.

An estimate of the vibration frequency, based on the mass of the Plateau border and surface tension, was found to grossly exceed the observed value. It would therefore appear to be necessary to include the mass of the displaced air to obtain a frequency of the order of that observed, as for example done by [13].

We also investigated the effect of viscosity on the growth of the emerging Plateau border by varying the value of the bulk viscosity of our soap solution. Fig. 7 shows the initial 13×10^{-3} seconds of the growth of the Plateau border. Clearly the growth is slowed down with increasing viscosity. In table 2 we present the slope of the linear part of the growth (first 2×10^{-3} seconds) of the plots averaged over different measurements of the same viscosity.

Glycerol conc. [volume %]	Viscosity [mPa s] (from [8])	Growth rate [m/s]	No. of measurements
0	1.00	0.539 ± 0.08	7
20	1.77	0.379 ± 0.05	7
70	11.30	0.307 ± 0.01	7

Table 2

The initial growth rate of a Plateau border for different values of bulk viscosity.

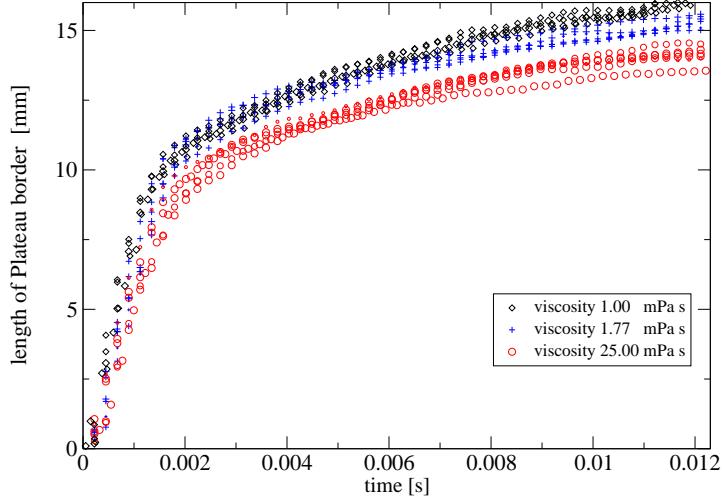


Fig. 7. The first 13×10^{-3} seconds after the formation of the PB for three different values of bulk viscosity indicates the reduction of growth-rate with an increase in viscosity. For all of the experiments in this figure we used plastic frames with a strut diameter of 4.30 mm.

4. The stability of small faces with more than three sides: further computations

We now consider prismoidal wire frames with more than three sides, and denote by n the number of sides of the frame. The pre-emptive instability occurs for other frames with up to five sides, but the critical size of the polygonal face which vanishes becomes very small (Fig. 8), and hardly worth pursuing experimentally. Fig. 9 shows Surface Evolver simulations of the two configurations for square ($n = 4$) and pentagonal ($n = 5$) prisms,

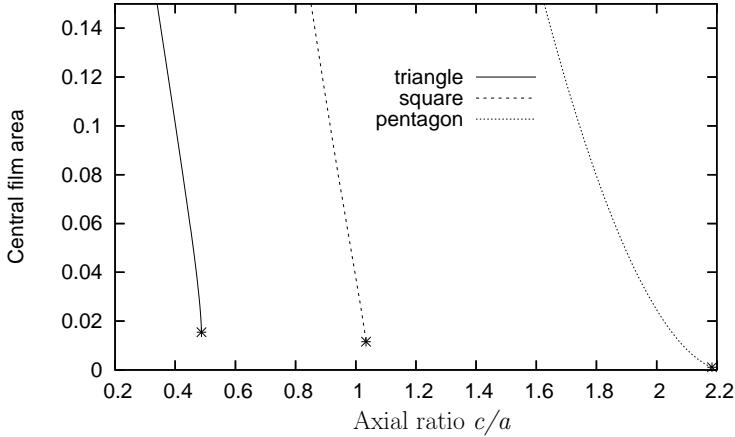


Fig. 8. As the aspect ratio of a four- or five-sided frame increases, the area of the central film shrinks in a manner analogous to the $n = 3$ case. Surface Evolver simulations show that the instability predicted by Plateau's Laws is triggered pre-emptively in these cases, while the central film retains finite area [5]. The critical area becomes smaller as the number of sides of the frame increases.

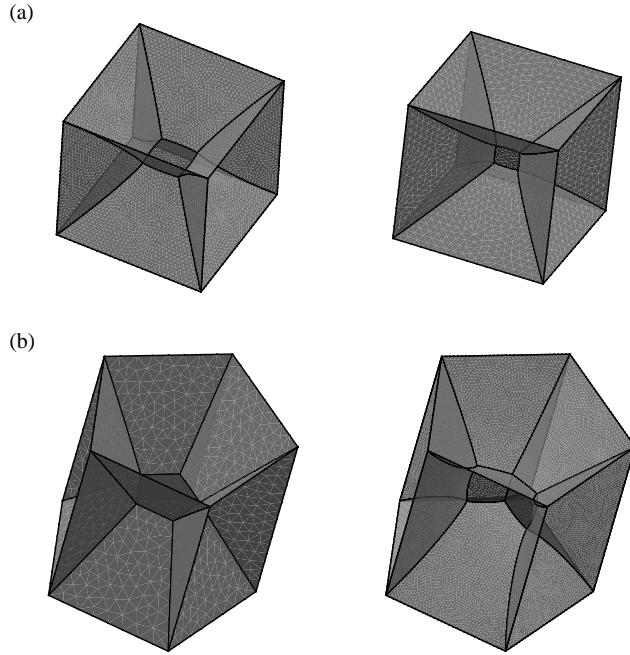


Fig. 9. Two possible configurations of soap films spanning square and pentagonal prisms. (a) Square frame, with $a = 1$ and $c = 0.960$. The critical value is $c/a = 1.043$, when the area of the square face is close to $0.01a^2$. This is the topological change familiar from the cubic frame [10], with two possibilities for the new face by symmetry. (b) Pentagonal frame, with $a = 1$ and $c = 1.60$. The critical value is $c/a = 2.184$, when the area of the pentagonal face is close to $0.001a^2$. In the second configuration, there are two vertical faces.

where the topological transition is more complex than in the case of the triangular prism. Here a is defined to be the length of each edge of the base of the frame, and c its height; in the simulations we vary c and keep a fixed and equal to one.

Beyond $n = 5$, the scenario changes. The case $n = 6$ is marginal and we will return to it below. For $n > 6$, the configuration with the single central film, which is analogous to that of Fig. 1 (configuration I), remains stable for an indefinite increase of the aspect ratio, and the area of the film tends to a constant as $c/a \rightarrow \infty$. This is because the adjoining angled Plateau borders connect to the sides rather than the corners of the frame (see Fig. 10). We have verified this behaviour experimentally in the case $n = 8$, for a frame made of thin copper wire (Fig. 10).

For the marginal case $n = 6$ (see Fig 11(a)), the edges at the ends of the frame make an angle of 120° . The Plateau borders now remain attached to the corners, but make an angle with the edges of the frame which tends to zero as $c \rightarrow \infty$. The calculations presented in Fig. 11 indicate that the area of the central face goes to zero as c^{-4} , in agreement with the calculations of Huff [6,11].

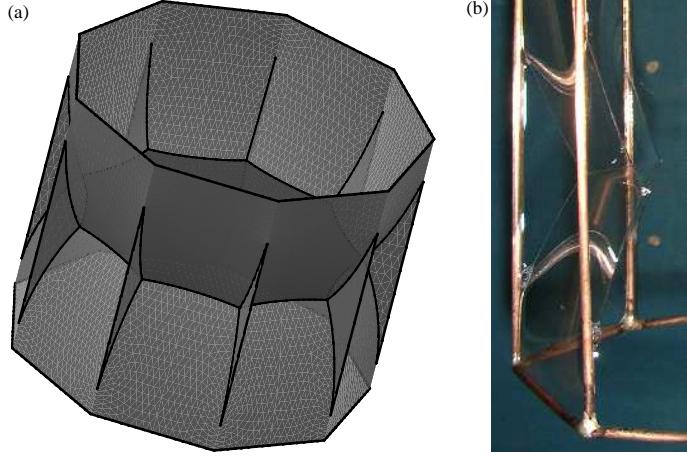


Fig. 10. (a) The (only) soap film configuration that spans a frame with nine sides, from simulation, with $c/a = 2.30$. This configuration remains stable when the aspect ratio is increased because of the way in which the Plateau borders join the sides of the frame rather than the corners. (b) Image from an experiment with an eight-sided frame ($c = 120\text{mm}$, $a = 40\text{mm}$), magnifying the region where the Plateau border meets the frame.

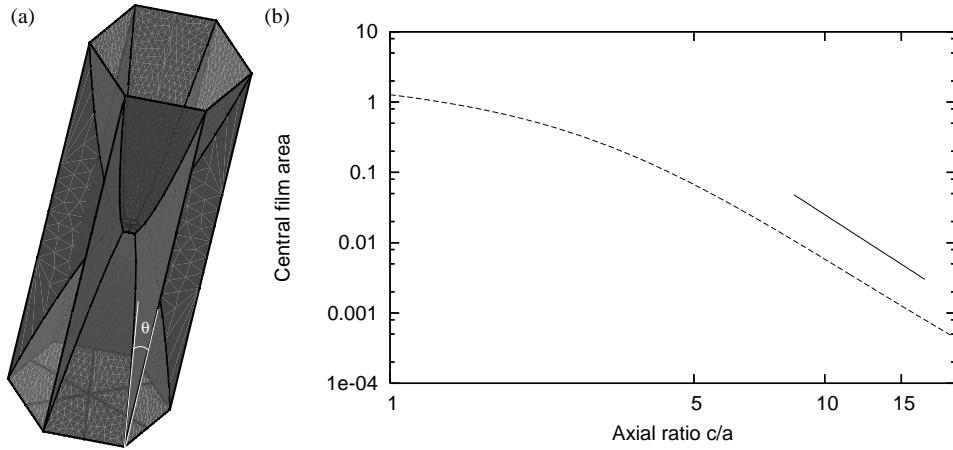


Fig. 11. (a) The (only) soap film configuration that spans a frame with six sides, from simulation, with $c/a = 5.70$. The angle that the corner of the film makes where it meets the frame, marked θ , decreases to zero as $c/a \rightarrow \infty$. (b) As the aspect ratio is increased the area of the central face tends to zero. Note that the data is plotted on a double logarithmic scale; the solid line is provided as reference and has a slope of -4 .

5. Conclusions

Interest in the physics of foams has moved from static to dynamic properties [12], posing fresh challenges for both theory and experiment. In this work we have taken a case which, in addition to its intrinsic appeal, offers a good test for future theory. At present this is limited as much by our lack of understanding of the physical factors that are at work as by technical problems of simulation or experiment.

Our experimental data suggests a different value for the exponent β in equation 1, which governs the disappearance of a triangular face, than was previously reported [5]. Power laws close to a critical point are known to be very hard to establish from experimental data. Here it is important to note that we have increased the time resolution of our new measurements by a factor of at least 100, so we expect the current data to be better suited for power law fits. Also, we have now used frames consisting of solid struts, while previously we used hollow struts. Soap solution was trapped in the hollow struts, and consequently released, which is no longer possible in the current set-up.

The topological transition was triggered from about 10 seconds to a minute after the formation of the soap film configuration (the films burst after about one minute). We found that both the values of pre-factor α and exponent β of equation 1 remained almost constant, without a clear trend. This indicates that gravitationally induced drainage did not play a role in our experiments. As expected for a power law relation, the exponent β is also independent of the size of the frame, only the coefficient α scales with frame size. Future experiments should also examine the role of the thickness of the struts of the frame, and thus the role of the thickness of the Plateau borders connecting struts and films.

The effect of liquid viscosity on the time scale of the topological change requires closer inspection. While viscosity plays a small role in the disappearance of the triangular face, it clearly slows down the growth of the emerging Plateau border. This could reflect the different time scales involved in the two processes that make up the topological change. The power laws fits for the change in area of the triangular face were obtained for a time range of about 20×10^{-3} s, whereas the linear growth of the Plateau border happens within about 1.5×10^{-3} s.

Recently Durand and Stone [14] studied the dynamics of a topological change for a two-dimensional foam, i.e. a single layer of bubbles confined between two glass plates, with a spacing of 1 cm. In particular, they recorded the growth of a film in time, after a topological change (figure 3 of [14]). Their surprising finding was that the bulk liquid viscosity plays a minor role in comparison with surface viscoelasticity. The time scale for the topological change was of the order of seconds, i.e. much larger than in our experiments. This is another clear indication of the role of friction [9], imposed by the confining plates in such 2d foam experiments.

In the light of these observations it is clear that future work should focus on obtaining a better understanding of the various dissipative mechanism that occur in the course of a (3D) topological change. In particular this should include an experimental study of the role of surface viscosity and theorizing on the value of the exponent β . The application of sound waves to excite soap film configuration (as was demonstrated in this work, see also [15]), and possibly trigger topological changes, might prove as a useful experimental tool.

Appendix A

It is evident from Fig. 2 and the previous discussion of Hutzler *et al.* [5] that the central film area A , as a function of c/a , has a re-entrant form, with a branch of unstable solutions for which A increases with c/a . It is a common phenomenon in physics that solutions for which some parameter has the “wrong” dependence on a control variable

are unstable, and that this can be proven. Here we offer a demonstration for the present case.

Consider an equilibrium surface of energy (total area) E_1 for a certain value of c/a , the axial ratio of the frame, and another of energy E_2 for a frame of slightly greater c/a and the same volume. These are to be superimposed, with the same orientation and central point. Concentrating on the curved part of the surface, we see that under the stated condition, these two surfaces must cross somewhere. It is therefore possible to construct two new trial solutions for cases 1 and 2, by cutting up and rejoining the surfaces, with surface 1 either coincident with or lying outside surface 2 everywhere. The energies of the new surfaces are distinguished by primes. In the stated procedure the combined total area is conserved, so

$$E'_1 + E'_2 = E_1 + E_2 \quad (2)$$

It follows that it is impossible that both of the new trial solutions have energies greater than the original ones, and so the latter cannot both be (strictly) stable. Obviously it follows that no solution on the anomalous branch can be stable. (Strictly speaking, the argument as it stands leaves room for a single stable state, but this presumably can be dispensed with, by considerations of continuity.)

Acknowledgements:

This research was supported by the European Space Agency (contracts 14914/02/NL/SH and 14308/00/NL/SH), Science Foundation Ireland. (RFP 05/REP/PHY00/6), and the EU program COST P21 (The Physics of droplets). SJC acknowledges support from EPSRC (EP/D071127/1). MS is supported by the Irish Higher Education Authority (PRTLI-IITAC). We also wish to thank the reviewers for the critical reading of the manuscript.

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