

Comment on “Foam imbibition in microgravity. An experimental study” by H. Caps, H. Decauwer, M.-L. Chevalier, G. Soyez, M. Ausloos and N. Vandewalle

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Abstract. In a recent paper, Caps et al. [1] described parabolic flight experiments showing the movement of liquid into the foam during the microgravity phase. In this comment, we present a detailed theory of this process, supported by numerical calculations, confirming their conclusion that the wetting front moves with the square root of time. We further show that this diffusion process is similar for different surfactant systems, which allows us to provide bounds on the value of the diffusion coefficient.

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The properties of aqueous foams are particularly well understood, at least in the limit of low liquid fraction [2]. Much of this understanding is due to the development of *drainage equations*, which describe the motion of liquid through a foam under various conditions [3, 4, 5]. In the case of microgravity, analytic solutions for the diffusive spreading of a pulse of liquid have been given [6] and solutions for various other experimental situations of interest also exist [7].

Caps et al. [1] performed experiments on aqueous foams during the microgravity phase of a parabolic flight, similar to those first described by Noever and Cronise [8]. Their results show that liquid moves rapidly upwards into the foam when the acceleration due to gravity is reduced, in broad agreement with their analysis of a drainage equation. This “capillary wetting” is a well-known phenomenon, even under terrestrial condi-

tions [6], but it is more pronounced, and more amenable to analysis, when viewed in microgravity conditions.

We will expand upon the details of the theoretical approach to analyzing such capillary-rise experiments using drainage equations. The effects of surface rheology on the drainage process are poorly understood, leading us to consider two limiting cases which provide bounds on the rapidity with which liquid moves into the foam. We complement the analysis of Caps et al. [1] by providing detailed scaling laws for the position of the wetting front that invades a dry foam, without recourse to free fitting parameters.

The standard drainage models are straightforwardly adapted to the microgravity case. They are expressed in nonlinear diffusion equations which describe the spreading of liquid through the network of narrow channels, or Plateau borders (PBs) that make up the bulk of the foam’s liquid [7]. We shall consider two one-dimensional diffusion equations, representing PB interfaces that are *rigid*

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or *mobile* [9, 10] due to the surface chemistry of the surfactant solution used to produce the foam. The former equation is based upon Poiseuille flow through the PBs, while the latter assumes plug flow in the PBs, with viscous dissipation occurring in the vertices where they meet. These are recognised as two limiting cases, with real foams often lying close to one or other of the limits. However, the details of the interpolation between these limits remains to be fully understood.

Our measure of liquid content is the cross-sectional area A of a PB, directly analogous to the channel-width parameter l used in [1]: $l = (8A/(4 - \pi))^{1/2}$.

An important change to the familiar drainage equations is entailed by the change in geometry – the experiments were two-dimensional (i.e. a single layer of bubbles), so the liquid-transporting PBs are pressed against the walls of the Hele-Shaw cell; we shall refer to them as surface PBs.

The diffusion equations express the variation of PB area A with position x and time t . We denote the liquid surface tension by σ and its viscosity by η , using the values given in [1]: $\sigma = 3.0 \times 10^{-2} \text{N/m}$ and $\eta = 1 \times 10^{-3} \text{Ns/m}^2$. The Hele-Shaw cell is 2mm thick and for our purposes it is sufficient to take parameter values for a monodisperse, hexagonal bubble structure: the length of each surface PB, $L = 1 \text{mm}$, is taken as the relevant length-scale. The liquid fraction Φ_l is then approximately $1.35A/L^2$.

The time-scale is given by

$$T = \frac{\eta L}{\delta \sigma K}$$

where the geometrical constant is $\delta = (\frac{1}{2}(4 - \pi))^{1/2} \approx 0.655$ while the permeability constant K depends upon the model. In the rigid-interface case our calculations [11] suggest that K varies little between the bulk and the surface Plateau borders. We replace the factor of $\frac{1}{3}$, due to averaging the possible orientations of a PB, with a factor of $\frac{1}{2}$, since we are now averaging in two dimensions, and use the value $K_c = 9.9 \times 10^{-3}$. The value of K in the mobile interface case is obtained empirically [10, 12] for bulk vertices (agreeing with a naive order-of-magnitude estimate based upon flow through a packed bed of spheres [10]), and in the absence of experi-

ments on the dissipation in surface vertices, we must here assume that it also does not vary greatly and use $K_n = 2.3 \times 10^{-3}$ for these large bubbles. The relevant time-scales are then $T_c = 5.14 \times 10^{-3} \text{s}$ and $T_n = 2.21 \times 10^{-2} \text{s}$.

In each case we reduce the equations to dimensionless form, using $\alpha = A/L^2$, $\xi = x/L$ and $\tau = t/T$, to give

$$\frac{\partial \alpha}{\partial \tau} = \frac{\partial}{\partial \xi} \left(\frac{\sqrt{\alpha}}{2} \frac{\partial \alpha}{\partial \xi} \right) \quad \text{RIGID-INTERFACES} \quad (1)$$

and

$$\frac{\partial \alpha}{\partial \tau} = \frac{\partial}{\partial \xi} \left(\frac{1}{2} \frac{\partial \alpha}{\partial \xi} \right) \quad \text{MOBILE-INTERFACES} \quad (2)$$

which is the diffusion equation.

The experiment of interest is that of a dry foam which is wetted at one end. The capillary forces will cause liquid to move into the foam. We set $\alpha(\xi = 0, \tau) = \alpha_0$, where the parameter $\alpha_0 \approx 35\%$ represents the critical liquid fraction at which the foam becomes a bubbly liquid. The appropriate solutions of (1) and (2) are shown in figure 1 and explained in the following sections. In each case we extract a diffusion coefficient and compare it with the one measured by [1] in their experiments: $D_w = 1.19 \text{cm}^2 \text{s}^{-1}$.

Rigid Interfaces

The most straightforward way to solve (1) is to look for a similarity solution, as Koehler et al. [6] did for the case where gravity is included. This is necessarily of the form

$$\alpha(\xi, \tau) = y(s), \quad s = \frac{\xi}{\tau^{1/2}}. \quad (3)$$

This shows that the wetting front moves in the positive ξ -direction with a rate proportional to the square root of time. The similarity function y satisfies

$$s y' + \frac{2}{3} \left(y^{3/2} \right)'' = 0. \quad (4)$$

The amount of liquid in the foam also increases with $\sqrt{\tau}$. The diffusion coefficient is

$$D = \frac{L^2}{T_c} = \frac{\sigma L K_c \delta}{\eta} \approx 1.94 \text{cm}^2 \text{s}^{-1}.$$

This is larger than the experimental value, and provides an upper bound. A numerical solution of (4) for the liquid fraction, in dimensional variables, is shown in figure 1, scaled according to (3).

Mobile Interfaces

This is the case considered by Caps et al. [1]; here we give the analytic solutions for the liquid fraction and edge-length, and include the parameters appropriate for the two-dimensional geometry.

The appropriate scaling is again given by (3) and the solution of (2) is an error function:

$$\alpha(\xi, \tau) = \alpha_0 \operatorname{erfc}\left(\frac{\xi}{\sqrt{2\tau}}\right). \quad (5)$$

This is shown in figure 1; comparison with the case of rigid interfaces suggests that liquid moves more quickly into the dry foam here, since the foam is made slightly wet throughout immediately. The diffusion coefficient is

$$D = \frac{L^2}{T_n} = \frac{\sigma L K_n \delta}{\eta} \approx 0.452 \text{cm}^2 \text{s}^{-1},$$

which is a lower bound for the diffusivity of the wetting process. It should be compared with the theoretical value determined by Caps et al. [1], $D \approx 0.72 \text{cm}^3 \text{s}^{-1}$, which neglects the actual two-dimensional geometry of the foam sample.

The volume of liquid in the foam also increases in proportion to the square root of time. Moreover, the solution shows that at a given position in the foam, the edge-width l increases as

$$l \sim \operatorname{erfc}^{1/2}\left(t^{-1/2}\right)$$

which is rather more complicated than the ansatz adopted in eq. (1) of Caps et al. [1].

Summary

For both of the equations studied, the liquid advances into the dry foam with the square root of time, showing generic diffusion behaviour. In neither case is there a sharp wetting front; identification of such a front in an

experiment is therefore rather arbitrary and it would be better to fit a measured profile, wherever possible.

The diffusion coefficient measured in the experiments, $D_w = 1.19 \text{cm}^2 \text{s}^{-1}$, lies midway between the values calculated here. Our upper bound, $D = 1.94 \text{cm}^2 \text{s}^{-1}$ is a factor of four greater than the lower bound $D = 0.452 \text{cm}^2 \text{s}^{-1}$. It is not clear to which limit, if either, the surfactant solution used in the experiments should belong.

Microgravity experiments on foams present many new opportunities for studying the behaviour of these ubiquitous and remarkably useful materials. In the next generation of such experiments, it is to be hoped that a greater control over factors such as bubble size (the ability to obtain monodisperse foams for example) and the influence of vibration will be possible. As we have shown, there is already a close correspondence between theory and experiment, but it is to be hoped that carefully performed experiments will lead to further advances in theory. Moreover, a good choice of experiment, such as the constant addition of liquid to a dry foam at a point, should allow for the identification of the correct theoretical model to apply, as determined by the surface chemistry of the surfactant solution.

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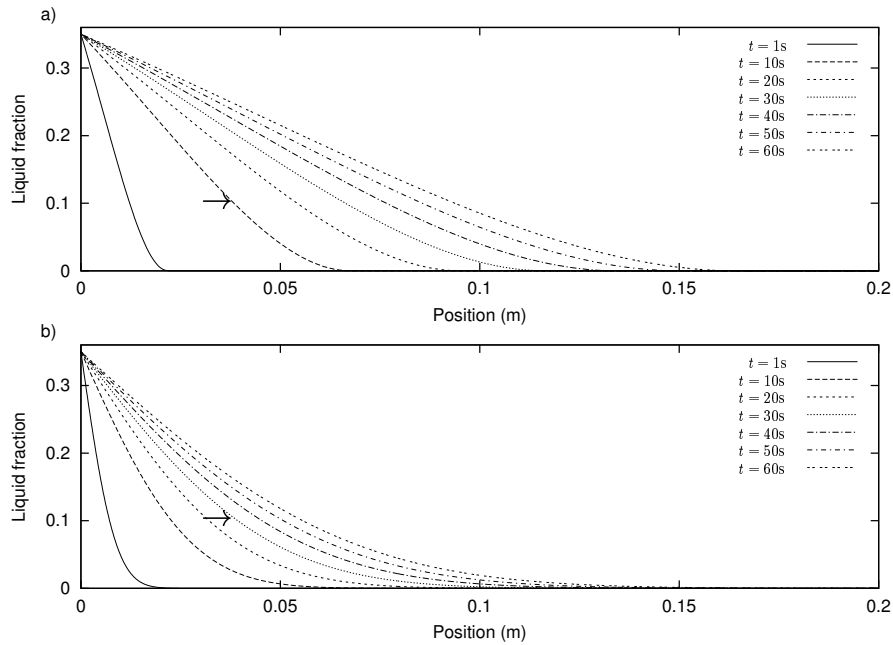


Fig. 1. The zero-gravity wetting from one end, with $\alpha_0 = 35\%$, of a one-dimensional dry foam. a) Numerically obtained profiles for a foam with rigid interfaces are shown at various times t . b) For the case of mobile interfaces, profiles are obtained from (5). The volume of liquid in the foam increases with the square root of time in both cases, but the constant of proportionality is lower in the mobile-interface case.

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