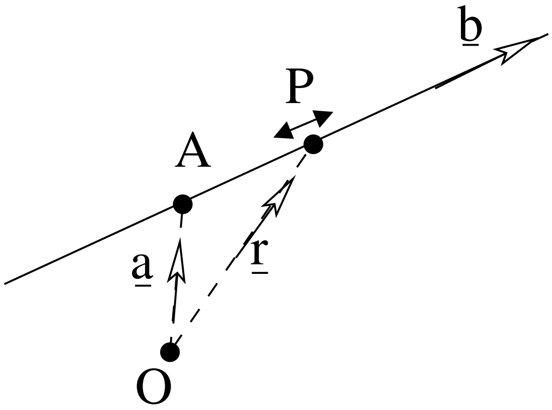
2.3 Vector equation of a straight line

Consider a line which passes through a point A with position vector $\underline{a}$, parallel to a given direction $\underline{b}$, in three dimensional space. Let P be any point on this line, with position vector $\underline{r}$.



Since $\overrightarrow{AP}$ is parallel to $\underline{b}$, it is equal to $\lambda\underline{b}$ for some $\lambda \in \mathbb{R}$. Then the equation of the line is

$$\underline{r} = \overrightarrow{OP} = \overrightarrow{OA} + \overrightarrow{AP} = \underline{a} + \lambda\underline{b}.$$

For each value of λ we get a different point on the line; we say that λ *parametrizes* the line. In components, we have that the position of any point (x, y, z) on the line is given by $x = a_1 + \lambda b_1$, $y = a_2 + \lambda b_2$ and $z = a_3 + \lambda b_3$.

We can find the line through two points A, at $a_1\underline{i} + a_2\underline{j} + a_3\underline{k}$, and B, at $b_1\underline{i} + b_2\underline{j} + b_3\underline{k}$, in the same way, since the line is parallel to

$$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = (b_1 - a_1)\underline{i} + (b_2 - a_2)\underline{j} + (b_3 - a_3)\underline{k}.$$

Example 2.8. *Find the vector equation of the line through points A and B with position vectors $\underline{a}$ and $\underline{b}$ respectively.*

Example 2.9. *Give the vector equation of the line through A, with position vector $-3\underline{i} + 2\underline{j} - 3\underline{k}$, and B, with position vector $\underline{i} - \underline{j} + 4\underline{k}$.*

Example 2.10. *Find the position vector of the point of intersection of the two lines $\underline{r} = \underline{i} + \underline{j} - \underline{k} + 2t\underline{j}$ and $\underline{r} = \underline{i} + s\underline{k}$, where t and s parametrize the lines.*

Example 2.11. *Given two distinct non-zero vectors $\underline{a}$ and $\underline{b}$, find the position vector of the point of intersection of the two lines $\underline{r} = \underline{a} + t\underline{b}$ and $\underline{r} = (2\underline{a} + b) + s(\underline{a} - \underline{b})$, where t and s parametrize the lines.*