

MT10110 Aseiniad 3: Fectorau – Llinellau a Lluosymiau
DATRYSIADAU

1. (a) $\overrightarrow{AB} = \underline{b} - \underline{a}$; (b) $\overrightarrow{AC} = \frac{3}{4}(\underline{b} - \underline{a})$; (c) $\overrightarrow{CB} = \frac{1}{4}(\underline{b} - \underline{a})$; (d) $\overrightarrow{OC} = \underline{a} + \frac{3}{4}(\underline{b} - \underline{a}) = \frac{1}{4}\underline{a} + \frac{3}{4}\underline{b}$. [2,2,2,2]
2. (a) $\overrightarrow{OB} = \underline{c} + 3\underline{a}$; (b) $\overrightarrow{AB} = -\underline{a} + \underline{c} + 3\underline{a} = 2\underline{a} + \underline{c}$;
(c) $\overrightarrow{OD} = \underline{a} + \frac{1}{2}\overrightarrow{AB} = 2\underline{a} + \frac{1}{2}\underline{c}$; (d) $\overrightarrow{CD} = 3\underline{a} - \frac{1}{2}\overrightarrow{AB} = 2\underline{a} - \frac{1}{2}\underline{c}$. [2,2,2,2]
3. (a) $\overrightarrow{CA} = \overrightarrow{CO} + \overrightarrow{OA} = -\underline{c} + \underline{a}$;
(b) $\overrightarrow{AB} = \overrightarrow{AC} + \overrightarrow{CB} = \underline{c} - \underline{a} + 2\underline{a} = \underline{c} + \underline{a}$;
(c) $\overrightarrow{ED} = \overrightarrow{EB} + \overrightarrow{BD} = \frac{1}{2}\overrightarrow{CB} + \frac{1}{2}\overrightarrow{BA} = \frac{1}{2}(2\underline{a}) + \frac{1}{2}(-\underline{c} - \underline{a}) = \frac{1}{2}(-\underline{c} + \underline{a})$.
Felly mae $\overrightarrow{CA} = 2\overrightarrow{ED}$, fel sydd ei angen.
4. (a) $\overrightarrow{OC} = \frac{1}{2}\underline{b}$, $\overrightarrow{OD} = \underline{a} + \frac{1}{2}\overrightarrow{AB} = \frac{1}{2}\underline{a} + \frac{1}{2}\underline{b}$, ac $\overrightarrow{AC} = -\underline{a} + \frac{1}{2}\underline{b}$.
Yna gallwn gyfrifo $\overrightarrow{AF}$ mewn dwy ffordd: $\overrightarrow{AF} = h(-\underline{a} + \frac{1}{2}\underline{b})$ ac $\overrightarrow{AF} = -\underline{a} + k(\frac{1}{2}\underline{a} + \frac{1}{2}\underline{b})$.
Mae cymharu cyfernodau o $\underline{a}$ a $\underline{b}$ yn rhoi $-h = -1 + \frac{k}{2}$ a $h = k$, sy'n rhoi $h = k = \frac{2}{3}$.
(b) $\overrightarrow{BF} = -\underline{b} + \frac{2}{3}\overrightarrow{OD} = \frac{1}{3}\underline{a} - \frac{2}{3}\underline{b} = \frac{2}{3}(\frac{1}{2}\underline{a} - \underline{b}) = \frac{2}{3}\overrightarrow{BE}$, fel bod $\overrightarrow{BF}$ a $\overrightarrow{BE}$ yn baralel ac felly mae B , F , ac E yn gyddlinol. [6,4]
5. (a) $\overrightarrow{AB} = -\underline{i} - \underline{j} + 2\underline{k}$; (b) $|\overrightarrow{AB}| = \sqrt{1^2 + 1^2 + 2^2} = \sqrt{6}$; (c) $\hat{v} = -\frac{1}{\sqrt{6}}\underline{i} - \frac{1}{\sqrt{6}}\underline{j} + \frac{2}{\sqrt{6}}\underline{k}$;
(d) $\underline{r} = 2\underline{i} + 3\underline{j} - 5\underline{k} + \lambda(-\underline{i} - \underline{j} + 2\underline{k}) = (2 - \lambda)\underline{i} + (3 - \lambda)\underline{j} + (2\lambda - 5)\underline{k}$. [2,2,2,2]
6. (a) $\overrightarrow{AB} = 4\underline{i} + 3\underline{j}$, felly $\underline{r} = 2\underline{i} + 4\underline{j} + \lambda(4\underline{i} + 3\underline{j}) = (2 + 4\lambda)\underline{i} + (4 + 3\lambda)\underline{j}$. Pan mae $\lambda = 3$, mae gennym bod $\underline{r} = 14\underline{i} + 13\underline{j}$ fel sydd ei angen.
(b) $\overrightarrow{OC} = \mu(3\underline{i} - 4\underline{j})$; mae'n orthogonol i $\overrightarrow{AB}$ os yw $(3\underline{i} - 4\underline{j}) \cdot (4\underline{i} + 3\underline{j}) = 0$, a dyma'r achos.
(c) Mae'r llinellau'n croestorri pan fod $2\underline{i} + 4\underline{j} + \lambda(4\underline{i} + 3\underline{j}) = \mu(3\underline{i} - 4\underline{j})$; ac yna $2 + 4\lambda = 3\mu$ a $4 + 3\lambda = -4\mu$, felly $\lambda = -\frac{4}{5}$ a $\mu = -\frac{2}{5}$ ac mae gan N y fector safle $-\frac{6}{5}\underline{i} + \frac{8}{5}\underline{j}$.
Hyd $\overrightarrow{ON}$ yw $\sqrt{\frac{6^2 + 8^2}{25}} = 2$, a dyma'r pellter perpendicwlar o AB oddi wrth O .
7. $\overrightarrow{BA} = -\underline{b} + \underline{a}$, ac felly'r llinell yma yw $\underline{r} = \underline{a} + \lambda_1(\underline{a} - \underline{b}) = (1 + \lambda_1)\underline{a} - \lambda_1\underline{b}$.
 $\overrightarrow{CB} = 3\underline{a} - 2\underline{b}$, ac felly'r llinell yw $\underline{r} = -3\underline{a} + \lambda_2(3\underline{a} - 2\underline{b}) = 3(\lambda_2 - 1)\underline{a} - 2\lambda_2\underline{b}$.
Maent yn croestorri pan fod $(1 + \lambda_1)\underline{a} - \lambda_1\underline{b} = 3(\lambda_2 - 1)\underline{a} - 2\lambda_2\underline{b}$, sy'n rhoi $\lambda_1 = 8$, $\lambda_2 = 4$.
Mae hwn yn rhoi fector safle y pwynt L fel $\underline{a} + 8(\underline{a} - \underline{b}) = 9\underline{a} - 8\underline{b}$. [6,4,2]
8. $\overrightarrow{AC} = -5\underline{a} + 5\underline{b}$, felly $\underline{r} = 3\underline{a} + 2\underline{b} + \lambda(\underline{b} - \underline{a}) = (3 - \lambda)\underline{a} + (2 + \lambda)\underline{b}$.
Rydym angen dangos fod B yn gorwedd ar $\overrightarrow{AC}$: rhowch $\lambda = 4$ i gael $\underline{r} = -\underline{a} + 6\underline{b}$, sef B .
 $\overrightarrow{AB} = -4\underline{a} + 4\underline{b}$ ac $\overrightarrow{BC} = -\underline{a} + \underline{b}$, so $\overrightarrow{AB} = 4\overrightarrow{BC}$ ac felly $AB : BC = 4 : 1$. [3,2,2]
9. Ysgrifennwn $\underline{a} = a_1\underline{i} + a_2\underline{j} + a_3\underline{k}$, $\underline{b} = b_1\underline{i} + b_2\underline{j} + b_3\underline{k}$ a $\underline{c} = c_1\underline{i} + c_2\underline{j} + c_3\underline{k}$.
(a) Felly, gan fod adiad sgalarau yn gymudol, gallwn ysgrifennu
$$\underline{c} + \underline{b} = (c_1 + b_1)\underline{i} + (c_2 + b_2)\underline{j} + (c_3 + b_3)\underline{k} = (b_1 + c_1)\underline{i} + (b_2 + c_2)\underline{j} + (b_3 + c_3)\underline{k} = \underline{b} + \underline{c}.$$

(b) Yn yr un ffordd, gan fod sgalarau yn gysylltiadol, gallwn ysgrifennu

$$\begin{aligned}(\underline{a} + \underline{b}) + \underline{c} &= ((a_1 + b_1) + c_1)\underline{i} + ((a_2 + b_2) + c_2)\underline{j} + ((a_3 + b_3) + c_3)\underline{k} \\&= (a_1 + (b_1 + c_1))\underline{i} + (a_2 + (b_2 + c_2))\underline{j} + (a_3 + (b_3 + c_3))\underline{k} \\&= \underline{a} + (\underline{b} + \underline{c}).\end{aligned}$$

(c) Yn olaf

$$\lambda(\underline{a} + \underline{b}) = \lambda(a_1 + b_1)\underline{i} + \lambda(a_2 + b_2)\underline{j} + \lambda(a_3 + b_3)\underline{k} = (\lambda a_1 + \lambda b_1)\underline{i} + (\lambda a_2 + \lambda b_2)\underline{j} + (\lambda a_3 + \lambda b_3)\underline{k} = \lambda \underline{a} + \lambda \underline{b}.$$

[4,4,4]

10. Os yw $\underline{a} = a_1\underline{i} + a_2\underline{j} + a_3\underline{k}$, $\underline{b} = b_1\underline{i} + b_2\underline{j} + b_3\underline{k}$ a $\underline{c} = c_1\underline{i} + c_2\underline{j} + c_3\underline{k}$, yna mae

- (a) $\underline{a} \cdot \underline{b} = a_1b_1 + a_2b_2 + a_3b_3 = b_1a_1 + b_2a_2 + b_3a_3$, gan fod sgalarau yn gymudol; mae hwn yn hafal i $\underline{b} \cdot \underline{a}$ fel sydd ei angen.
- (b) $\underline{a} \cdot (\underline{b} + \underline{c}) = a_1(b_1 + c_1) + a_2(b_2 + c_2) + a_3(b_3 + c_3) = a_1b_1 + a_1c_1 + a_2b_2 + a_2c_2 + a_3b_3 + a_3c_3 = a_1b_1 + a_2b_2 + a_3b_3 + a_1c_1 + a_2c_2 + a_3c_3 = \underline{a} \cdot \underline{b} + \underline{a} \cdot \underline{c}$.
- (c) Ystyriwch y gydran $\underline{i}$ o $\underline{a} \wedge (\underline{b} + \underline{c})$, gan fod pob cydran yn debyg. Mae gennym $[\underline{a} \wedge (\underline{b} + \underline{c})]_{\underline{i}} = a_2(b_3 + c_3) - a_3(b_2 + c_2) = a_2b_3 + a_2c_3 - a_3b_2 - a_3c_2 = (a_2b_3 - a_3b_2) + (a_2c_3 - a_3c_2) = [\underline{a} \wedge \underline{b}]_{\underline{i}} + [\underline{a} \wedge \underline{c}]_{\underline{i}}$.

$$11. |\underline{a}| = \sqrt{5^2 + 1^2 + 2^2} = \sqrt{30}, |\underline{b}| = \sqrt{2^2 + 3^2 + 1^2} = \sqrt{14}, \underline{a} \cdot \underline{b} = 10 + 3 + 2 = 15; \text{ so } \cos \theta = \frac{15}{\sqrt{30}\sqrt{14}} = \sqrt{\frac{15}{28}}.$$

$$12. (a) |\underline{a}| = \sqrt{x^2 + y^2} = 5 \Rightarrow x^2 + y^2 = 25.$$

$$(b) x = 3\lambda \text{ ac } y = 2\lambda \text{ felly } \frac{y}{x} = \frac{2}{3} \Rightarrow y = \frac{2}{3}x.$$

$$(c) \underline{a} \cdot \underline{b} = 3x + 2y = 0, \text{ felly } y = -\frac{3}{2}x.$$

$$(d) (\underline{a} - \underline{b}) \cdot (4\underline{i} + \underline{j}) = ((x - 3)\underline{i} + (y - 2)\underline{j}) \cdot (4\underline{i} + \underline{j}) = 4(x - 3) + y - 2 = 4x + y - 14 = 0 \Rightarrow y = 14 - 4x.$$

[2,2,2,4]

$$13. (a) \underline{a} \cdot \underline{b} = 2 - 2 + 3 = 3; \quad (b) \underline{b} \wedge \underline{c} = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 2 & 1 & -1 \\ 1 & 3 & -2 \end{vmatrix} = \underline{i} + 3\underline{j} + 5\underline{k};$$

$$(c) \underline{a} \cdot (\underline{b} \wedge \underline{c}) = 1 - 6 - 15 = -20; \quad (d) \underline{a} \wedge (\underline{b} \wedge \underline{c}) = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 1 & -2 & -3 \\ 1 & 3 & 5 \end{vmatrix} = -\underline{i} - 8\underline{j} + 5\underline{k}.$$

14. Ysgrifennwn $\underline{a} = \underline{i} + 2\underline{j} + 2\underline{k}$ a $\underline{b} = 2\underline{i} - 3\underline{j} + 6\underline{k}$. Yna $\hat{\underline{a}} = \frac{1}{3}\underline{i} + \frac{2}{3}\underline{j} + \frac{2}{3}\underline{k}$, a

$$\text{proj}_{\hat{\underline{a}}} \underline{b} = \frac{\underline{a} \cdot \underline{b}}{|\underline{a}|} \hat{\underline{a}} = \frac{2 - 6 + 12}{3} \hat{\underline{a}} = \frac{8}{3} \hat{\underline{a}} = \frac{8}{9} \underline{a} = \frac{8}{9} \underline{i} + \frac{16}{9} \underline{j} + \frac{16}{9} \underline{k}.$$

$$15. \text{Cyfaint} = |\underline{a} \cdot (\underline{b} \wedge \underline{c})|. \quad \underline{b} \wedge \underline{c} = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 1 & 3 & 4 \\ 2 & 1 & -2 \end{vmatrix} = -10\underline{i} + 10\underline{j} - 5\underline{k}, \text{ so } \underline{a} \cdot (\underline{b} \wedge \underline{c}) = -30 - 20 + 5 = -45 \text{ a'r cyfaint yw}$$

45.