MA10110 Assignment 4: Vectors – Planes and Kinematics
Attempt all questions

These questions will not be assessed directly. Instead, you should answer the multiple choice test on Blackboard.

1. Give the equation of the plane passing through the point $(0, 0, 0)$, normal to $-\underline{k}$. [1]

2. Find the equation of the plane passing through the point $(5, 1, 2)$, normal to $\underline{n} = 2\underline{i} - 2\underline{j} + 4\underline{k}$. How does the equation change if the sign of the normal is reversed, $\underline{n} \to -\underline{n}$? [2,1]

3. (a) Find the equation of the plane passing through the points $(5, 1, 2)$, $(2, 1, 2)$ and $(4, 4, 4)$. (b) Find the equation of the plane that is parallel to this one and passes through the point $(10, 3, 1)$. (c) Compute the shortest distance between the two planes. [4,2,2]

4. Find the equation of the plane passing through the points A, B, C with position vectors $3\underline{i} - 2\underline{j} - \underline{k}$, $\underline{i} + 3\underline{j} + 4\underline{k}$, $2\underline{i} + \underline{j} - 2\underline{k}$, respectively, relative to an origin O. Find the distance from the origin to the plane. [3,2,3]

5. Consider the planes A: $3x - 4y + z = 2$, B: $-2x + y - 3z = 1$ and C: $x - 5y + 2z = 5$. (a) Find the equation of the line formed by the intersection of A and B. (b) Find the equation of the line formed by the intersection of A and C. (c) Find the cosine of the angle between B and C. [5,5,2]

6. (a) Find a unit vector that is at right angles to both of the vectors $\underline{i} - 3\underline{j} + \underline{k}$ and $2\underline{i} + \underline{j} - \underline{k}$. (b) Obtain the equation of the plane passing through the point $(1, 1, 1)$ which is perpendicular to the line of intersection of the planes $x - 3y + z = 5$, $2x + y - z = 4$. [3,3]

7. Give the shortest distance of the point $(3, 2, 1)$ from the plane $z = 0$. [1]

8. Calculate the shortest distance of the point $(2, \sqrt{15}, -1)$ from the plane $3x - \sqrt{15}y + 5z = 12$. [3]

9. Calculate the closest distance of the plane $20x + 7y + z = 55$ from the origin. [3]

10. A turkey walks with velocity $-\underline{i} - 3\underline{j}$ cm/s. If the turkey is initially at position $2\underline{i} + 6\underline{j}$ m, find the time at which it reaches its nest at the origin.

11. Santa's sleigh is climbing after take-off with a position vector given by $\underline{r} = 10t\underline{i} + 10t\underline{j} + t^2\underline{k}$. Find the sleigh's velocity at time $t = 10$ and the magnitude of its acceleration at time $t = 20$.

12. Two of Santa's elves are observed at time $t = 0$ to have positions (measured in kilometres) $\underline{r} = \underline{i} + 5\underline{j}$ and $\underline{r} = 3\underline{i} - \underline{j}$. The first elf walks due east with speed 1 km/h while the second elf walks due north with speed 2 km/h. Find the time at which they are closest, and the distance between them at that time.

13. Two children, Doris and Boris, simultaneously notice that Santa has dropped a gift by the chimney at position $\underline{r}_G = 7\underline{i} + 3\underline{j}$. Doris has position vector $\underline{r}_D = \underline{i} - 3\underline{j}$ and Boris has position vector $\underline{r}_B = 8\underline{j}$. Calculate the direction in which each child must move to get to the present, and, assuming that they both move at the same speed, determine who arrives first. [5]

14. Santa is trying to drop a present down the funnel of HMS Express. His sleigh is moving north-east at 30 mph and the ship is steaming north-west at 10 mph. Find the velocity of the ship relative to Santa's sleigh.

15. A slice of nut roast is slid across a horizontal table so that at time $t \geq 0$ its position vector is $\underline{r} = \frac{1}{2}(t^2 - 3t)\underline{i} + 2t\underline{j}$. Find its velocity and acceleration vectors and show that its minimum speed is two.

16. A radar station is located at the origin, and Santa's sleighport P is at position vector $60\underline{i} + 90\underline{j}$ km. On a day when the wind has constant velocity, two sleighs A and B leave P at the same time, flying at the same speed of v km/h relative to the wind. The velocity of sleigh A relative to the wind is in the direction $\frac{3}{5}\underline{i} + \frac{4}{5}\underline{j}$ and the velocity of sleigh B relative to the wind is in the direction $\frac{7}{25}\underline{i} - \frac{24}{25}\underline{j}$. One hour after they leave, the radar station records their position vectors to be $132\underline{i} + 175\underline{j}$ km and $100\underline{i} - \underline{j}$ km respectively. Find the sleighs' speed v and the wind velocity $\underline{w}$.