MA10110 Assignment 2: Coordinate Geometry – Circles, Polars, Loci and Conics
Attempt all questions

These questions will not be assessed directly. Instead, you should answer the multiple choice test on Blackboard.

1. Find the equation of the circle C in each of the following cases:

   (a) The end points of a diameter are $(-3, 8), (5, 2)$. [4]

   (b) C is in the system $x^2 + y^2 + \lambda y - 2 = 0$ and contains $(3, -1)$. [3]

2. (a) Sketch the circle $x^2 + y^2 - x - y - \frac{1}{2} = 0$.

   (b) Find the equation of the tangent to it at each of the points (i) $(\frac{3}{2}, \frac{1}{2})$ and (ii) $(\frac{1}{2}(1 + \sqrt{3}), 0)$. [3, 4]

3. Let C be the circle $x^2 + y^2 + 2x + 4y - 2 = 0$.

   (a) Find the centre and radius of C.

   (b) Determine the tangential distance of the point (4,3) from C. [3, 2]

4. Give the polar coordinates of the point with Cartesian coordinates $(5, 12)$.

5. Show that the polar equation of the curve $xy = x + y$ is $r = \sec\theta + \operatorname{cosec}\theta$. [3]

6. Find the Cartesian equation of the curve $r\cos 2\theta = \dfrac{1}{r} + \cos\theta$. [3]

7. Find a polar equation for the circle $(x-3)^2 + (y+4)^2 = 2$. [6]

8. A point P moves so that its tangential distance from a circle of radius 4 is always 3. Determine the locus of P.

9. A point P is on a fixed circle $\mathcal{C}$. If K is a fixed point, find the equation of the locus of the midpoint of PK. Describe the locus. (Hint: move $\mathcal{C}$ to have centre at the origin and unit radius.)

10. Describe, without proofs, the locus of the point P in the following cases, where A, B are fixed distinct points and θ a constant angle, $0° < \theta < 180°$.

    (a) $PA = PB$;

    (b) $\widehat{PAB} = \theta$;

    (c) $PA^2 + PB^2 = AB^2$;

    (d) P is at a constant distance 1 from the circle with diameter AB.

    (e) $\widehat{APB} = \theta$;

11. The point P is on the line $Ax + By + C = 0$ and the point Q is on the line $Ax + By + D = 0$. As P and Q vary, show that the locus of the midpoint of PQ is a line parallel to the other two lines and give its equation.

. . . *questions continue* . . .

12. A is the point $(2, 0)$. Find the locus of P, where P is the midpoint of KA, in each of the following:

    (a) K moves on the y-axis;

    (b) K moves on the line $y = x - 1$; [5]

    (c) K moves on the unit circle $x^2 + y^2 = 1$.

13. A conic has focus at $(0, 0)$ and directrix at $x = -3$. In each case the given point lies on the conic, and these are the nearest focus and directrix. Hence determine the eccentricity of the conic, and state whether it is an ellipse, parabola or hyperbola:

    (a) $(-1, 0)$

    (b) $(0, 3)$

    (c) $(1, 2)$

    (d) $(3, -4)$

14. A conic has focus F. A variable line ℓ through F meets the conic at points A and B. Prove that as ℓ varies, the value of $\frac{1}{AF} + \frac{1}{BF}$ is constant. (Hint: Use polar coordinates.)

15. Find the equation of the tangent to the parabola $y^2 = x$ at the point $(1, 1)$.

16. (a) Write down the equations of the tangent and normal at the point $(at^2, 2at)$ on the parabola $y^2 = 4ax$.

    (b) Show that there are two tangents to the parabola from any point on the directrix and that they are orthogonal.

    (c) Further, show that the line joining the points of contact with the parabola of these tangents passes through the focus.

17. The tangent at the point $P : (t, t^{-1})$ on the rectangular hyperbola $xy = 1$ meets the axes at points A and B. Prove that as P varies, the area of the triangle AOB remains constant.

18. Prove that the normal at any point $P : (a\cos\theta, b\sin\theta)$ on an ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

has slope $\dfrac{a}{b}\tan\theta$.