MA10110 Assignment 1: Coordinate Geometry – Lines

Attempt all questions

Solutions to questions marked with an asterisk ($*$) should be written out neatly, scanned, and uploaded to Blackboard as a single PDF file by 3pm on Monday 12th October.

Ensure that you include your name and student ID number.

1. Express the following line equations in the form $y = mx + c$ or $x = c$:

   (a) $3x - 2y + 7 = 0$, (b) $5x + 3y = 1$, (c) $7x + 3 = 0$.

   For which value of α are the line $\alpha x + 2y - 1 = 0$ and the line in (a) parallel? [1,1,1,2]

2. Find:

   (a) the length of the line segment AB with end points $A(7, -3)$ and $B(3, 9)$;

   (b) the coordinates of the midpoint M of AB;

   (c) the equation of the perpendicular bisector of AB;

   (d) the length MC, where C is on this perpendicular bisector such that the angle $\widehat{ACB}$ is a right angle.

3. $*$ Write down the general equation of a line ℓ through the intersection of the lines $3x - y + 3 = 0$ and $x + y - 1 = 0$ using the form $F(x, y) + \lambda G(x, y) = 0$. Give the equation of ℓ when ℓ has slope 1. [2,3]

4. What is the general equation of a line through the point $(-2, 3)$?

5. Write down the general equation of a line through the intersection of the lines $3x - 5y + 41 = 0$ and $5x + 6y - 8 = 0$ using the form $F(x, y) + \lambda G(x, y) = 0$. Determine the equation of the line joining this intersection and the point $(8, 3)$. [2,3]

6. Show that the equation of the line through the point (p, q) parallel to the line $ax + by + c = 0$ is $ax + by - (ap + bq) = 0$. [5]

7. Find the equation of the general line through the point of intersection of the lines $2x - 3y + 1 = 0$ and $5x + 8y - 3 = 0$ using the form $F(x, y) + \lambda G(x, y) = 0$. Determine the equation of the line joining $(-2, 1)$ to this point of intersection. [2,3]

8. $*$ Give the equation of the general line through the point of intersection of the lines $3x - 2y + 10 = 0$ and $4x + 7y - 3 = 0$ using the form $F(x, y) + \lambda G(x, y) = 0$. Determine the equation of the line joining $(0, 1)$ to this point of intersection. [2,3]

9. $*$ Find the equation of the line through the point $(3, -1)$ that is orthogonal to the line $3x - 6y + 1 = 0$. [5]

10. Find the equation of the perpendicular bisector of the line segment with end points $(5, -3)$ and $(3, 7)$. [5]

11. $*$ Find the equation of the line through the point $(3, -5)$ that is:

    (a) orthogonal to the line $3x - 6y - 5 = 0$;

(b) parallel to the line $3x - 6y - 5 = 0$. [3,2]

12. $*$ Find the shortest (perpendicular) distance between the point $(2, 3)$ and the line $4x - 3y + 5 = 0$. [5]

13. Determine whether the points $(-30, 8)$, $(4, 11)$ are on the same or opposite sides of the line $3x - 9y + 112 = 0$. Which of the points, if any, is on the same side as the origin? Determine the inequality that c must satisfy for the point $(5, c)$ to be on the same side of this line as the point $(4, 11)$. [2,1,2]

14. $*$ Find the range of values of c such that the points $(-3, -6)$ and $(7, 20)$ are on opposite sides of the line $y = 4x + c$. [5]

15. Find the equations of the angular bisectors of the lines $4x - 3y + 5 = 0$ and $x + 4 = 0$.

16. Find the angles between the lines $y = 3x - 5$ and $x + 4y = 1$.

Challenge Questions

17. The sides BC, AC, AB of the triangle ABC have equations $U_i = 0$ $(i = 1, 2, 3)$ respectively, where $U_i = a_i x + b_i y + c_i$. Show that the line through A parallel to BC has equation

$$\begin{vmatrix} a_1 & b_1 \\ a_3 & b_3 \end{vmatrix} U_2 + \begin{vmatrix} a_2 & b_2 \\ a_1 & b_1 \end{vmatrix} U_3 = 0.$$

18. Show that the general equation of the line orthogonal to the line ℓ with equation $ax + by + c = 0$ is of the form $bx - ay + d = 0$, where d can be any number. Further, show that the line through the point (p, q) orthogonal to ℓ has equation $bx - ay - (bp - aq) = 0$.

19. Show that the lines which meet $y = 2x - 1$ at 30° have slopes $-8 \pm 5\sqrt{3}$.

20. Evaluate the area of the triangle whose vertices are $(2, -3),\ (1, -1),\ (5, 1)$.