CANDIDATE SOLUTIONS TO PARTITION PROBLEMS: A SUMMARY
Latest update available at `http://users.aber.ac.uk/sxc/two_d_clusters.html`

- The conjectured least perimeter partition is given for bubbles of unit area, including the boundary, or a tiling of the unit sphere.
- We try to attribute the best candidates with citations.
- When giving the perimeter, we use a superscript $*$ to denote that there exists a proof of the optimality of this candididate, and a superscript † to indicate that there is a weaker demonstration of optimality (e.g. by enumeration of topologies, by assuming convexity and/or assuming connectedness).
- The colour scheme in the figures is as follows: defects are classified using the idea of *topological charge*. Bulk bubbles have a charge $q = 6 - n$, where n is the number of sides. Thus hexagons have zero charge and are not coloured. In the same way, peripheral bubbles have charge $q = 5 - n$. Bubbles are coloured red if $q = 1$, blue if $q \geq 2$ and yellow if $q = -1$.

References

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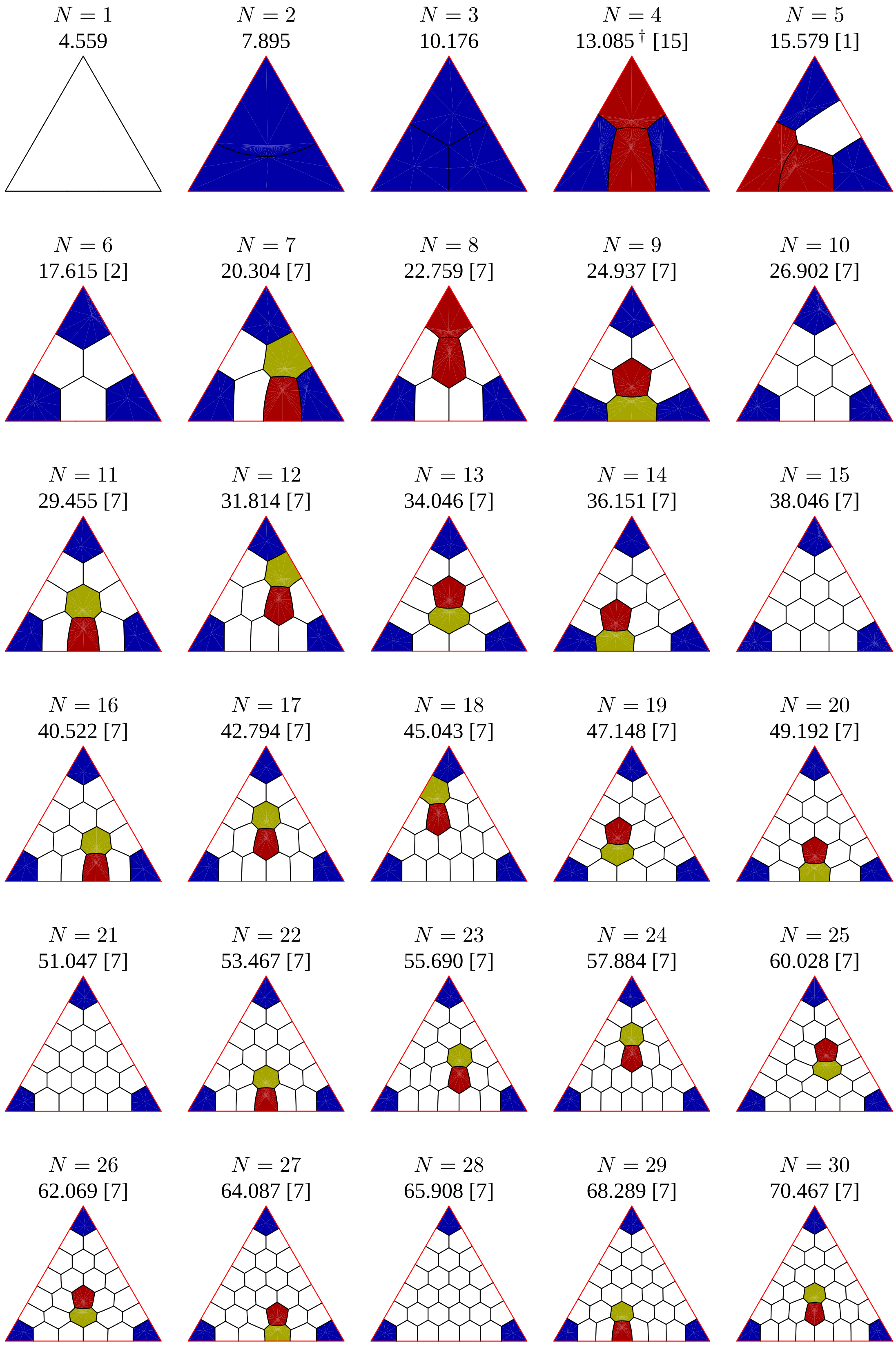

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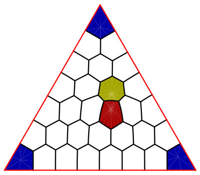

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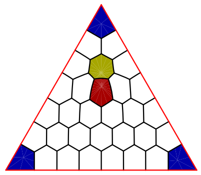

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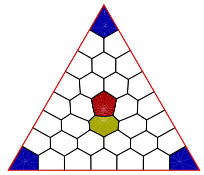

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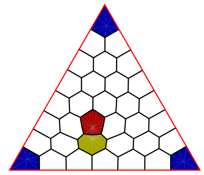

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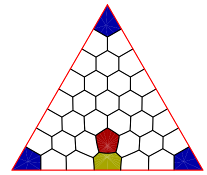

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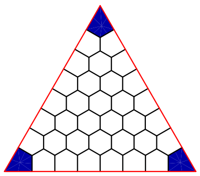

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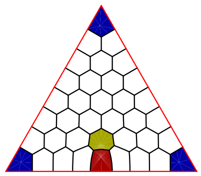

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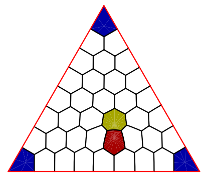

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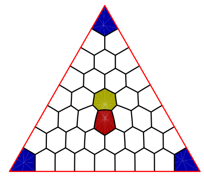

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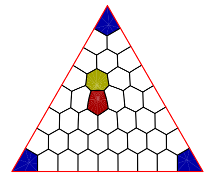

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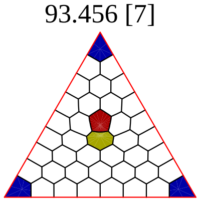

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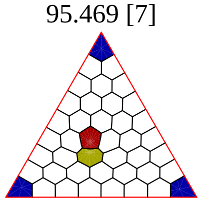

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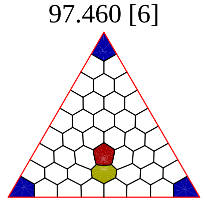

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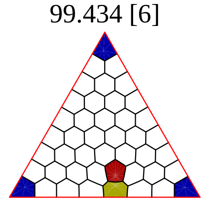

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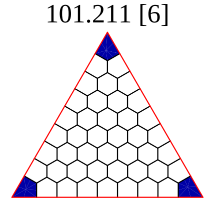

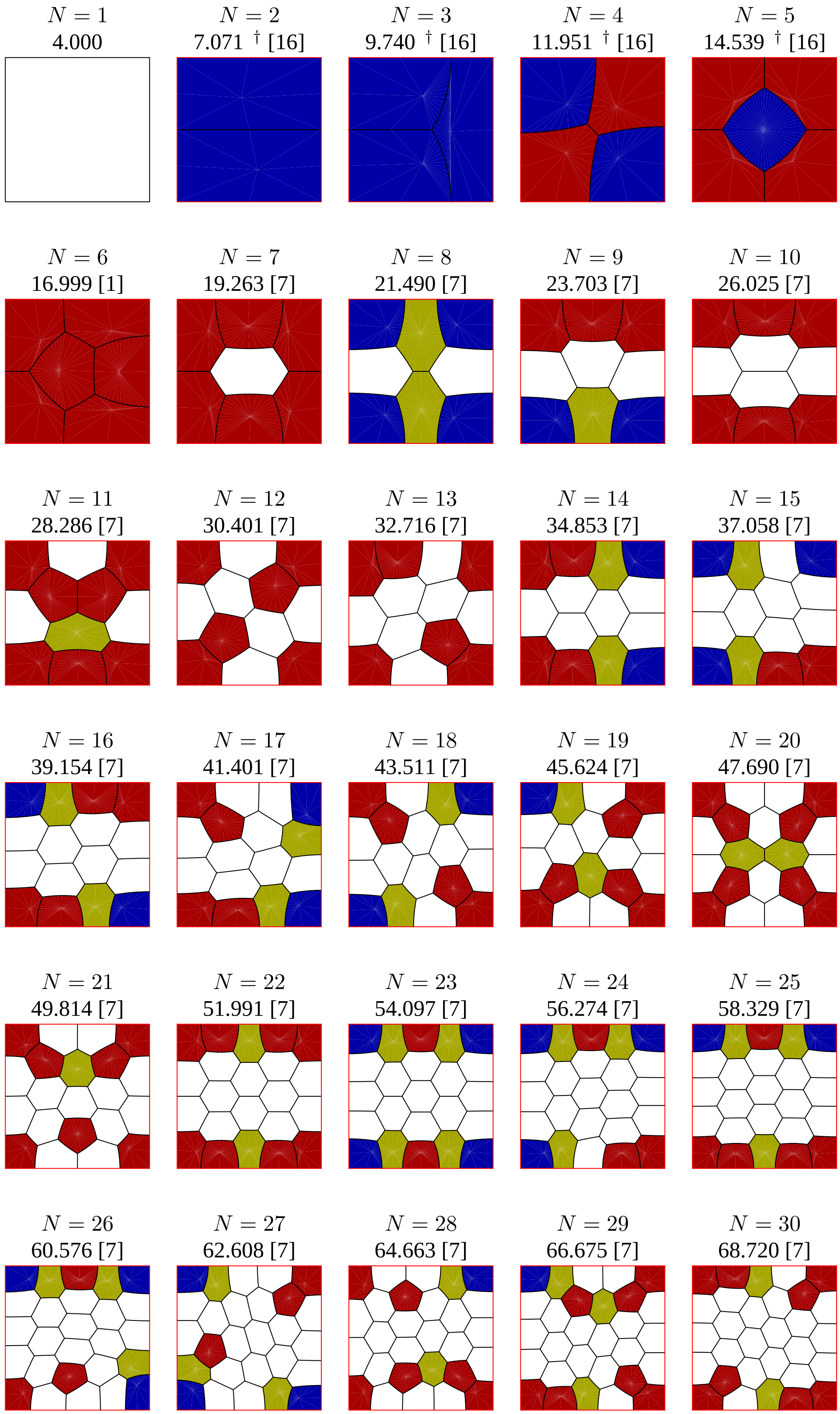

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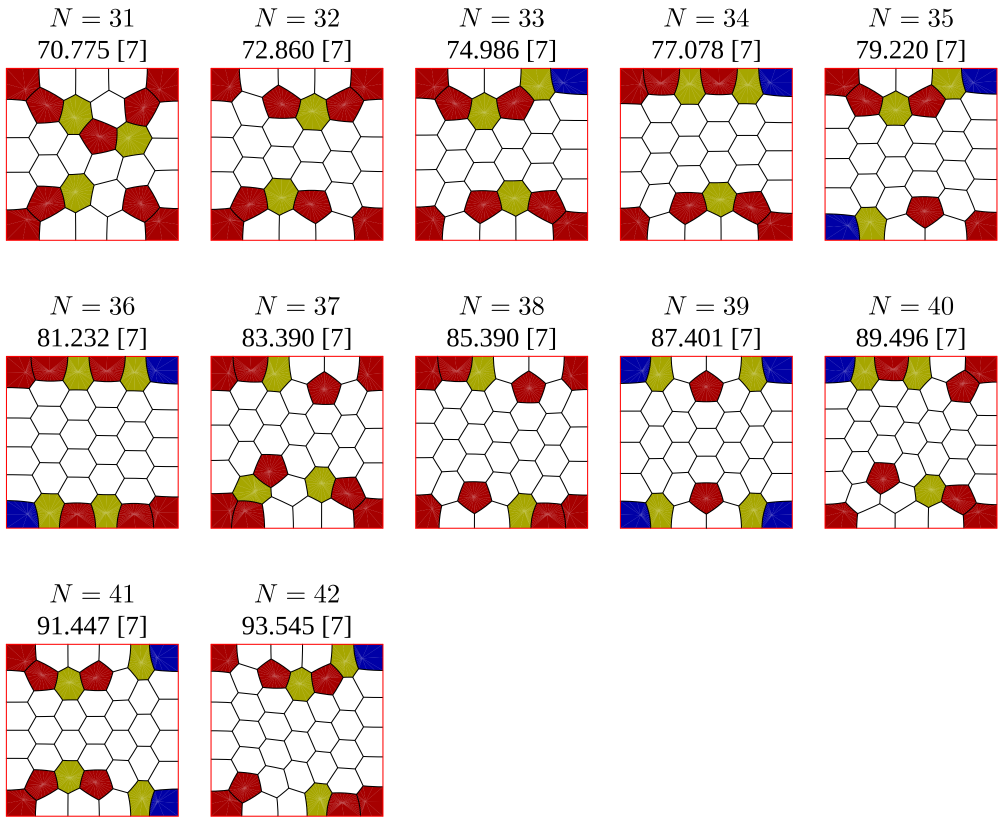

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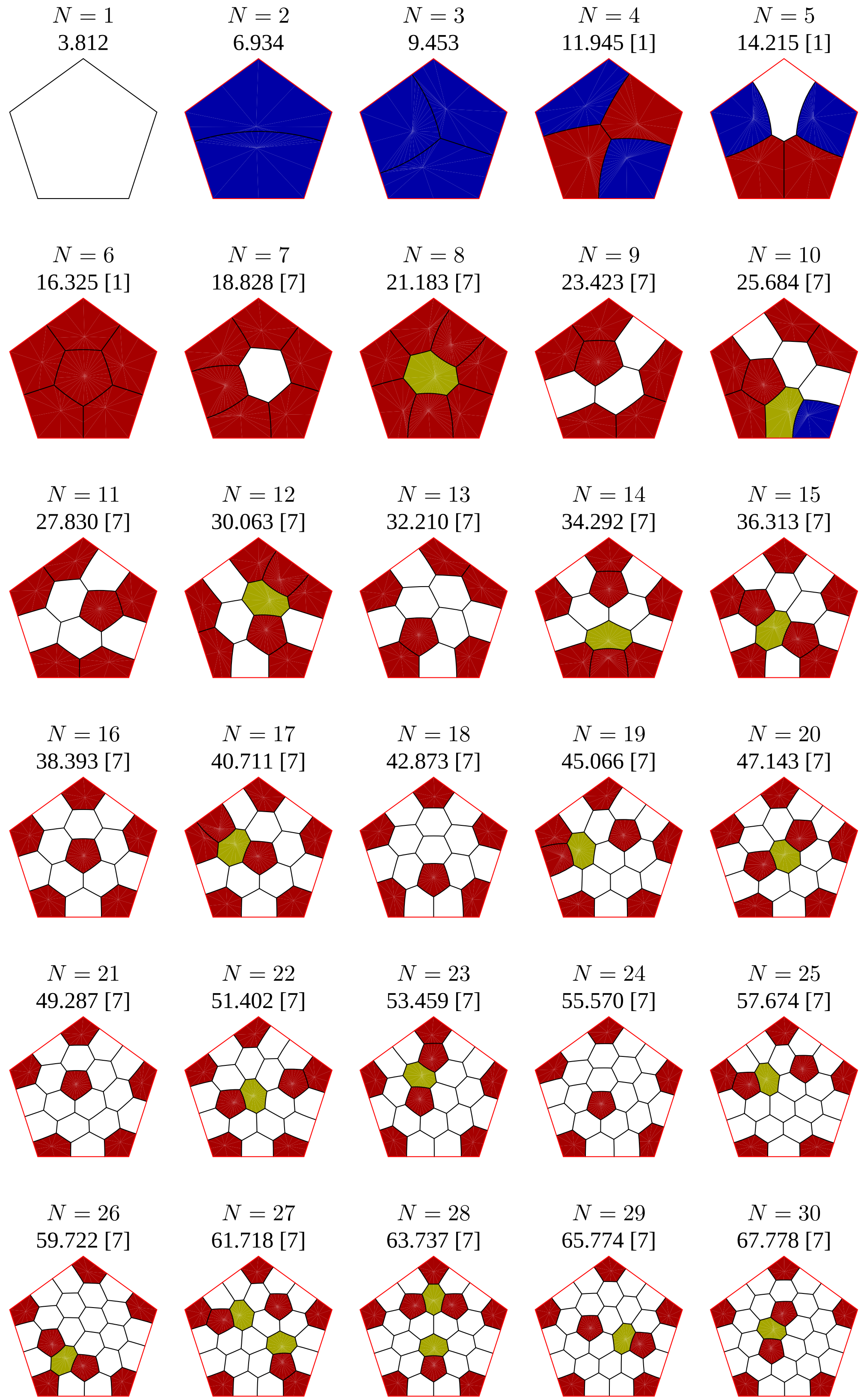

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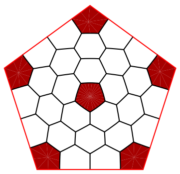

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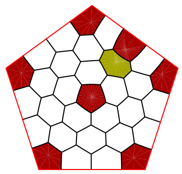

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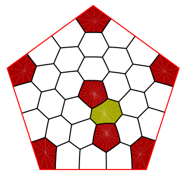

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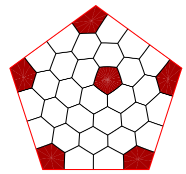

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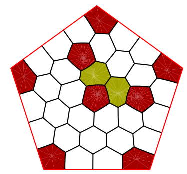

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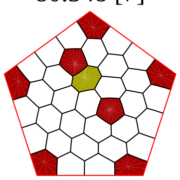

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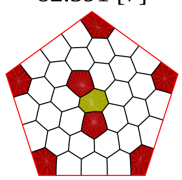

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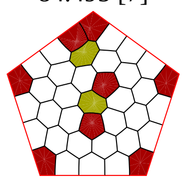

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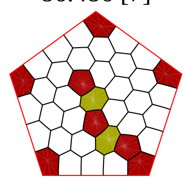

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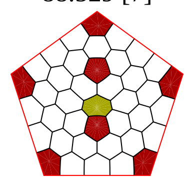

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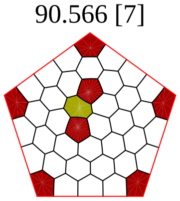

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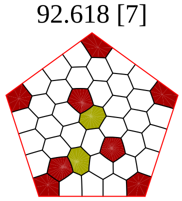

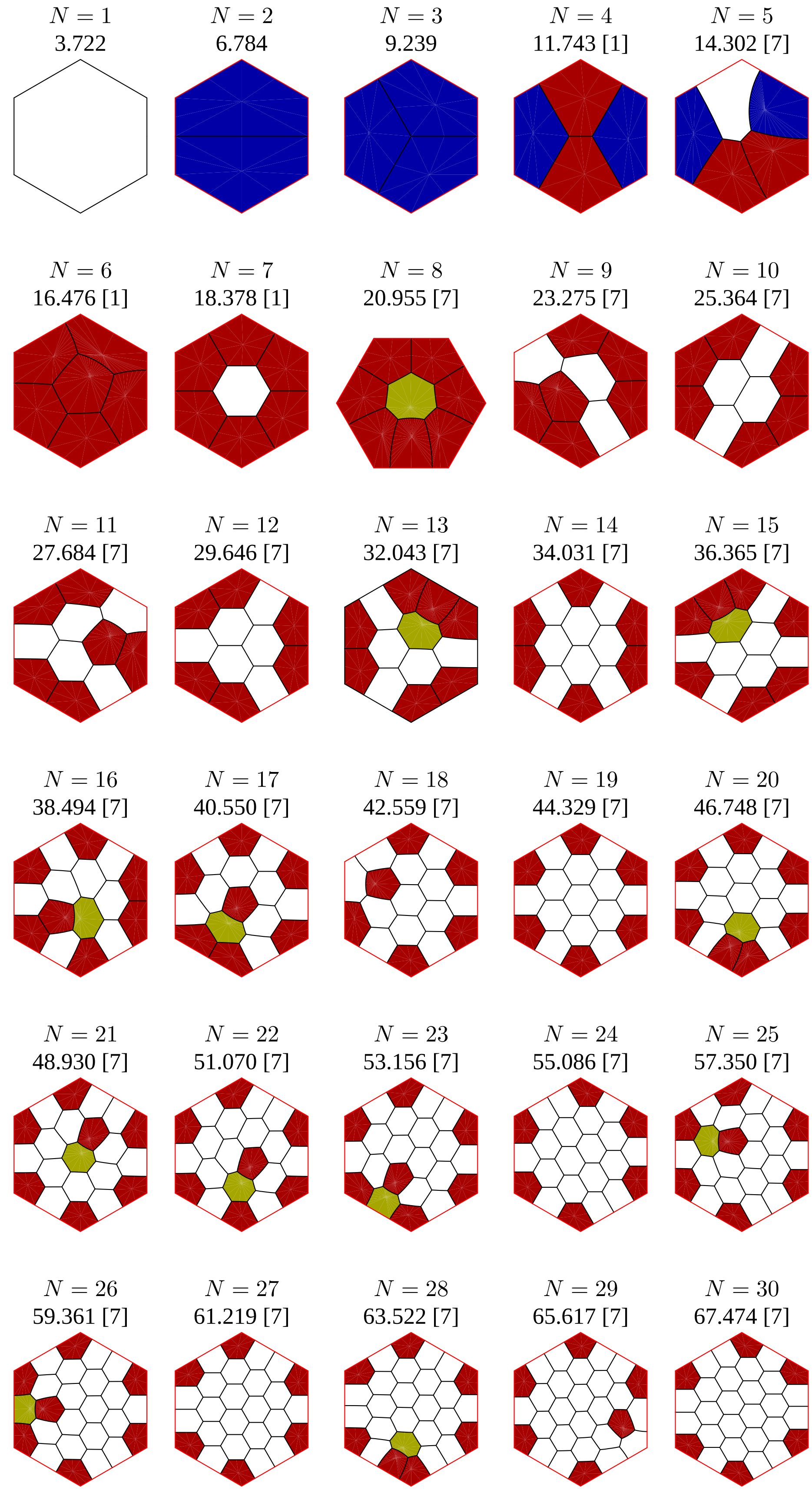
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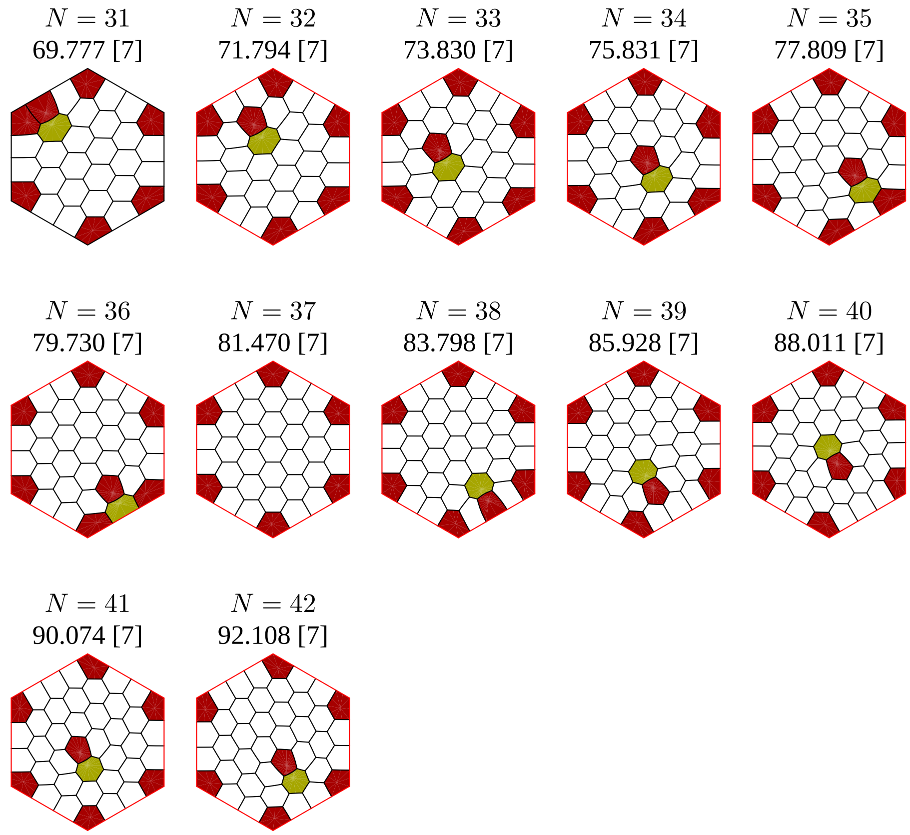

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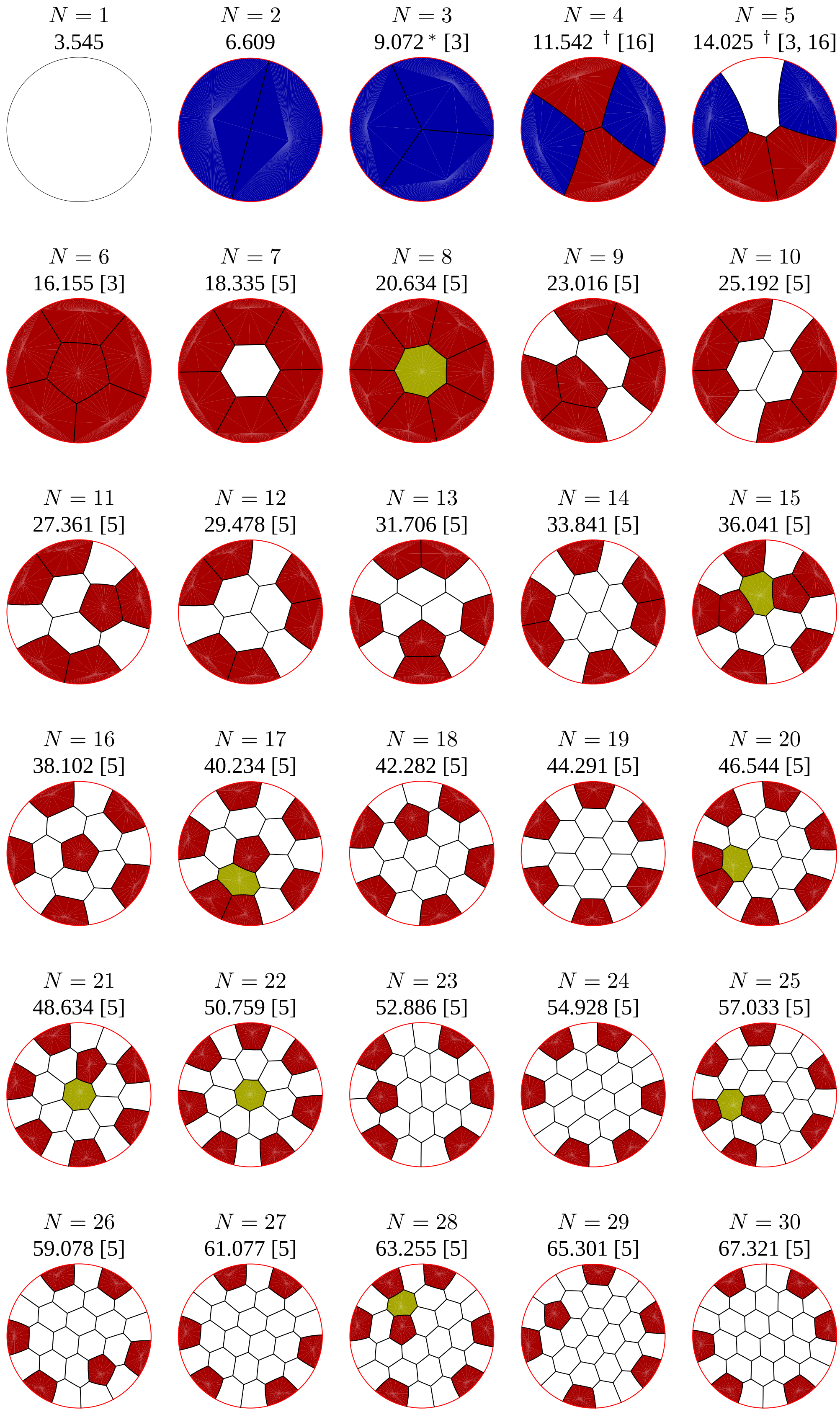
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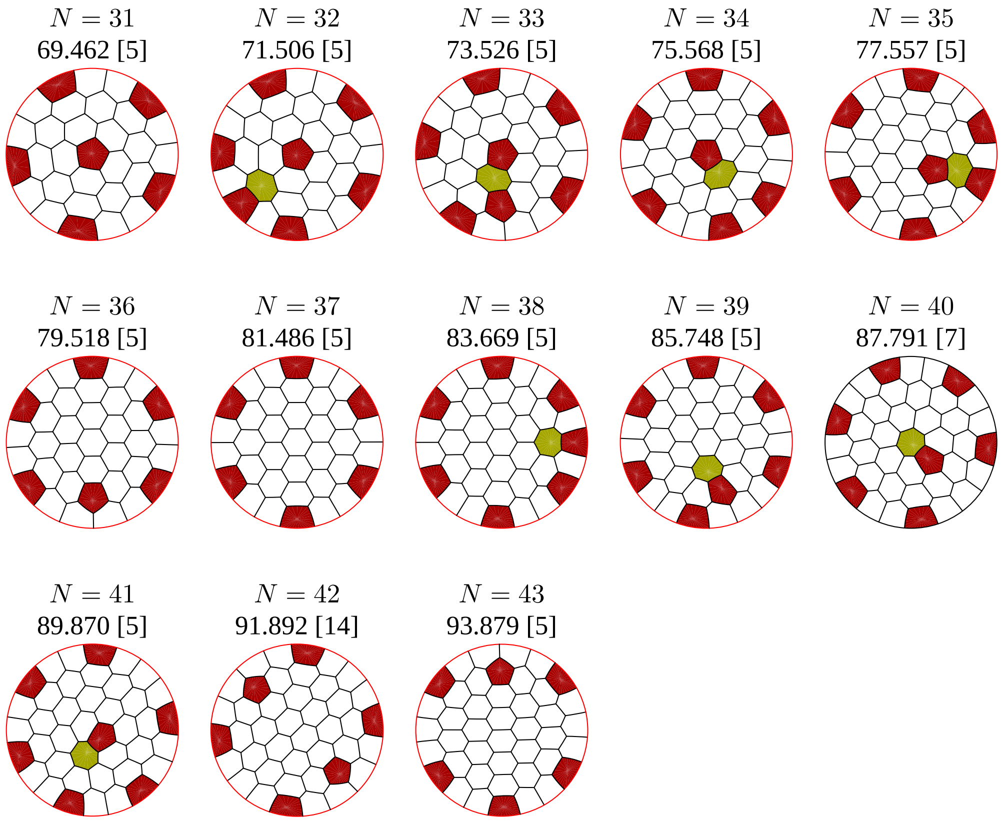
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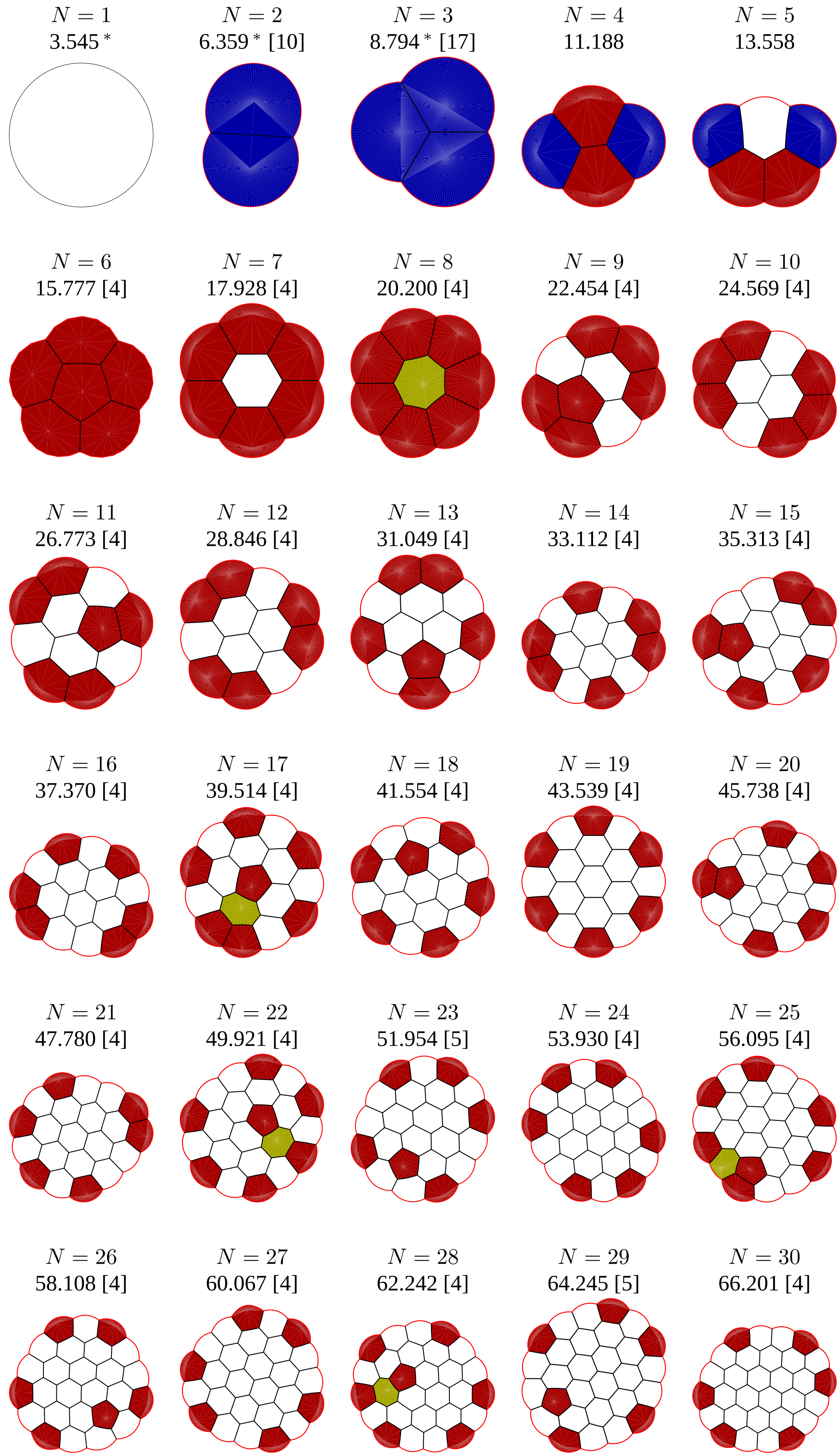

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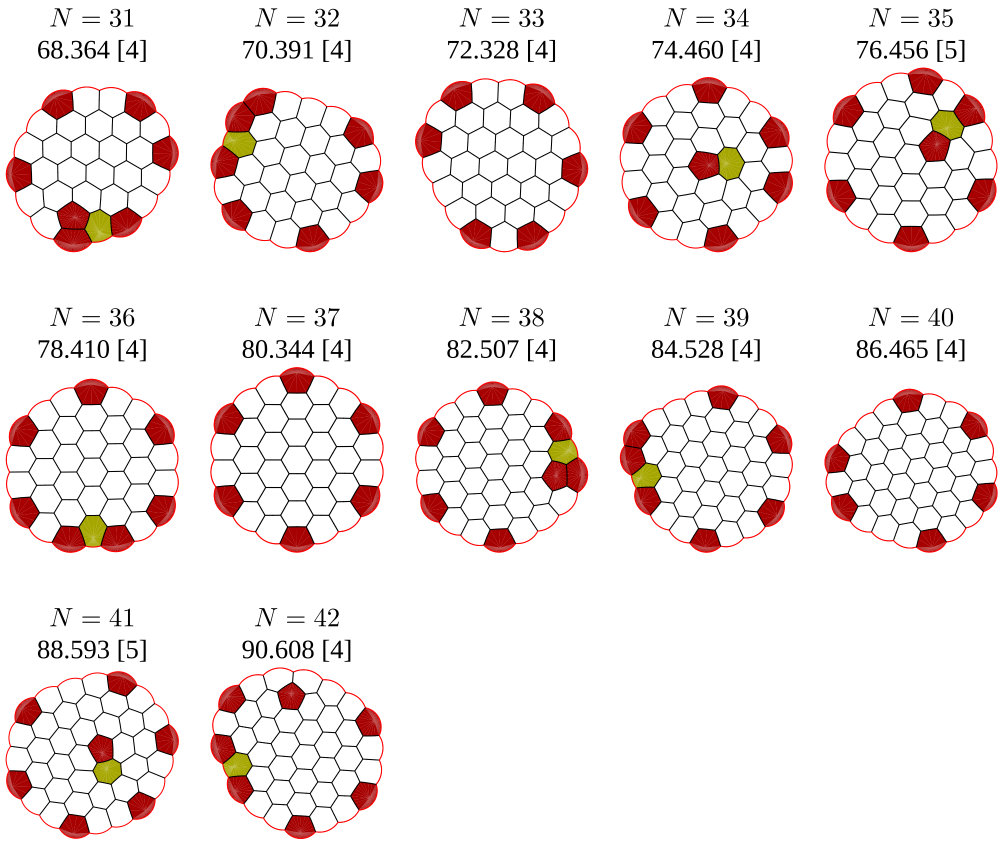
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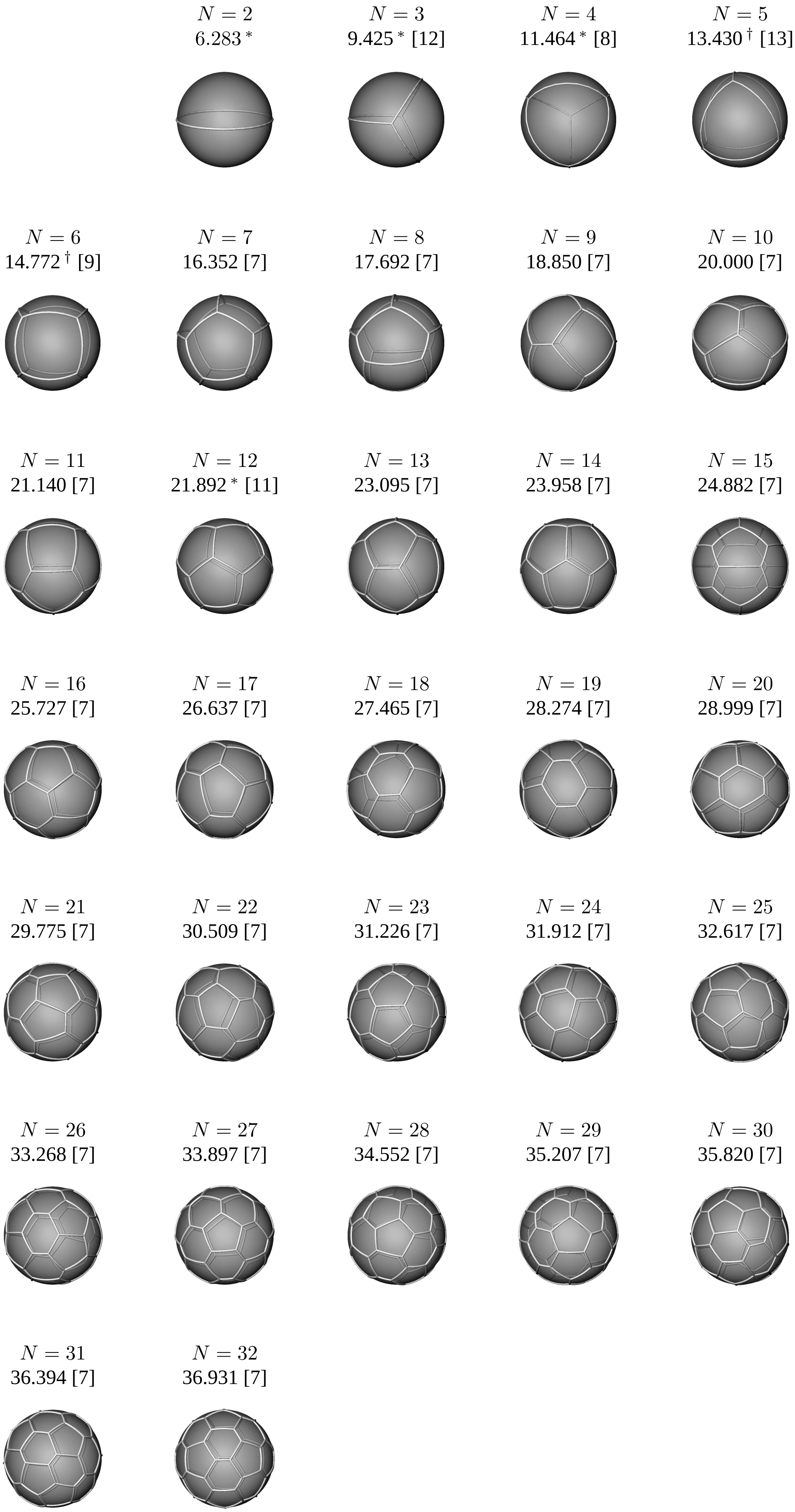
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