TRANSITIONS IN PLATEAU BORDERS

AND

TOPOLOGICAL INSTABILITIES OF DEFORMED BUBBLE CLUSTERS

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TRANSITIONS IN PLATEAU BORDERS

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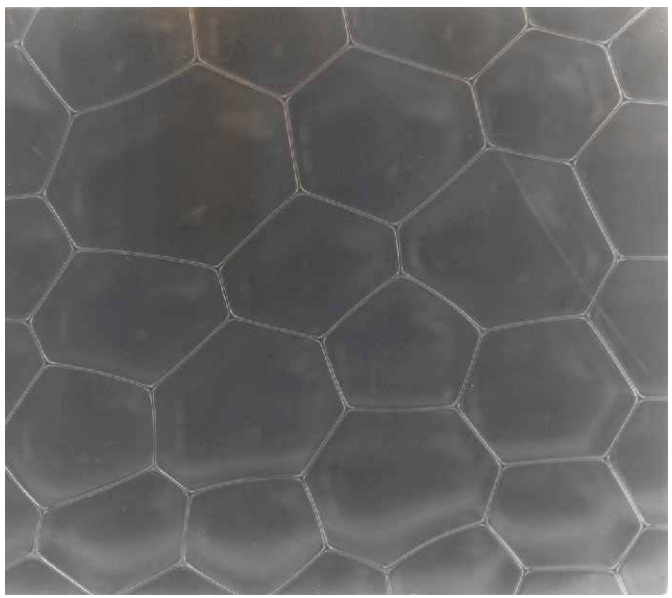
LIQUID FOAMS

Liquid Foams = clusters of bubbles

Bubbles - gas inside ; surrounded by liquid films

Edges meet at vertices

Liquid in Plateau borders = PBs



DRY FOAM

foams: dry ➡ liquid films of negligible thickness meet in PB

wet ➡ liquid films of finite thickness meet in PB

2D foams - PBs run perpendicular to the plane of the foam

PBs are polygonal regions with curved edges

PBs are bounded by liquid surfaces (=interfaces between the liquid and the gas) of tension γ_L

γ= film tension and $\gamma = 2\gamma_L$ (PB tangent to film)

Wet foam: internal PBs - surrounded by bubbles

peripheral PBs - border outside gas n=3

Internal PBs:

- Small liquid fraction ➡ - PBs are triangular n=3
- High liquid fraction ➡ - PBs with n=3, 4 or 5 (we did not study n>5)

- experiments with foam clusters –

  change the liquid fraction

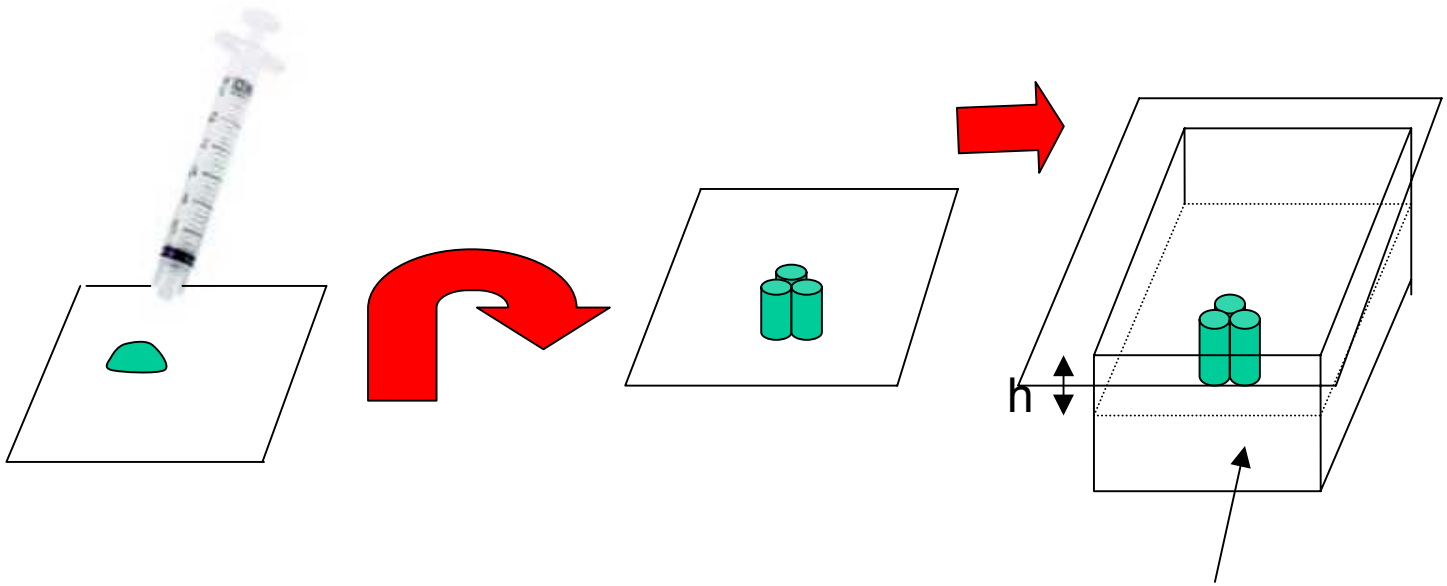
- calculations with the Surface Evolver

Experimental procedure

Monolayer technique (Smith 1952; Vaz and Fortes 1997):

- ☞ Vessel with detergent solution
- ☞ Blow air bubbles with a graduated syringe on a plate
- ☞ The plate was inverted onto the liquid in the vessel
- ☞ Bubbles make contact with the liquid pool; gap between liquid surface and the plate = height *h* of the bubbles
- ☞ the liquid surface height can be varied by adding or removing solution $\Rightarrow$ change the height *h* of the bubbles

experimental set-up



h = **gap between liquid surface and the plate or height of the cluster**

Bubble volumes, V_G = 320 or 400 mm^3

3, 4 and 5 bubble clusters of the same volume

Change the liquid fraction

❖**Small h** : h ≈ a (a =the capillary length= 1.85 mm) $a = (\gamma_L / \rho_L g)^{1/2}$

$\Rightarrow$ clusters with an internal PB surrounded by n bubbles + n peripheral 3-PBs

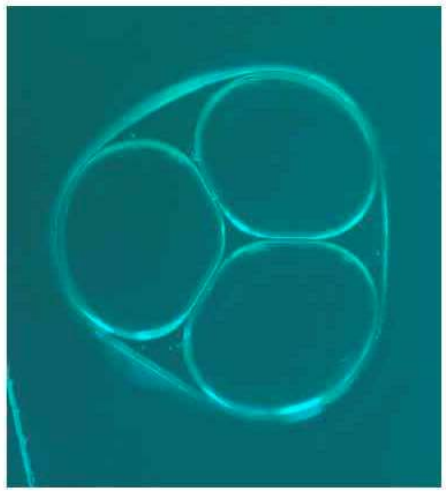
❖**Increase h; observe the evolution of PBs and measured with image analysis software** :

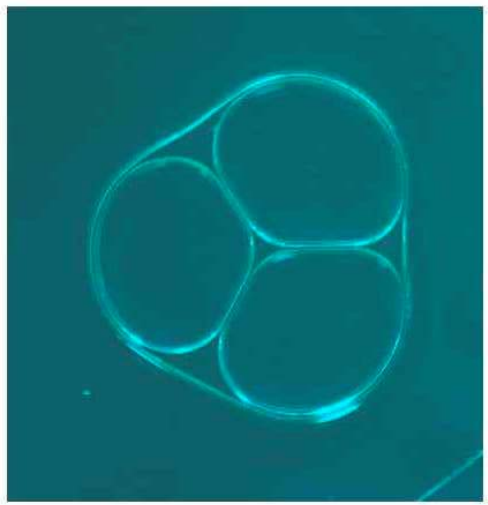
- A_L (total area of the internal PBs)
- A_G (area of one bubble)

Experimental results

Evolution of PB with n-fold symmetry surrounded by n identical bubbles:

Ex: n=3





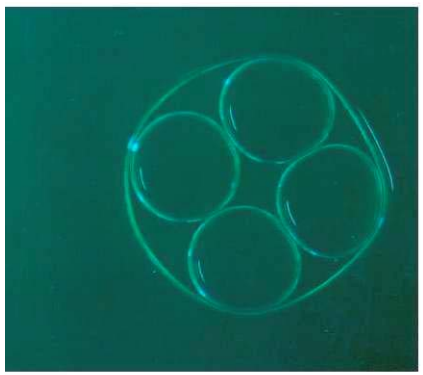
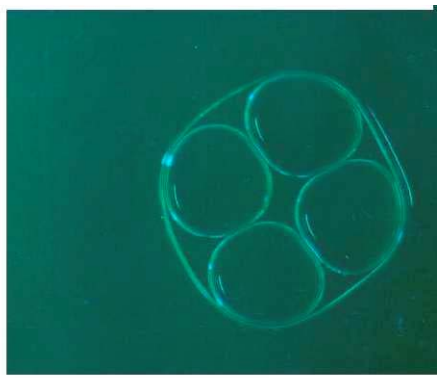
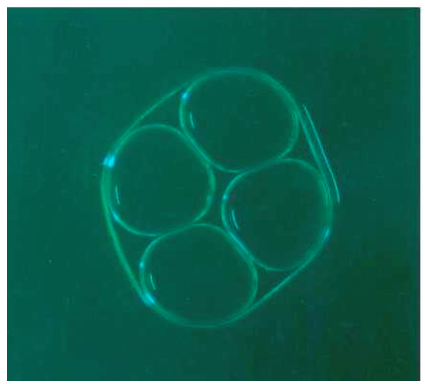
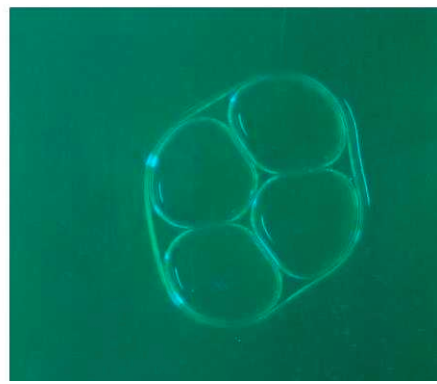
Note curvature of peripheral PB outer surface

V_G= 400 mm^3

h=1.56 mm **h=1.72mm**

$\left(A_L / A_G\right)^{1/2}$ **=0.18** $\left(A_L / A_G\right)^{1/2}$ **=0.15**

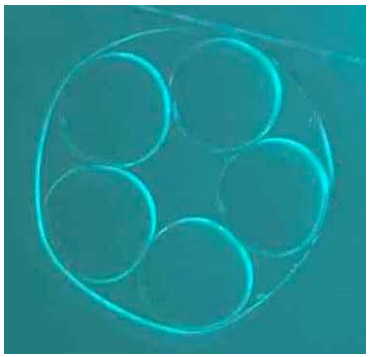
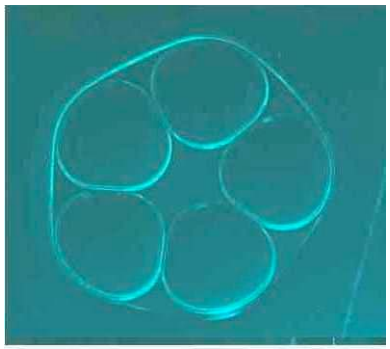
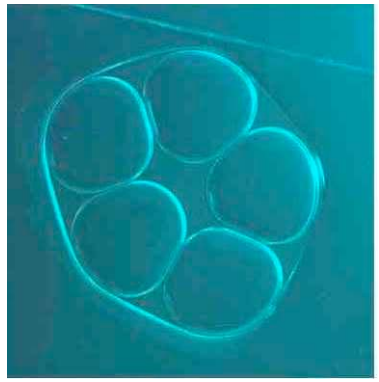
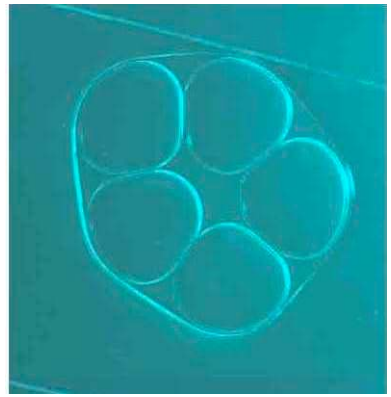
n=4

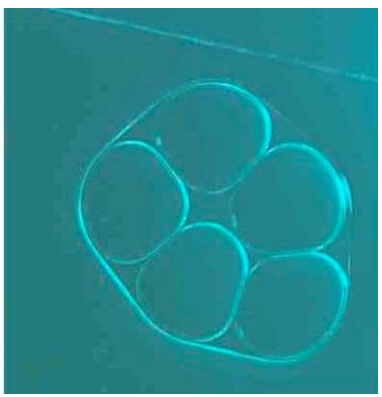
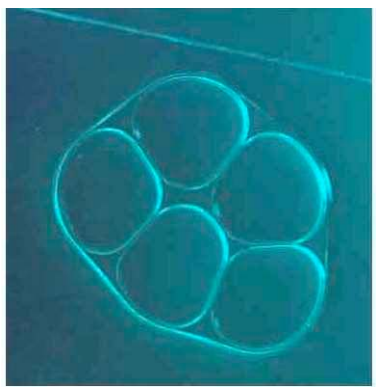
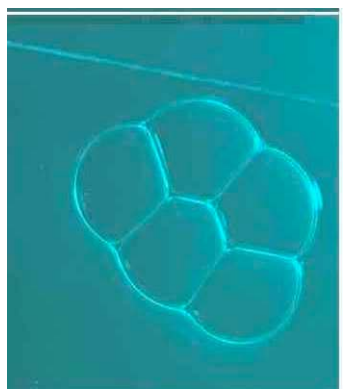
V_G= 400 mm³

Conf. 4	Conf. 4’	Conf. 4’	Conf. 3+3
h=0.94 mm	h=1.20mm	h=1.57 mm	h= 1.88 mm
$\left(A_L / A_G\right)^{1/2}$ =0.56	=0.47	=0.39	=0.29

n=5 V_G= 400 mm^3

	5	5’	5’’	5’’’
	h=1.98mm	h=2.11mm	h=2.17 mm	h= 2.14 mm
$(A_L / A_G)^{1/2}$	=0.81	=0.62	=0.57	=0.60

	4+3	(4+3)’	3+3+3
	h=2.16mm	h=2.18mm	h=3.0 mm
$(A_L / A_G)^{1/2}$	=0.57	=0.47	=0.12

Experimental results

- n= 3 – PB remains symmetrical at any h; for either positive or negative p_L-p_0
- n=4 – increasing h ➡

4 =square PB (4-fold axis of symmetry) changes p_L-p_0=0

⬇

into 4' a rectangular PB (two mirror symmetry planes)

⬇

two triangular (3+3) PBs of the same size (two mirror symmetry planes)

reversible sequence

❑n=5

small h ⇒ configuration 5 - 5 fold symmetric PBs
larger h ⇒ p_L-p_0<0 ⇒ configurations of lower symmetry

configuration 5' - mirror plane of symmetry
configuration 5'' – mirror plane of symmetry
configuration 5''' – no symmetry

dissociation of PB

configuration (4+3) mirror plane of symmetry
configuration (4+3)' no symmetry
configuration (3+3+3) mirror plane of symmetry

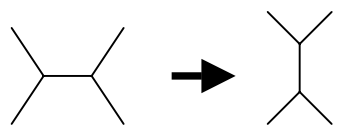
Sequence 1: 5 →5'→(4+3) →(4+3)' →(3+3+3)
Sequence 2: 5 →5' →5''' →5'' →(3+3+3)

Surface Evolver calculations

Ken Brakke (1992)

- **equilibrium configurations**
- **surface energy**
- **excess pressure**

1. **Network: coordinates of vertices; define edges and cells**
2. **Perturbations in vertices = shuffling (jiggle command)**
3. **Apply T1 (neighbor switching) in short side edges (topological changes)**
4. **Relaxation**

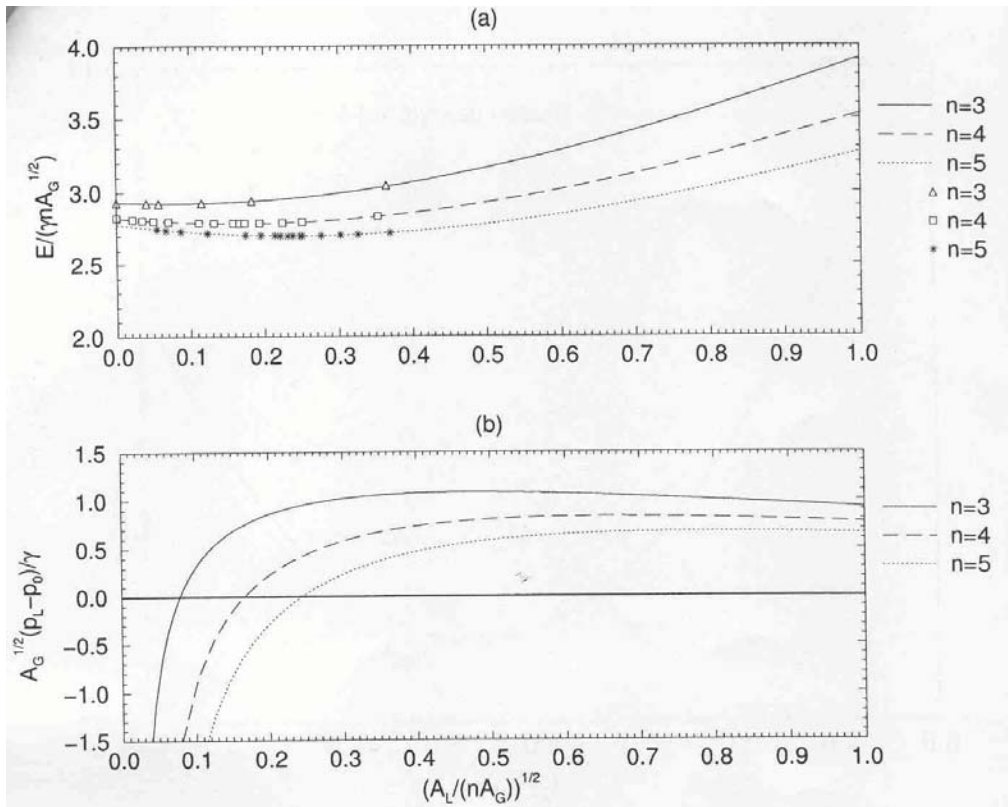


Analytical solutions for n-fold symmetrical clusters without peripheral PBs are easy to obtain (Eur.Phys.JE,2005)

Plots of reduced energy $E/(\gamma A_G^{1/2})$ **vs** $\left(A_L/A_G\right)^{1/2}$

for n-fold symmetrical clusters n=3,4,5

- **Analytically (curves)**
- **Surface Evolver (symbols)**



- ✓Curves exhibit a minimum
- ✓Surface Evolver reproduces the exact results to within 0.1%
- ✓Plots of the excess pressure p_L-p_0 in the internal PBs exhibits a maximum
- ✓Energy minimum occurs when p_L-p_0 =0

Lower symmetry clusters → requires Surface Evolver

Starting configuration:

- **-internal PBs surrounded by n quadrilateral bubbles**
- **-internal PBs = regular polygons with n straight sides**
- **-peripheral PBs = triangles**

Linear mode

↗Perturbations in vertices = shuffling (jiggle command)
↗Apply T1 (neighbor switching) in short side edges (topological changes)
↗Relaxation

- shuffling/relaxation procedure = 100

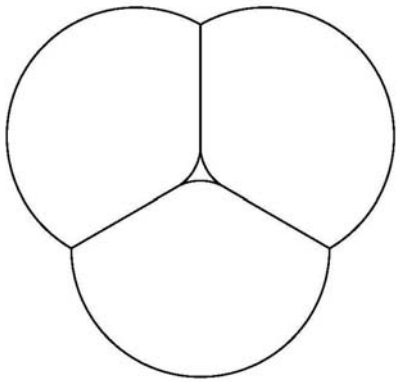
- 6 levels of refinement

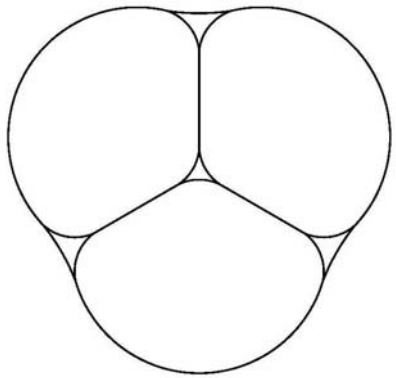
→ **MINIMISE ENERGY** $= \gamma_L L_L + \gamma_F L_F$ $\gamma = 2\gamma_L$

L_L and L_F are total lengths of the PBs surfaces and the films

SURFACE EVOLVER results:

n=3 Without and with peripheral PBs





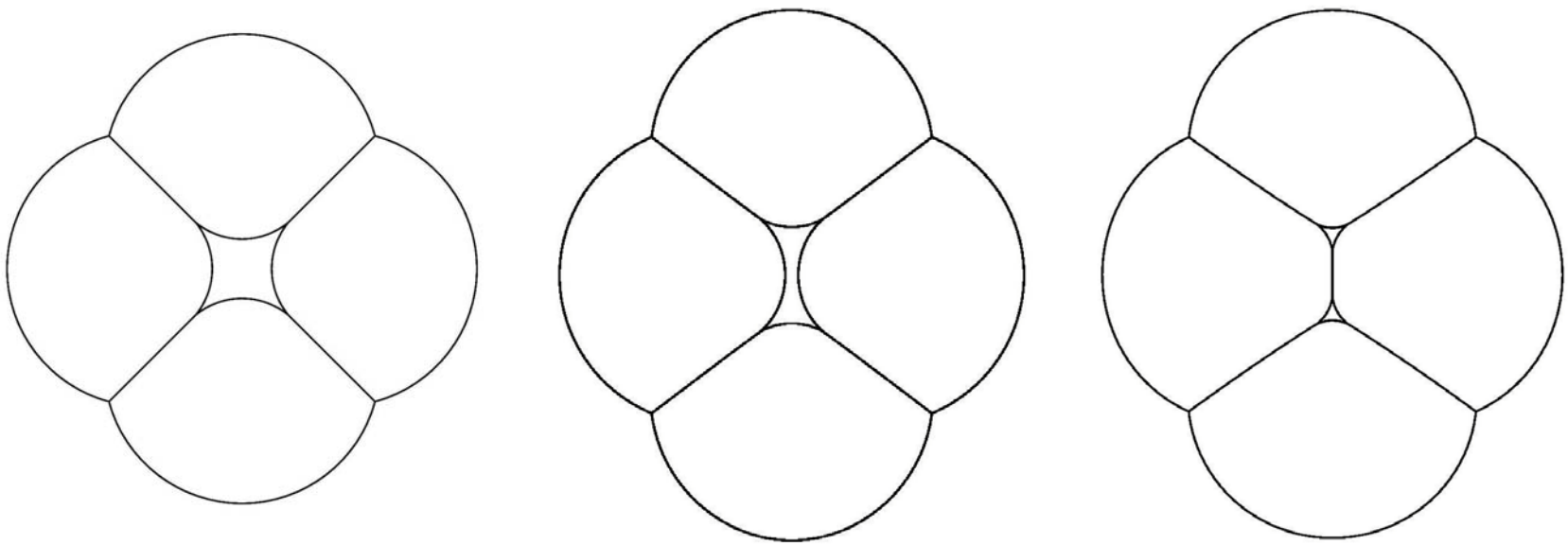
$\left(A_L / A_G\right)^{1/2}$ **=0.1** **=0.1**

$E/(\gamma A_G^{1/2})$ **= 8.768** **=8.656**

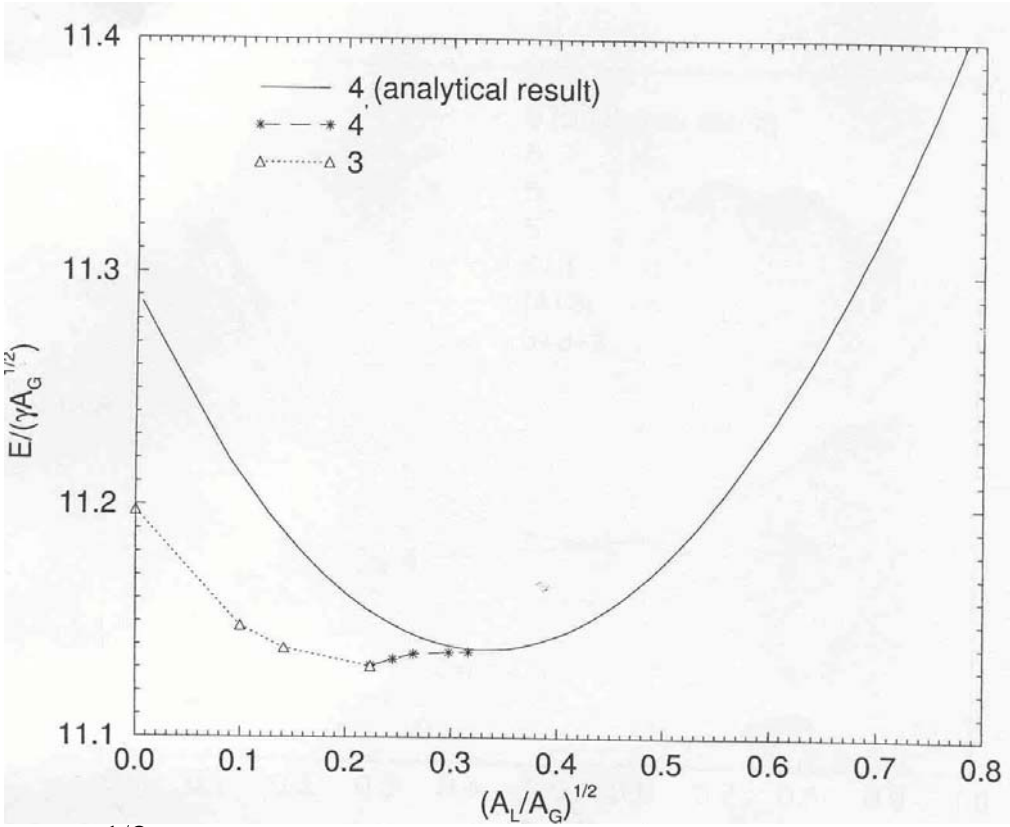
The peripheral PBs lower the cluster energy

SURFACE EVOLVER results: n=4

- **Surface Evolver yielded the configurations found in experiments**
- **Adding peripheral PBs does not change the geometry/topology of the internal PBs**
- **A_L/A_G > that of the minimum of the energy curve ⇒ higher energy configurations (not discussed here)**
- **A_L/A_G < that of the minimum of the energy curve ⇒ lower energy configurations**

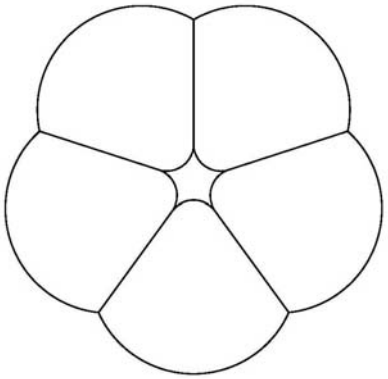


n=4

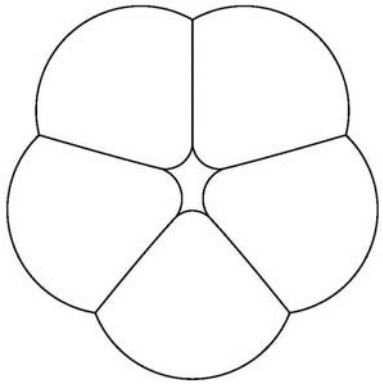


- $\left(A_L / A_G\right)^{1/2}$ < 0.22 $\Rightarrow$ configuration (3+3)
- $\left(A_L / A_G\right)^{1/2}$ =0.22 $\Rightarrow$ two triangles coalesce and give 4'

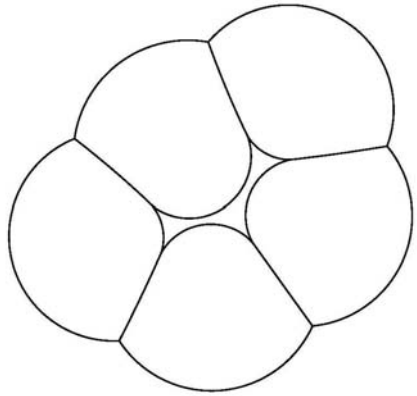
SURFACE EVOLVER results: n =5



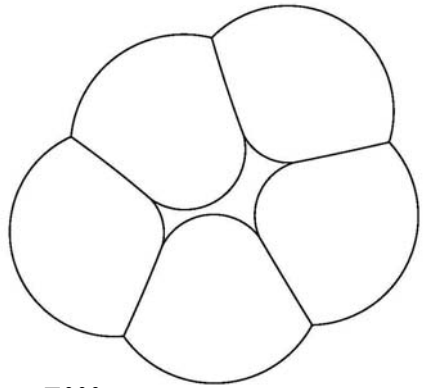
5



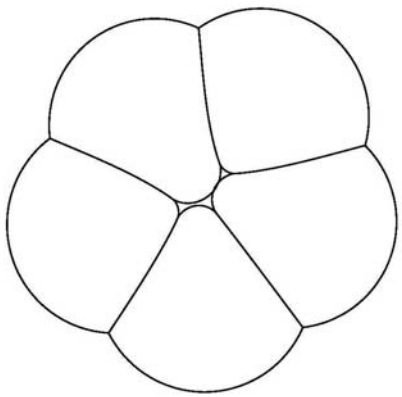
5'



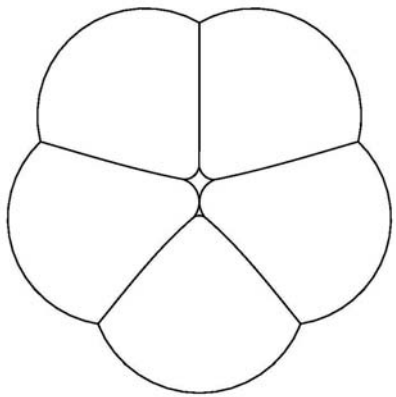
5''



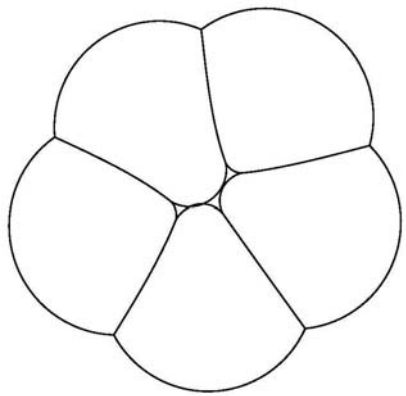
5'''



4'+3

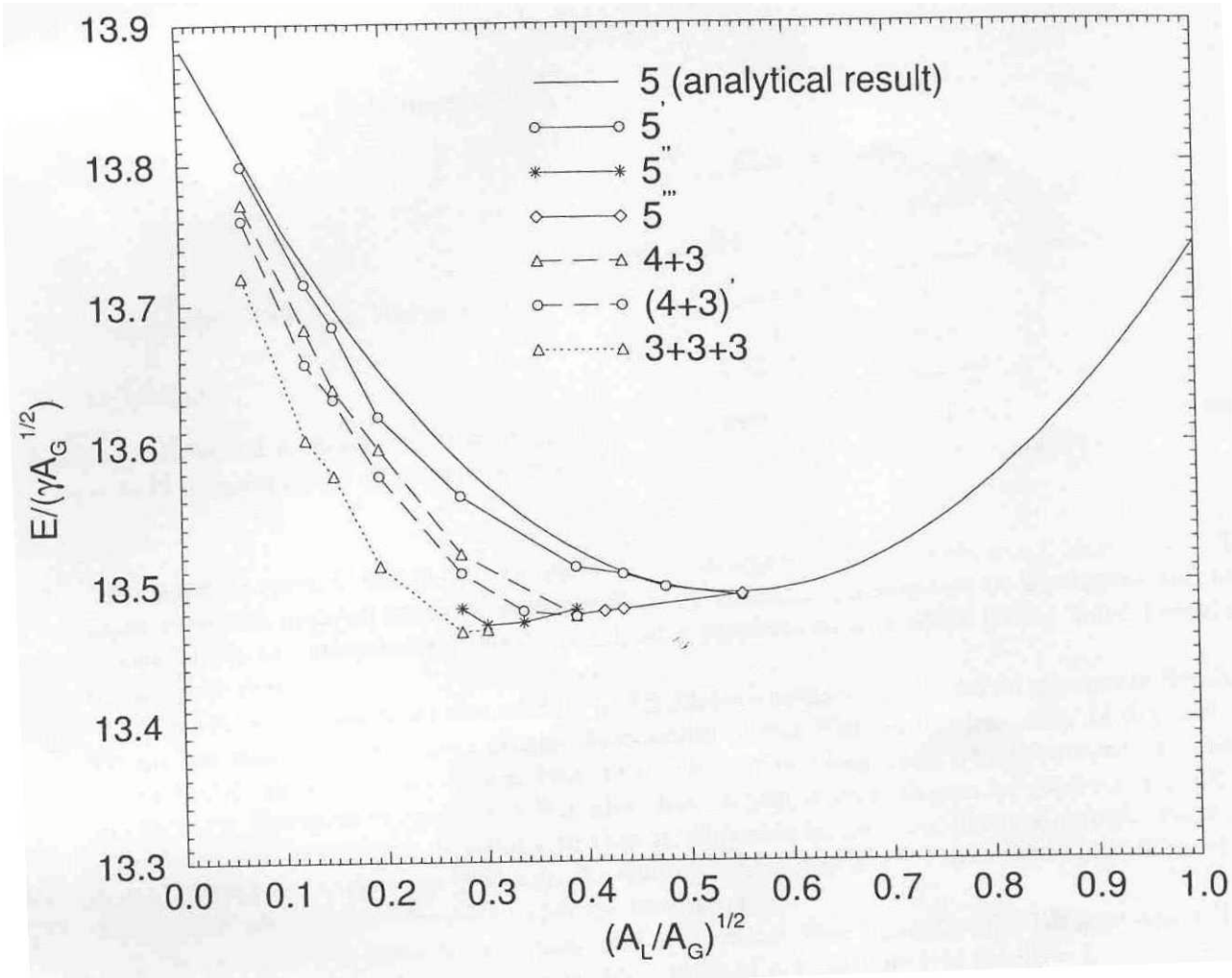


4+3



3+3+3

SURFACE EVOLVER results:



SURFACE EVOLVER results: n = 5

⌘ A_L/A_G < that of the minimum of the symmetric curve ⇒ lower energy configurations

⌘ close to the minimum configuration 5' is the one of lowest energy

⌘ it is beaten at lower A_L/A_G first by 5''' and then by 5'' (consistent with experimental sequence) : 5 →5' →5''' →5'' →(3+3+3)

⌘ configurations (4+3) and (4+3)' have approximately the same energy; none of these is ever that of lowest energy

(beaten by (3+3+3) or 5'')

⌘ the Surface Evolver sequence on increasing A_L/A_G is the experimental sequence 2; sequence 1 was not found

Discussion

- n=4 : the sequence of equilibrium configurations on decreasing A_L/A_G is $4 \rightarrow 4' \rightarrow (3+3)$ on experiments and with the Surface Evolver
- The transitions do not occur at the A_L/A_G predicted by Surface Evolver due to deviations from 2D
- n=5 ; various configurations of a 5-sided PB;

  two experimental sequences but only one Surface Evolver sequence
- 4- and 5- bubble clusters break their symmetry when the PB excess pressure vanishes. The energy is minimal at this point

  For liquid fractions smaller than these minima there are alternative lower energy configurations
- similar to petal clusters; break symmetry (Weaire, Graner , Cox) at energy minimum