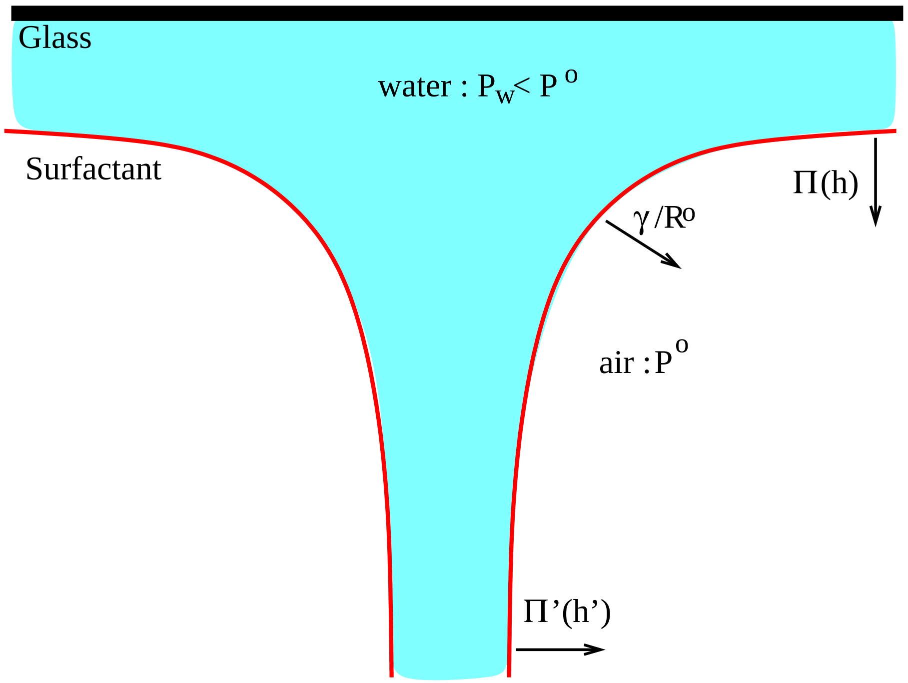
# Dissipation in bubble trains flowing through narrow channels.

**I. Cantat, R. Delannay, J. Etrillard, E. Terriac (PhD)**
collaboration with N. Kern (Trinity College Dublin)

GMCM, UMR 6626, université Rennes I

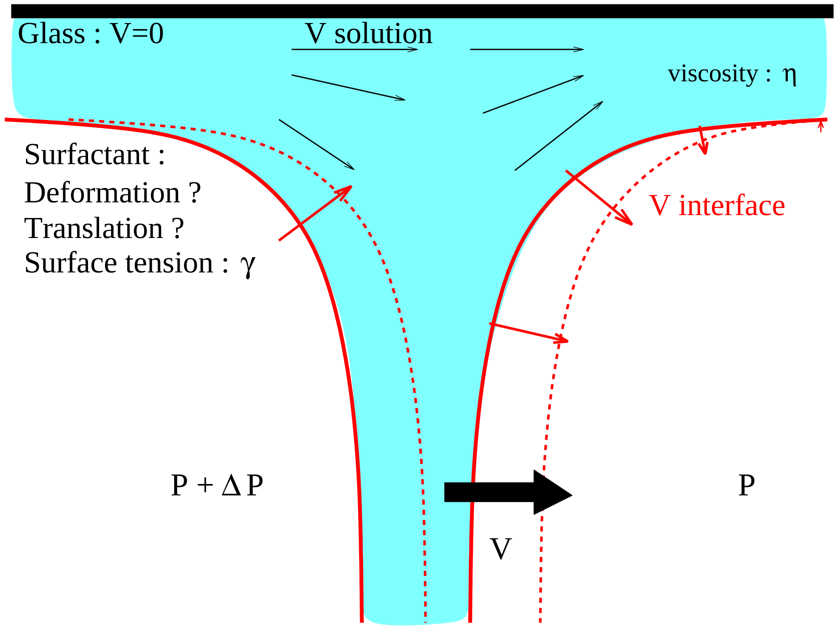
# Equilibrium properties




The pressure gap between air and water is compensated by

- surface tension and curvature
- disjoining pressure and very small thickness

# Flow properties

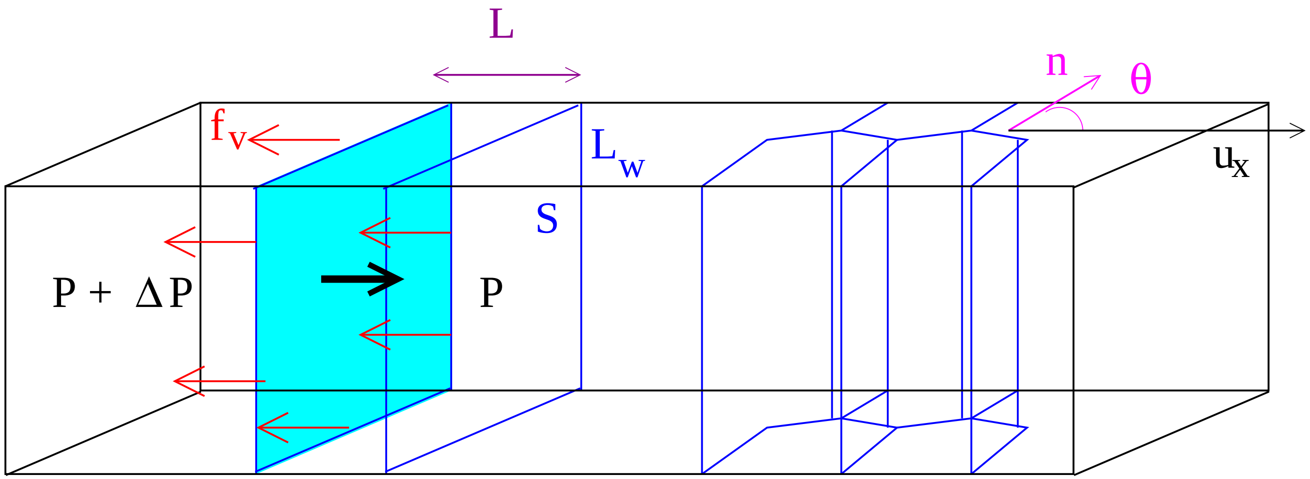



- Inertial terms are negligible.
- Injected power is dissipated instantaneously
- Forces localized around Plateau borders.

## Which pressure difference produces a given velocity ?

Stokes law without free boundary : $\Delta P \sim \eta V$ $\rightarrow$ not true in this situation.

# Force balance on a film




$$S\Delta P = L_w\, f_v(V, r, L, \eta, \gamma, \theta)$$

- $f_v$ force per unit length of PB, $S$ tube section, $L_w$ wetting length
- $V$ film velocity, $r$ PB radius, $L$ distance between 2 films, $\eta$ bulk viscosity, $\gamma$ surface tension, $\theta$ angle between the velocity and the PB normal.
- Other rejected parameter for $f_v$ : tube radius $R$, equilibrium thin film thickness.

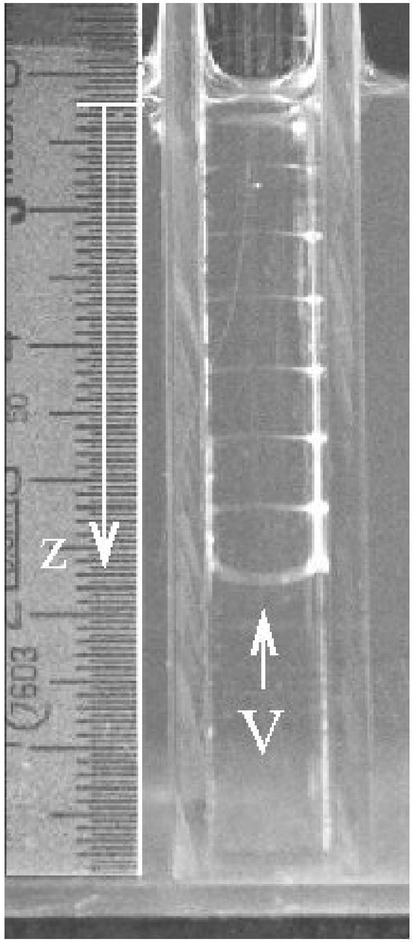
# Dimensional analysis

$$f_v \sim \gamma f(Ca, r/L, \theta) \qquad Ca = \eta V/\gamma$$

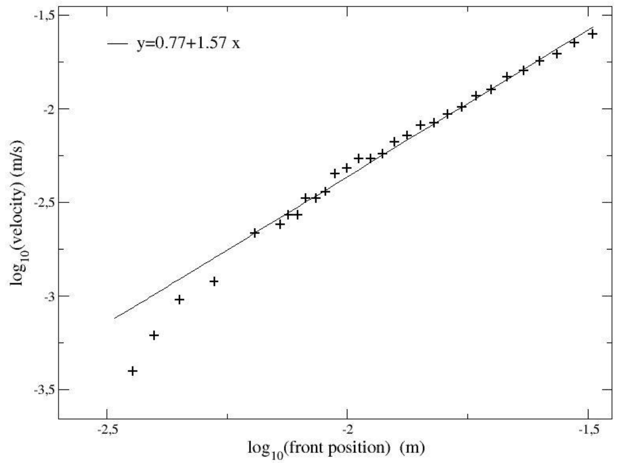
• The liquid fraction $\phi \sim r^2/(RL)$ is not a relevant parameter (it depends on the tube radius)

• The amount of water is characterized by the Plateau border curvature and must appear divided by another length : only L remains.

# Experiments




• The bubble train is produced in the channel by N2 blowing in a dish washing product solution.
• The channel is quickly submerged into a bowl and the foam rises in the channel pushed by the water.

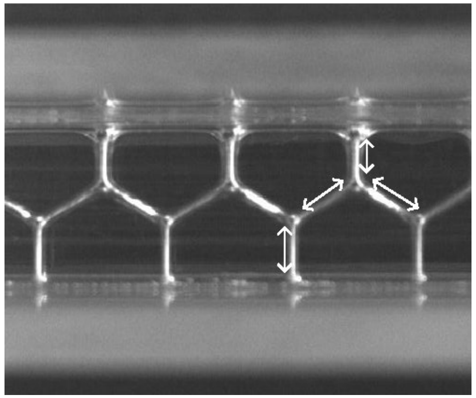



$$S\Delta P = S\rho g z = L_w f_v(dz/dt) \quad (L_w \text{ wetting length})$$

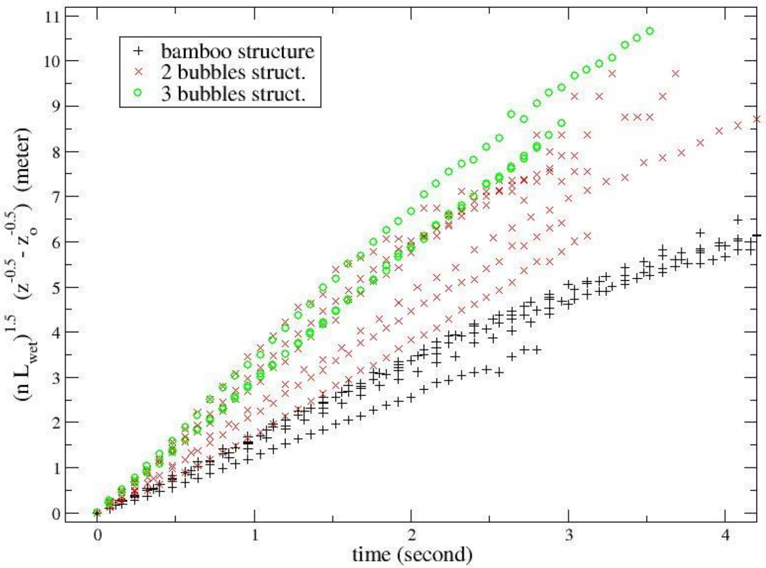
$$f_v(dz/dt) = K dz/dt^{2/3} \quad \text{(large velocities)}$$

$$z = L_w K/(S\rho g)\ dz/dt^{2/3}$$

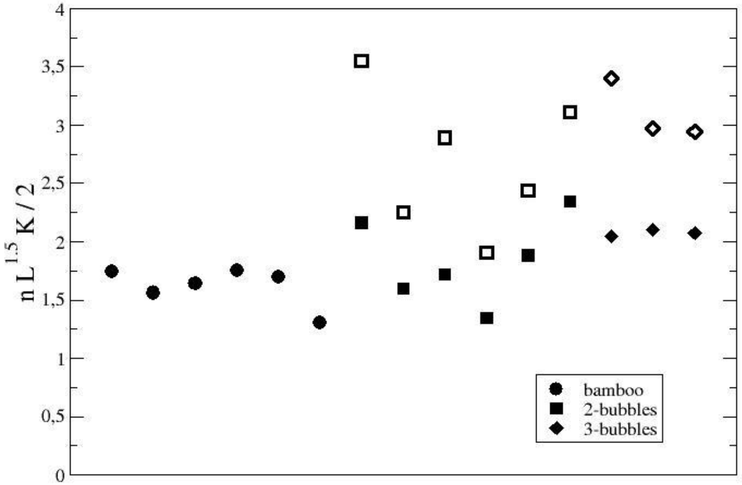
# Influence of the film orientation



Various regular structures have been investigated.

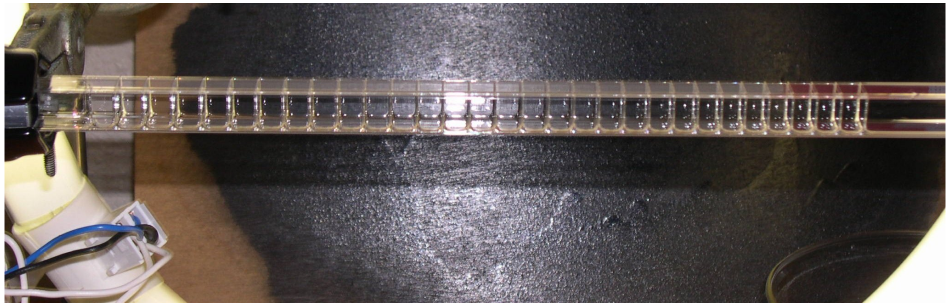


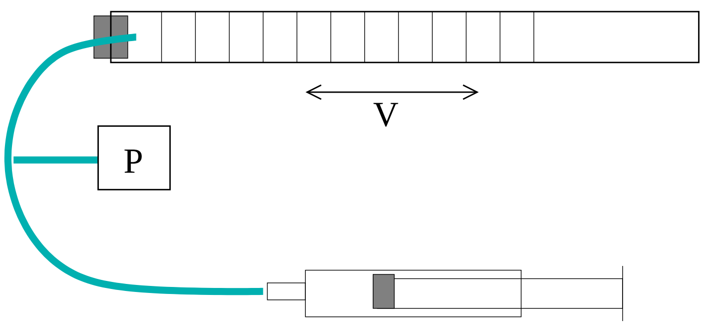

Rescaled results using $L_w$ and assuming a 2/3 power law.




Prefactor of the previous graph, using $L_w$ or $L_w \cdot u_y$

## Influence of the liquid fraction and of the bubble size



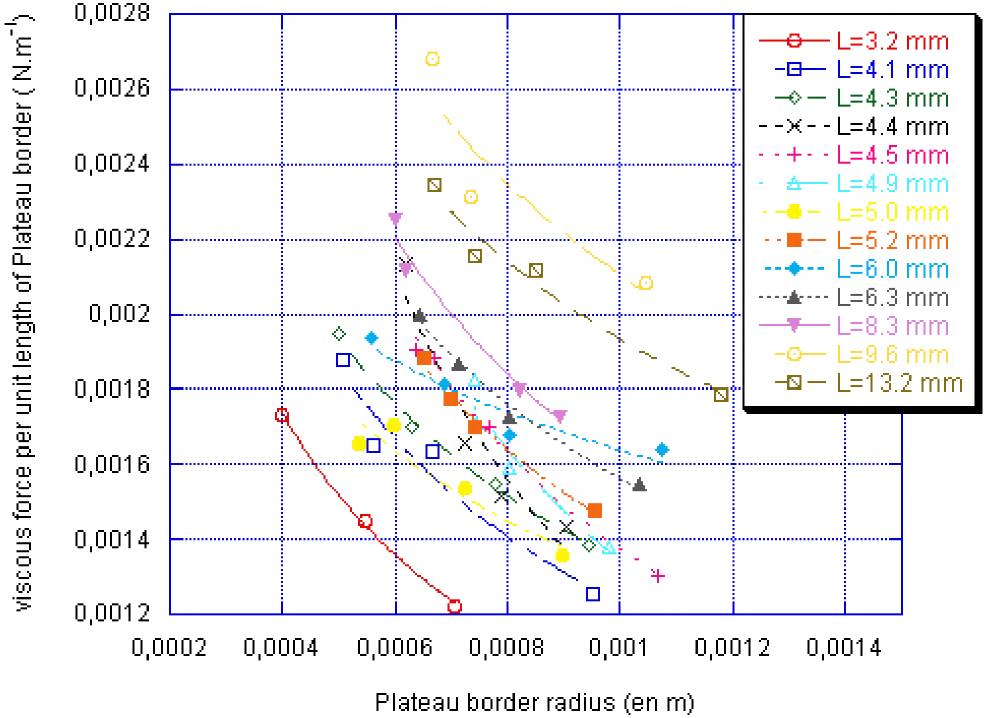


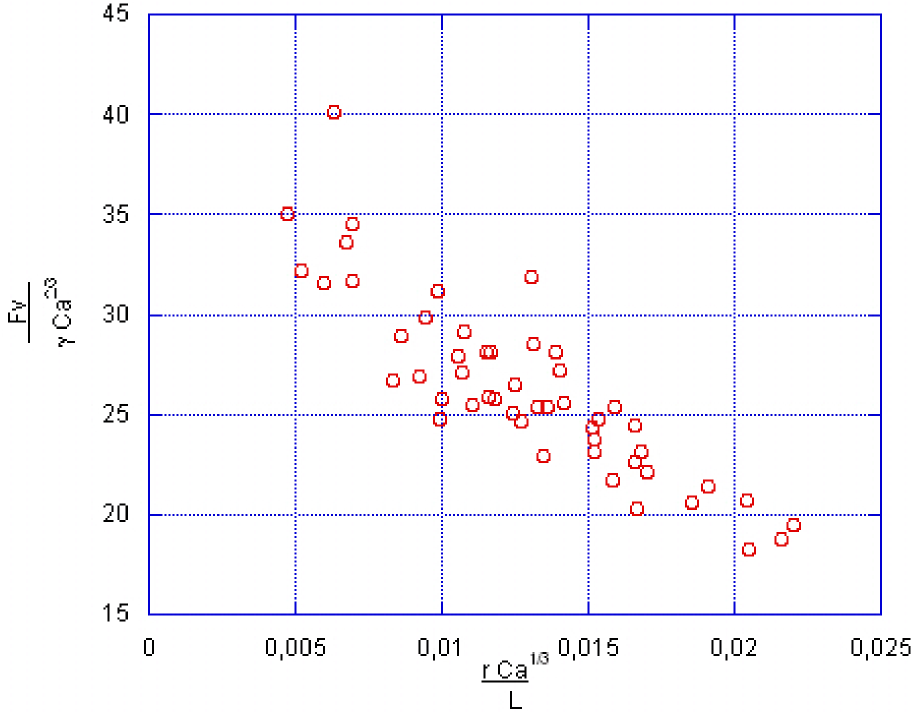

*Experimental set-up*

- dish washing product.
- $\rho L_w r^2 = M$
- $\Delta P$ constant after the first shift : reproducible wetting film.
- Film number n constant.
- Square ($1\text{cm}^2$) or circular (d=3mm) channel section.

# Results and rescaling

square tube, influence of L and r, V = 3 mm/s

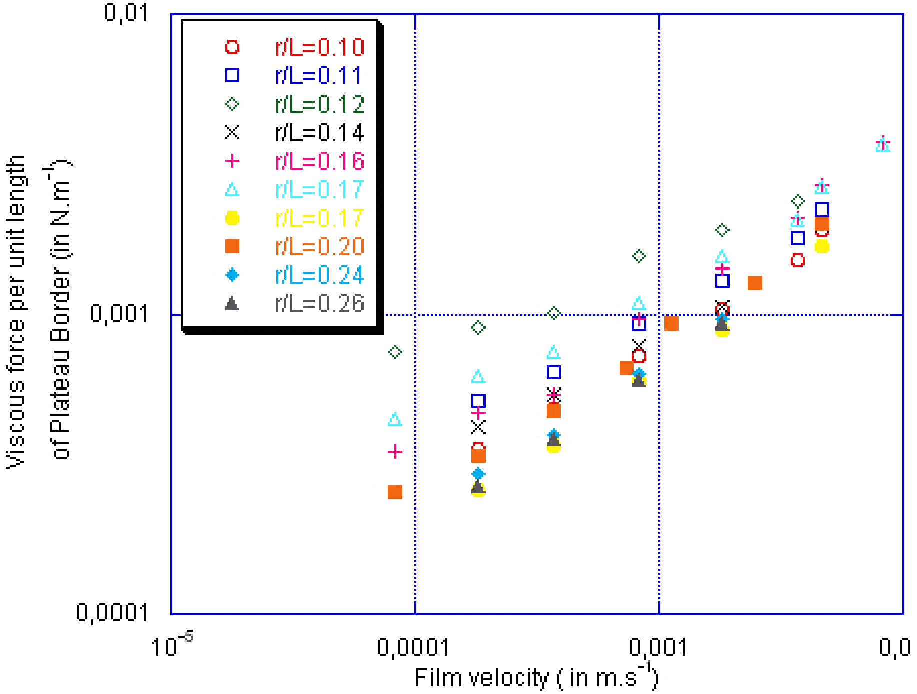


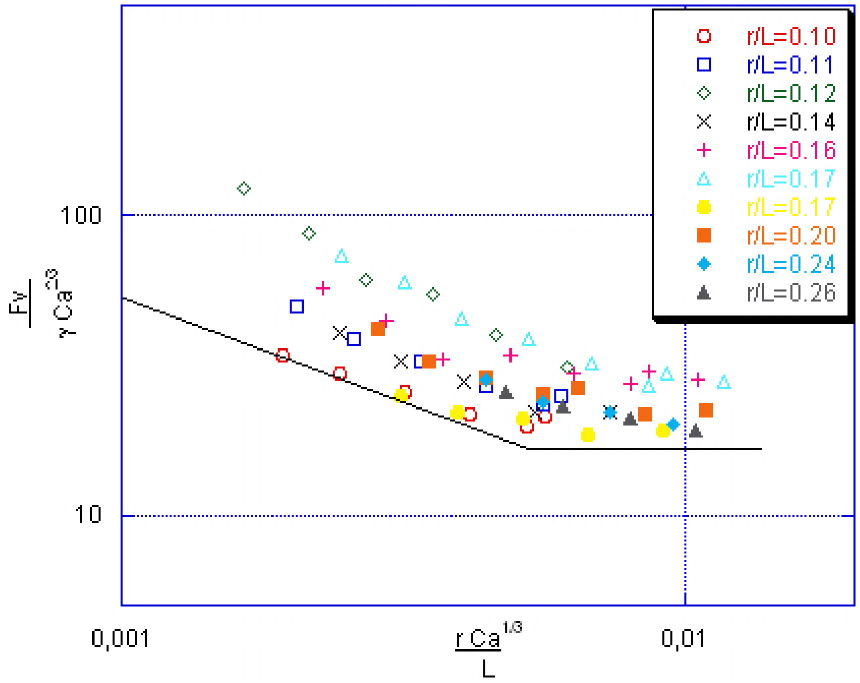

Large L : top , small L, bottom.

$\Rightarrow f_v(r, L) = f_v(r/L)$

# Influence of the velocity



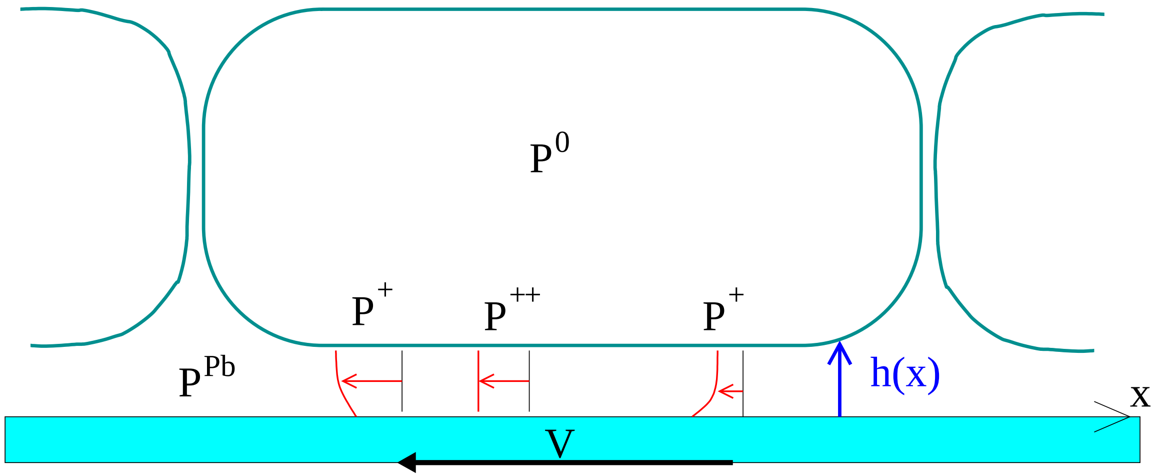

Various power laws, same solution.




slope (1) : $v^{1/2}$; slope (2) : $v^{2/3}$.

$$f_v(Ca, r/L) \stackrel{?}{=} Ca^{2/3} \bar{f}_v(Ca^{1/3} r/L)$$

# Lubrication theory




| | | |
|---|---|---|
| Pressure | $P = \frac{\gamma}{R}$ | $P = -\gamma h_{xx}$ ; $\frac{\partial P}{\partial x} = -\gamma h_{xxx}$ |
| Velocity | Poiseuille + linear term | $v_x = \frac{1}{2\eta}\frac{\partial P}{\partial x} y(y - 2h(x)) - V_0$ |
| Flux | $Q = \int_0^h v_x dy$ | $Q = \frac{\partial P}{\partial x} h^3/(3\eta) + V_0 h$ |

$$\boxed{\frac{\partial^3 h}{\partial x^3} = \frac{3Ca}{h^2}\left(1 - \frac{Q}{V_0 h}\right)}$$

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*Boundary conditions*

curvature :$h_{xx}(0) = P^0/\gamma$ , $h_{xx}(L) = P^0/\gamma$ *origin choice :* $h(0) = h(L)$

*Integral conditions*

*Force* :

$$\frac{F_{l/p}}{\gamma} = \frac{\mu}{\gamma}\int_0^L \frac{\partial v_x}{\partial y} dx = \int_0^L \frac{3Ca}{h}\left(1 - \frac{Q}{V_0 h}\right) dx$$

*Energy* :

$$\frac{F_{l/p} V_0}{\gamma} = \frac{\mu}{\gamma}\int_0^L \int_0^h \left(\frac{\partial v_x}{\partial y}\right)^2 dx dy = V_0 \int_0^L \frac{3Ca}{h}\left(1 - \frac{Q}{V_0 h}\right)^2 dx$$

We deduce the missing condition on the liquid flux :

$$\frac{Q}{V_0} = \frac{\int_0^L h^{-2} dx}{\int_0^L h^{-3} dx}$$

# Rescaled equation

$$\bar{h} = hCa^{-1/3}/L, \qquad \bar{x} = x/L, \qquad \alpha = Q/(V_0 L Ca^{1/3}), \qquad \xi = Ca^{1/3} r/L$$

$$\begin{aligned}
h^{(3)} &= \frac{3}{h^2}\left(1 - \frac{\alpha}{h}\right) \\
h^{(2)}(0) &= h^{(2)}(1) = 1/\xi \\
h(0) &= h(1) \\
\alpha &= \frac{\int_0^1 h^{-2} dx}{\int_0^1 h^{-3} dx} \\
\frac{F}{\gamma Ca^{2/3}} &= 3\int_0^1 h^{-1}\left(1 - \frac{\alpha}{h}\right) dx
\end{aligned}$$

$$\boxed{f_v = \gamma Ca^{2/3} H(Ca^{1/3} r/L)}$$

Denkov prediction : $H(x) = \sqrt{x}$

# Conclusion

• For a given solution, various power laws may be obtained for the force/velocity relation.

• Lubrication analysis allows to partially rescale the various experimental data. The relevant parameter is $\xi = Ca^{1/3} r/L$.

• The monolayer dynamics is probably responsible for the remaining disagreement between theory and experiment.