Coherent-Feedback Formulation of a Continuous Quantum Error Correction Protocol

arXiv:0907.0236

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Talk plan

- Reminder of quantum error correction (QEC).
- Continuous QEC.
- Description of a scheme for coherent-feedback QEC for a simple 3 qubit bit flip code.
- Concluding remarks.

Quantum error correction (QEC)

- Quantum error correction is essential for quantum information processing since qubits are susceptible to decoherence.
- When decoherence alters the state of a qubit, a QEC algorithm acts to restore it to the state prior to decoherence.

QEC principles

- Main ingredient is to introduce reduncancy by encoding a logical qubit into a number of physical qubits.
- Simple 3 qubit code: A *logical* qubit is encoded by 3 *physical* qubits. Logical qubit $|0>_L$ encoded by 3 physical qubits $|000>$ and $|1>_L$ encoded by $|111>$.
- A logical state $a|0>_L + b|1>_L$ is encoded as $a|000> + b|111>$. The 3 qubit codespace is $C = \text{span}\{|000>, |111>\}$.

The 3 qubit bit flip code

- This simple code belongs to a class of QEC codes called *stabilizer codes* (Gottesman, PRA 54, 1862)
- $C = \text{span}\{|000\rangle, |111\rangle\}$
- Correctable errors are single qubit bit flips X_1, X_2 or X_3
- Error is determined by measuring the parity Z_1Z_2 between qubits 1 and 2, and Z_2Z_3 between qubits 2 and 3. Measurement results called the error syndrome.
- Error syndromes are: {1,1} (no error), {-1, 1} (qubit 1 has flipped), {1,-1} (qubit 3 has flipped) and {-1,-1} (qubit 2 has flipped)

Continuous QEC

- Most proposed QEC schemes are discrete. Error detection and recovery operations are done periodically with a sufficiently small period.
- In continuous QEC, the idea is to detect and correct errors continuously as they occur. Suitable for low level continuous time differential equation based models.
- Continuous QEC using continuous monitoring and measurement-feedback: Ahn, Doherty & Landahl et al, PRA 65, 042301; Ahn, Wiseman & Milburn, PRA 65, 042301; Chase, Landahl & Geremia, PRA 77, 032304.

Why coherent-feedback?

- Not necessary to go up to the “macroscopic” level and have interfaces to electronic circuits for measurements. Not limited by bandwidth of electronic devices.
- No classical processing required and avoids challenges imposed by the requirement of such processing; e.g., numerical integration of nonlinear quantum filtering equations in real-time.
- Entirely “on-chip” implementation; a controller can be on the same hardware platform as the controlled quantum system. In particular, in solid state monolithic circuit QED.

Quantum network notation and operations

- Open Markov quantum system G = (S,L,H).
- Concatenation product

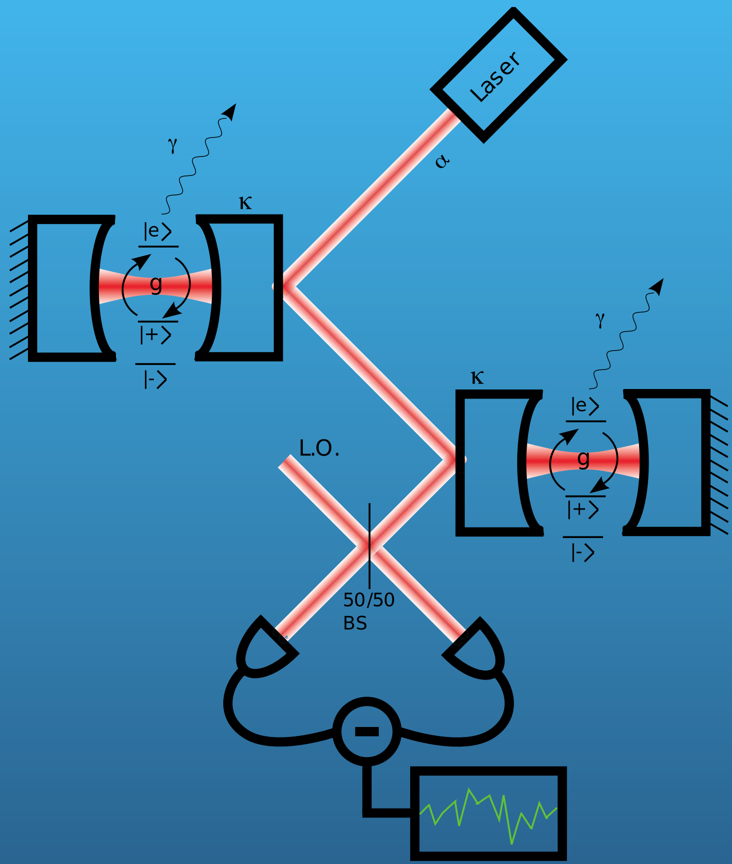
$$G_2 \boxplus G_1 = (S_2, L_2, H_2) \boxplus (S_1, L_1, H_1) = \left(\begin{bmatrix} S_2 & 0 \\ 0 & S_1 \end{bmatrix}, \begin{bmatrix} L_2 \\ L_1 \end{bmatrix}, H_1 + H_2 \right)$$

- Series product

$$\begin{aligned} G_2 \triangleleft G_1 &= (S_2, L_2, H_2) \triangleleft (S_1, L_1, H_1) \\ &= (S_2 S_1, S_2 L_1 + L_2, H_1 + H_2 + \Im\{L_2^\dagger S_2 L_1\}) \end{aligned}$$

Gough & James, IEEE-TAC (to appear), 2009, arXiv:0708.4483

Continuous parity measurement



$$
\begin{aligned}
dU(t) \;=\; & \Big((\sqrt{\kappa} b_1 + \sqrt{\kappa} b_2 + \alpha) dA^*(t) - h.c. \\
& - \frac{(\sqrt{\kappa} b_1 + \sqrt{\kappa} b_2 + \alpha)^* (\sqrt{\kappa} b_1 + \sqrt{\kappa} b_2 + \alpha)}{2} dt \\
& + \sum_{k=1}^{2} \Big(\sqrt{\gamma} \sigma^{(i)} dB^*(t) - h.c. - \frac{\gamma}{2} \sigma^* \sigma dt \Big) \\
& + \frac{\kappa}{2} (b_1^* b_2 - h.c.) dt + g \sum_{i=1}^{2} (\sigma^{(i)*} b_i - h.c.) dt \\
& + \Big(\frac{\bar{\alpha} \sqrt{\kappa}}{2} (b_1 + b_2) - h.c. \Big) dt \Big) U(t)
\end{aligned}
$$

$$
\begin{aligned}
d\bar{U}(t) \;=\; & \big((\Pi_{12} - I) d\Lambda(t) + \alpha \Pi_{12} dA^*(t) - \bar{\alpha} dA(t) \\
& - \frac{|\alpha|^2}{2} dt \Big) \bar{U}(t)
\end{aligned}
$$

$$
\lim_{\substack{\alpha, \kappa \to \infty \\ \frac{\alpha}{\sqrt{\kappa}} = \text{const}}} \; \lim_{g \to \infty} \|(U(t) - \bar{U}(t))\psi\| = 0 \text{ for all } \psi
$$

in the reduced Hilbert space.

Kerckhoff, Bouten, Silberfarb & Mabuchi, PRA 79, 024305

But cannot include bit flip errors!

Modified parity measurement model

- Full single atom-cavity-field model with bit flip errors:

$$
\begin{aligned}
dU(t) &= \Big((\sqrt{\kappa} b + \alpha) dA_1^*(t) - h.c. - \frac{1}{2}(\sqrt{\kappa} b + \alpha)^* (\sqrt{\kappa} b + \alpha) dt \\
&\quad + \sqrt{\Gamma} \sigma_X dA_2^*(t) - h.c. - \frac{\Gamma}{2} \sigma_X^* \sigma_X dt \\
&\quad + \sqrt{\gamma} \sigma dA_3^*(t) - h.c. - \frac{\gamma}{2} \sigma^* \sigma dt + g(\sigma^* b - h.c.) dt \Big) U(t)
\end{aligned}
$$

- Reduced atom-field model with bit flip errors:

$$
\begin{aligned}
d\bar{U}(t) &= \Big((Z - I) d\Lambda_{11}(t) + \alpha Z dA_1^*(t) - \bar{\alpha} dA_1(t) \\
&\quad + \sqrt{\Gamma} X dA_2^*(t) - h.c. - \frac{1}{2}(|\alpha|^2 + \gamma) dt \Big) \bar{U}(t)
\end{aligned}
$$

$$
\lim_{\substack{g, \kappa \to \infty \\ \frac{g}{\kappa} = \text{const}}} \|(U(t) - \bar{U}(t))\psi\| \text{ for all } \psi \text{ in the reduced Hilbert space.}
$$

Modified parity measurement model

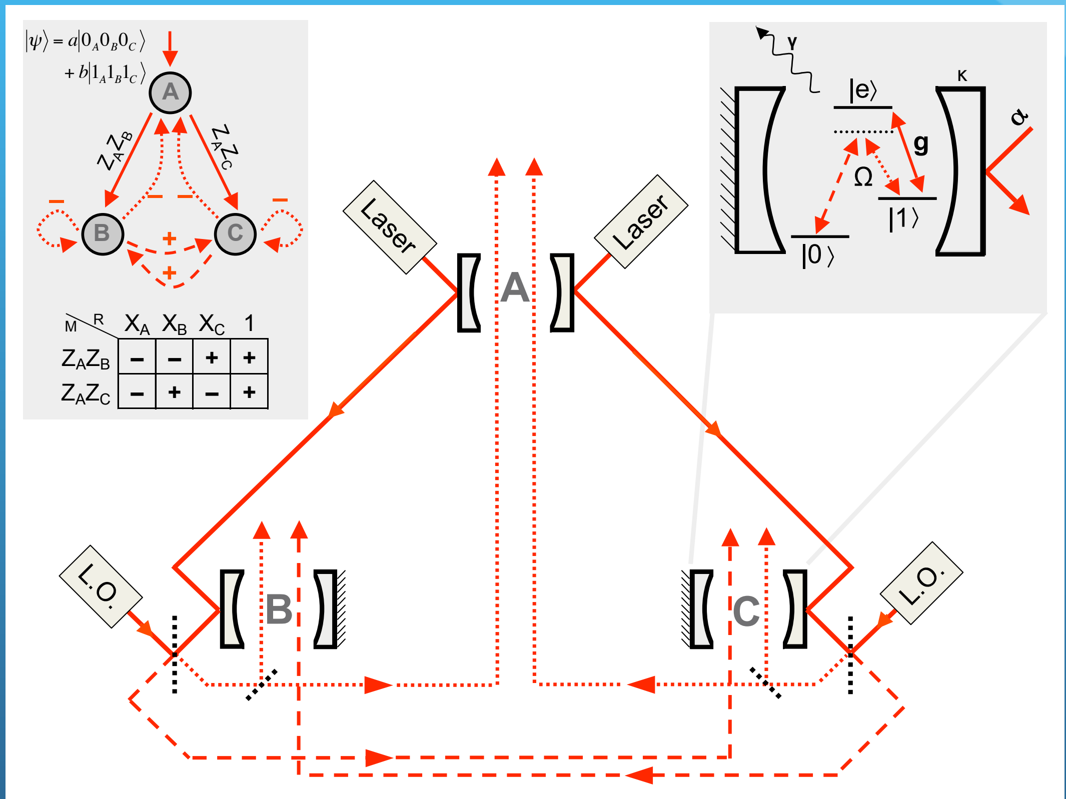
- The reduced model can be written as:

$$\left(\begin{bmatrix} Z & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 0 \\ \sqrt{\Gamma} X \end{bmatrix}, 0 \right) \triangleleft \left(I, \begin{bmatrix} \alpha \\ 0 \end{bmatrix}, 0 \right)$$

- Reduced model for two coherently driven cavities:

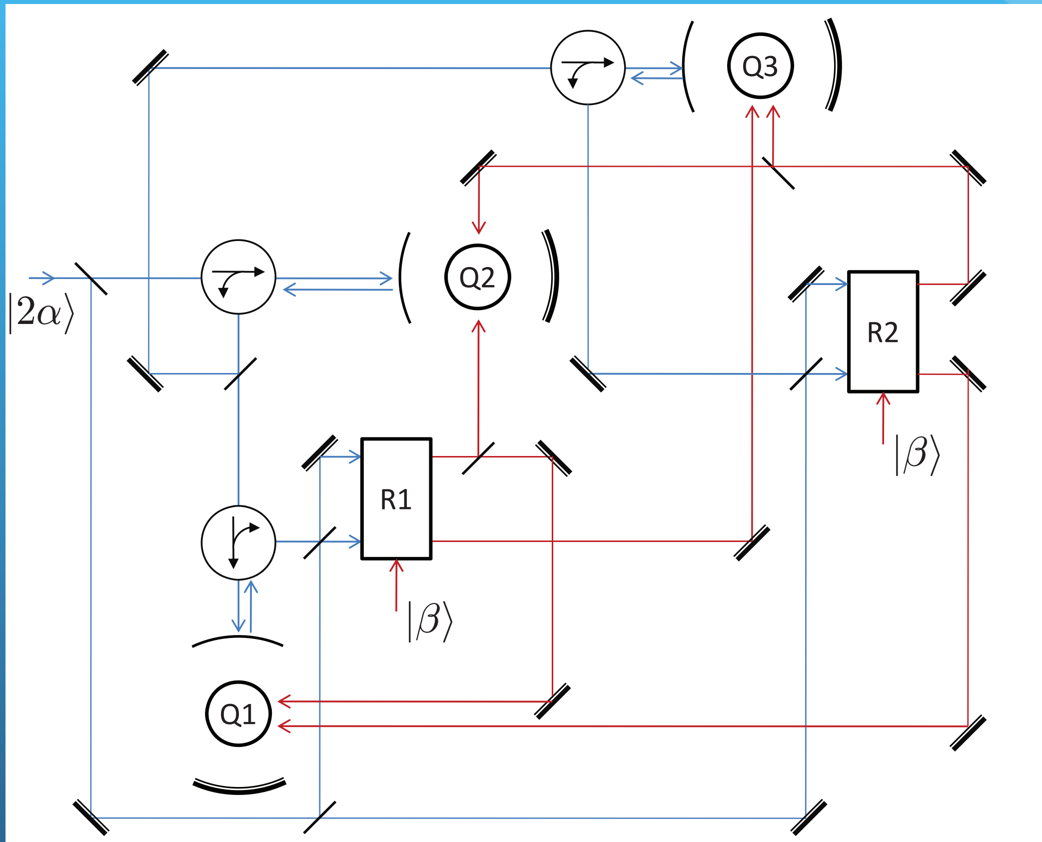
$$\left(\begin{bmatrix} Z_2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ \sqrt{\Gamma} X_2 \end{bmatrix}, 0 \right) \triangleleft \left(\begin{bmatrix} Z_1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \begin{bmatrix} 0 \\ \sqrt{\Gamma} X_1 \\ 0 \end{bmatrix}, 0 \right) \triangleleft \left(I, \begin{bmatrix} \alpha \\ 0 \\ 0 \end{bmatrix}, 0 \right)$$
$$= \left(\begin{bmatrix} Z_2 Z_1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \begin{bmatrix} \alpha Z_2 Z_1 \\ \sqrt{\Gamma} X_1 \\ \sqrt{\Gamma} X_2 \end{bmatrix}, 0 \right)$$

The “bare bones” of it



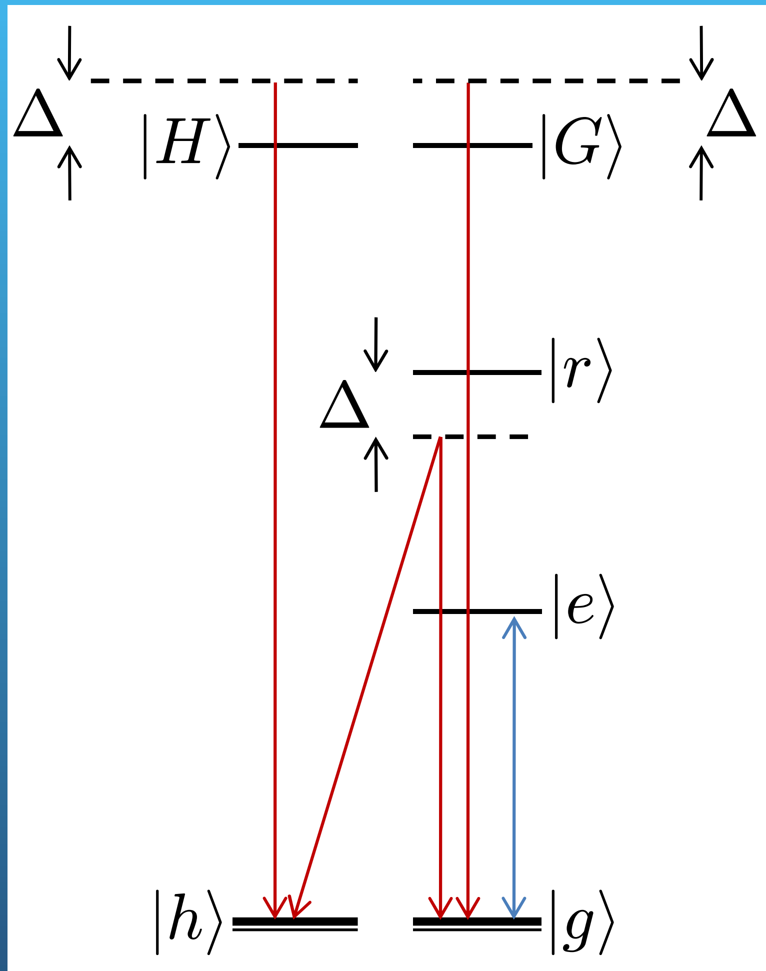
- Coherent lasers drive atomic Raman transitions between the two ground states of the atom to correct bit flips.
- To make it work, need more than this …

The actual scheme



- Quantum switches R1, R2 inserted to facilitate switching to higher amplitude bit flip correcting Raman lasers.

Qubit atomic level scheme



Raman transitions with ac Stark shift compensation

- Given by the terms:

ac Stark shift compensation

$$\left(1, \sqrt{\gamma}\left(\sigma_{gr} + \sigma_{gG}\right), \Delta\left(\frac{1}{2}\Pi_r - \Pi_G\right)\right)$$

Raman transitions

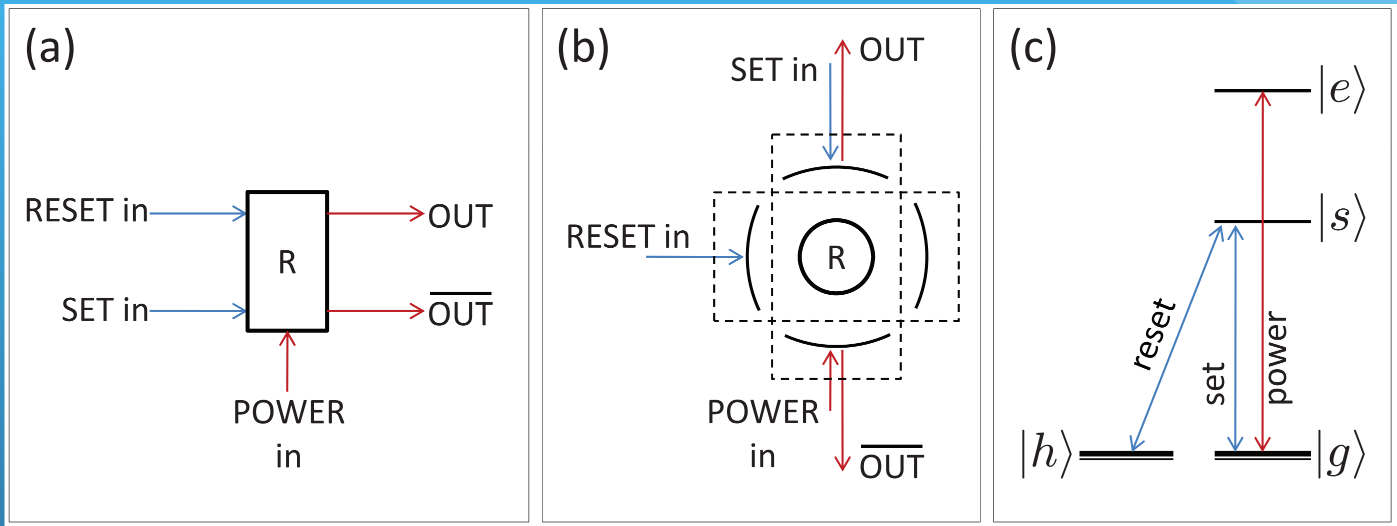
$$\boxplus \left(1, \sqrt{\gamma}\left(\sigma_{hr} + \sigma_{hH}\right), \Delta\left(\frac{1}{2}\Pi_r - \Pi_H\right)\right)$$

- Together with coupling to probe laser becomes:

$$\left(\begin{bmatrix} Z & 0 \\ 0 & 1 \end{bmatrix}, 0, 0\right) \boxplus \left(1, \sqrt{\gamma}\left(\sigma_{gr} + \sigma_{gG}\right), \Delta\left(\frac{1}{2}\Pi_r - \Pi_G\right)\right)$$

$$\boxplus \left(1, \sqrt{\gamma}\left(\sigma_{hr} + \sigma_{hH}\right), \Delta\left(\frac{1}{2}\Pi_r - \Pi_H\right)\right)$$

QSDE model for switch/relay

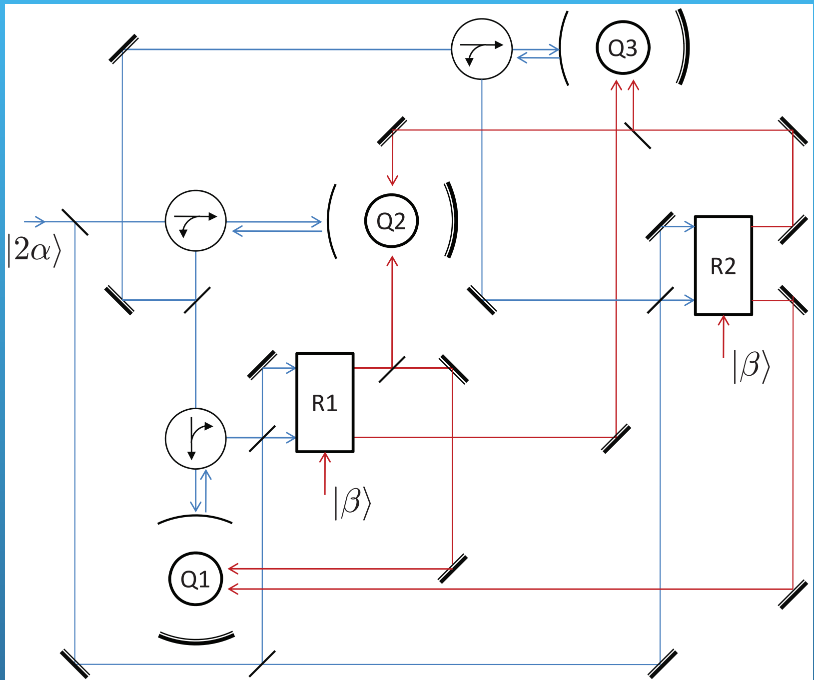


- Simple QSDE model:

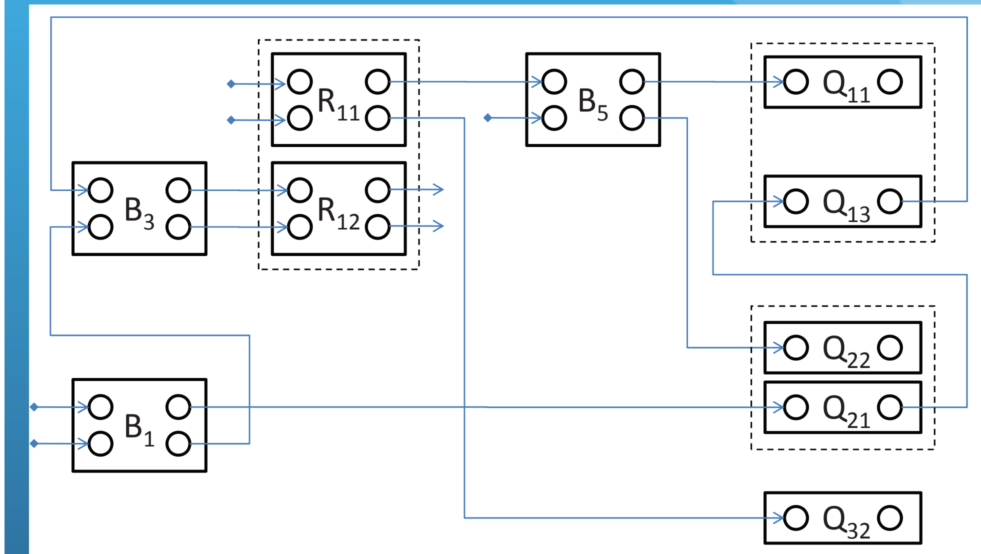
$$\left(\begin{bmatrix} \Pi_g & -\Pi_h \\ -\Pi_h & \Pi_g \end{bmatrix}, \begin{bmatrix} \beta\Pi_g \\ -\beta\Pi_h \end{bmatrix}, 0 \right) \boxplus \left(\begin{bmatrix} \Pi_h & -\sigma_{hg} \\ -\sigma_{gh} & \Pi_g \end{bmatrix}, 0, 0 \right)$$

Detailed physical model: H. Mabuchi, arXiv:0907.2720

QEC network description



- (Modified) half network diagram



Half network $G_p \boxplus G_f$ (p=probe path, f=feedback path):

$$G_p = R_{12} \lhd B_3 \lhd ((Q_{13} \lhd Q_{21}) \boxplus (1,0,0)) \lhd B_1,$$

$$G_f = (Q_{11} \boxplus Q_{32} \boxplus Q_{22}) \lhd (B_5 \boxplus_2 (1,0,0)) \lhd (R_{11} \boxplus (1,0,0))$$

QEC network master equation

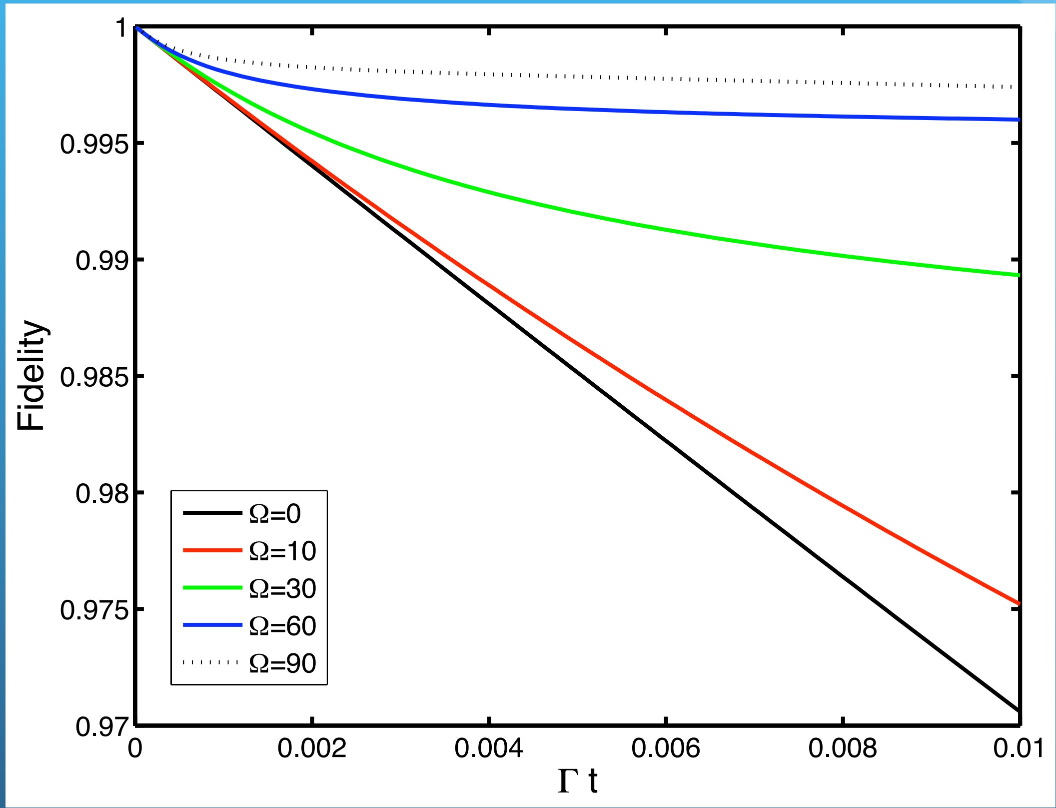
- The QEC network master equation in the limit that $\Delta, \beta \to \infty$ with β^2/Δ constant is:

$$\dot{\rho}_t = -i[H, \rho_t] + \sum_{i=1}^{7} \left(L_i \rho_t L_i^* - \frac{1}{2}\{L_i^* L_i, \rho_t\} \right),$$

where $(\Omega \equiv \beta^2/\gamma\Delta)$,

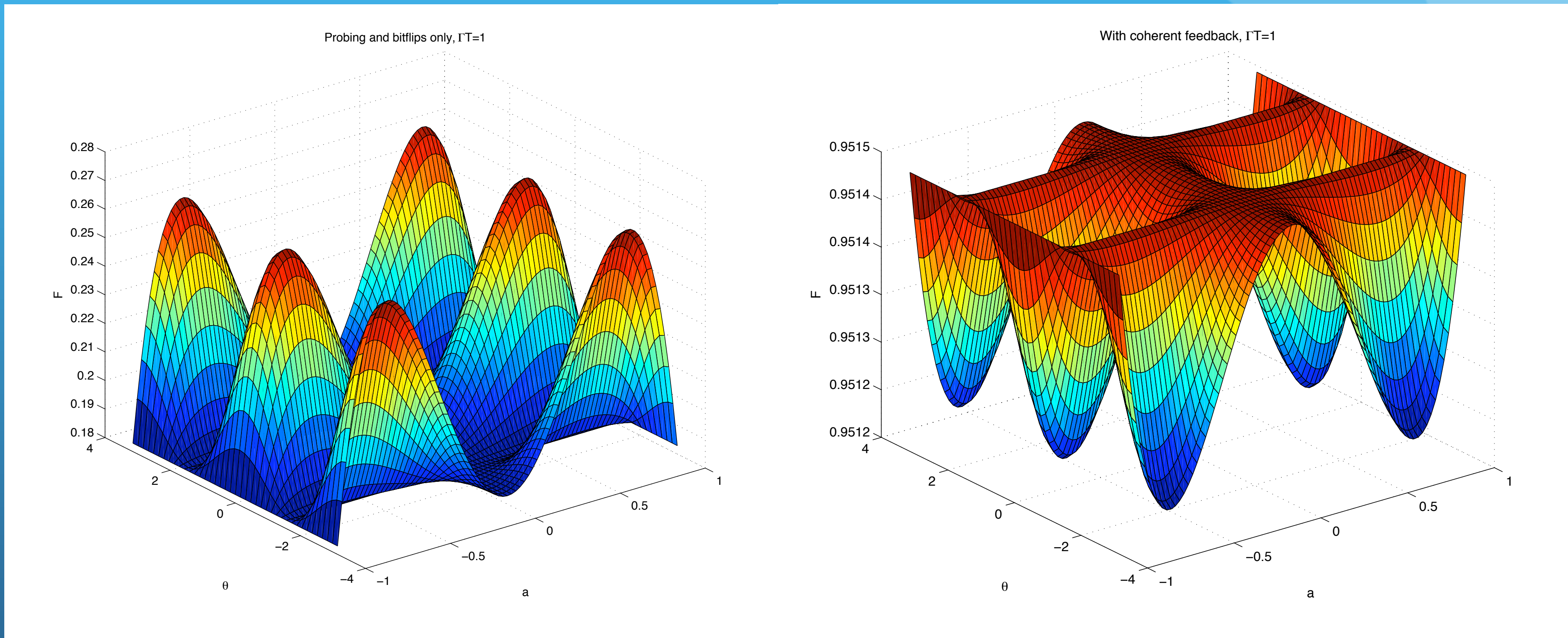
$$\begin{aligned}
H &= \sqrt{2}\Omega \Pi_g^{(R_1)} \Pi_h^{(R_2)} X_1 + \sqrt{2}\Omega \Pi_h^{(R_1)} \Pi_g^{(R_2)} X_3 \\
&\quad - \Omega \Pi_g^{(R_1)} \Pi_g^{(R_2)} X_2, \\
L_1 &= \frac{\alpha}{\sqrt{2}} \left\{ \sigma_{hg}^{(R_1)} (1 + Z_1 Z_2) + \Pi_h^{(R_1)} (1 - Z_1 Z_2) \right\}, \\
L_2 &= \frac{\alpha}{\sqrt{2}} \left\{ \sigma_{gh}^{(R_1)} (1 - Z_1 Z_2) + \Pi_g^{(R_1)} (1 + Z_1 Z_2) \right\}, \\
L_3 &= \frac{\alpha}{\sqrt{2}} \left\{ \sigma_{hg}^{(R_2)} (1 + Z_3 Z_2) + \Pi_h^{(R_2)} (1 - Z_3 Z_2) \right\}, \\
L_4 &= \frac{\alpha}{\sqrt{2}} \left\{ \sigma_{gh}^{(R_2)} (1 - Z_3 Z_2) + \Pi_g^{(R_2)} (1 + Z_3 Z_2) \right\}, \\
L_5 &= \sqrt{\Gamma} X_1, \quad L_6 = \sqrt{\Gamma} X_2, \quad L_7 = \sqrt{\Gamma} X_3.
\end{aligned}$$

Ideal QEC network performance



Fidelity = $< \psi_0 |\rho(t)| \psi_0 >$; Simulation parameters: $\Gamma = 0.01$, $\alpha = 10$, $|\psi_0> = (|ggg> - |hhh>)/\surd 2$; $\rho(0) = |\psi_0> < \psi_0|$.

Ideal QEC network performance



Fidelity = $\langle \psi_0 | \rho(T) | \psi_0 \rangle$; Simulation parameters: $\Gamma = 0.01$, $\alpha = 10$, $\beta = 30$,
$|\psi_0\rangle = a\,|ggg\rangle + \sqrt{(1-a^2)}e^{i\theta}|hhh\rangle$, a in $[-1,1]$, θ in $[-\pi,\pi]$; $\rho(0) = |\psi_0\rangle\langle\psi_0|$.

Concluding remarks

- We have proposed a coherent-feedback QEC scheme for a simple 3 qubit QEC code that protects against single bit flip errors; can be easily adapted to a 3 qubit phase flip code.
- Simulations of the QEC network master equation indicates the scheme can slow down decoherence due to single bit flips.
- Ideas for the future: Adaptation to more complex stabilizer codes, but necessarily also with more complex quantum circuits. Perhaps also to non-stabilizer codes (more challenging?).