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GNOMine: An Application of the Generic GNOME Model to the Representation of Mines

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Abstract

This report presents a mine model, which is based on the generic object model GNOME ([LHC96]). This model allows the representation of the mine geometry, but also of other mine's attributes that are surface roughness, rock type, obstacles (which are not included in geometry), and lighting. All the GNOME's formalism is applied to the mine. Each attribute is individually described, *i.e.* the decomposition tree and relation graph are given, and inter-attribute relations are described. These inter-attribute relations allow, for example, the association of surface roughness and rock type to, among others, mine walls and obstacles, or the positioning of obstacles and lightings into the mine.

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1 Introduction

To provide an example of an instance of GNOME, a generic object model introduced in [LHC96], we have developed a mine model: GNOMine. An instance of the model, *i.e.* a representation of a particular mine can be used to create road-books (a qualitative path) or plan paths following different criteria, such as lighting conditions or presence of obstacles.

This model has been used to create a virtual mine for the purpose of studying navigational problems such as exploration, obstacle detection, intersection recognition ([Bér96]), autonomous navigation, *etc.* This aspect is not addressed in this report.

2 The Different Attributes

The different represented attributes are geometry, surface roughness, rock type, obstacles, and lighting. The purpose of geometry is evident given the task to accomplish. It is used to represent the navigation environment and also to support all other informations. The surface roughness and rock type constitute a clue of the sensors' response quality. The (known) obstacles are represented in order to avoid them or to plan free paths. The lighting knowledge allows to plan paths based on this information (*e.g.* paths that are illuminated everywhere). Note that in a more precise context, such as navigation between a digging point and a dumping point, it may be interesting to have a "site" attribute to represent these important places.

3 Geometry

3.1 The Decomposition Tree

The first level of the tree is made of the *mine* (M). This attribute-object is present in order to serve as a root for the decomposition tree.

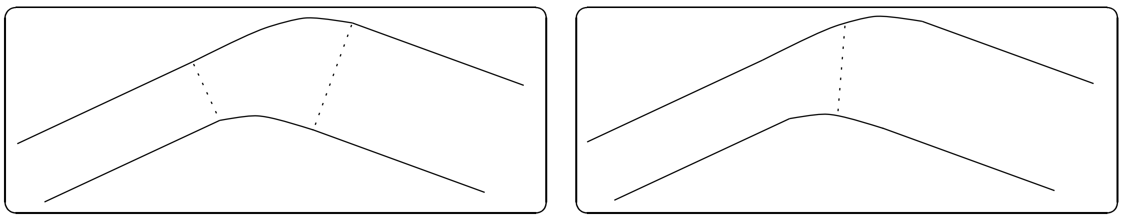
At the second level, the mine is decomposed into *galleries* (G_i), *rooms* (R_i), and *intersections* (I_i). Rooms are places where vehicles have tasks to accomplish: digging, shoveling, gas filling, *etc.* They must thus be represented as attribute-objects. In galleries, the only vehicles' task is to navigate. Galleries connect rooms and, incidently, intersections. However, since galleries have to be represented at a finer level for the control of the vehicles they contain, they have to be represented as attribute-objects and not as relations between attribute-objects. An intersection is a place where several (at least three at this level of the representation[1]) galleries meet.

At a finer level, galleries are decomposed into *gallery sections* (GS_i) and *intersections*[2]. The galleries' decomposition is necessary in order to describe their shape. Galleries can possess complex shape and representing them by a *single* part is generally not feasible. A

[1] We will see that intersections between only two gallery sections can also exist.

[2] So, it is possible to have an intersection between *two* gallery sections. It will be shown later that it is possible to have more than two gallery sections connected to an intersection: this is the case when at least three galleries are connected to an intersection (thus at a higher level in the decomposition). Thus, intersections do exist at several level in the hierarchy.

gallery's decomposition into gallery sections is thus guided by its shape (*i.e.* its width, height, and axis equation, *e.g.* straight, circular, or clothoidal, see Section 3.3)[3]. The decomposition thus implies that gallery sections will be represented by "simple" parametric equations (see Section 3.3.4). On the other hand, we have to represent the little part which links two gallery sections, whose geometry is (or can be) "complex." This little part is in fact an intersection and allows us to have a simple representation for gallery sections, in the sense that it is not necessary to have a complex equation to represent an essentially simple form, except for the extremities (Figure 1). The simple equation for the gallery sections allows a



(a) With intersection (b) Without intersection

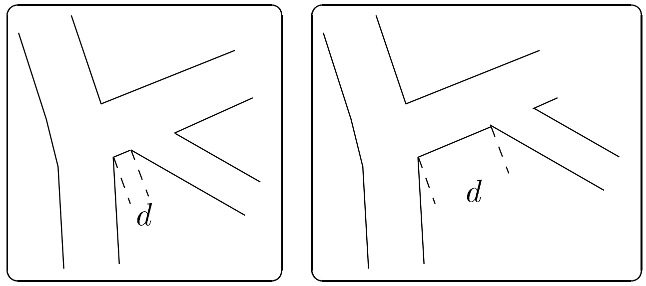
Figure 1: With or without intersection? Representing the transition between the two "simple" gallery's parts with an intersection allows us not to use a complex representation for these parts (in the current case, allows us not to represent an essentially straight form with, *e.g.*, splines or high degree polynomials for the little not straight section)

simple navigation mode in this part of the mine. Finally, the little part between two gallery sections should not be represented by an attribute-object, but only by a connection relation. In this case, it becomes impossible to associate to the corresponding mine geometric part an attribute-object like surface roughness. Thus, an intersection is placed between two gallery sections of a same gallery[4].

The intersections can be decomposed into other (more elementary) intersections and gallery sections or in a set of *walls* (W), a *floor* (F), and a *ceiling* (C). The first decomposition is interesting because there are cases where it is not clear whether several meeting galleries form *one* or *several* intersections. Often, the distinction is not unique and is subject to interpretation (Figure 2). Thus, it is interesting to have a hierarchy of intersections. In the case where one intersection is decomposed into two (or more) intersections, these ones are linked by gallery sections and not by galleries, because the distance between these intersections is "small". This decomposition can be arbitrary, *e.g.* based on a threshold on distance d (Figure 2), or based on an objective criterion like visibility ([Bér96]). The second decomposition, the one where an intersection is decomposed into walls, ceiling, and floor, takes place when the intersection is at the most elementary level, *i.e.* when it is not any more possible (or desirable) to decompose it into other intersections.

[3]The decomposition could also be guided by the roughness or the rock composition of the gallery's walls.

[4]This is not the case when there is no need for a little geometric part between two gallery sections, *i.e.* when the two sections have the same width and height and their axes are aligned.



(a) One intersection (b) Two intersections

Figure 2: One or two intersections? At what distance d are there two intersections?

Walls are vertical parts of the intersection joining the ends of adjacent galleries. There is thus as many walls as galleries connected to the intersection. As for elementary intersections, rooms are decomposed into a set of walls, a floor, and a ceiling.

Walls are split into *wall sections* (WS_i). The decomposition is based on criteria of representability of wall sections with a "simple" equation. In particular, a break must be present in places of high curvature (*i.e.* at wall's angles). A decomposition in sections with constant curvature could be considered (see, for example, [DT91] for such a decomposition). This leads to a representation based on circular arcs (see Section 3.3.3 for the application of quantification of wall sections). However, this introduces a large number of sections and it could be preferable to use more "complex" equations, like splines, for example. In this case, a decomposition such as in [LCF+94] (applied to 2D) can be done: data are approached by splines with a given discretization and, where the error is too large, a finer discretization is used. The wall sections provide a "precise" walls' quantification, which allows the planning of a path from one gallery section to an other.

In order to allow for various surface roughnesses and rock types on different gallery section faces, gallery sections are decomposed into two gallery walls (GW), a floor, and a ceiling.

Figure 3 shows the decomposition relation in a generic manner.

Figure 4 shows a mine example (the fine decomposition being not shown). Figure 5 shows the mine decomposition tree. For clarity reasons, gallery decomposition is only shown for gallery G_3, room decomposition for room R_3, wall decomposition for wall W_6, and gallery section decomposition for gallery section GS_1.

For the mine in Figure 4, the set Ω_0 of attribute-objects of geometric attribute implied by decomposition is thus:

$$
\begin{aligned}
\Omega_0 = \{ & \\
& \mathrm{M}, \\
& \mathrm{R}_1, \mathrm{R}_2, \mathrm{R}_3, \mathrm{G}_1, \cdots, \mathrm{G}_5, \mathrm{I}_1, \\
& \mathrm{GS}_1, \cdots, \mathrm{GS}_K, \mathrm{I}_2, \cdots, \mathrm{I}_N, \\
& \mathrm{F}_1, \cdots, \mathrm{F}_{N+2K+3}, \mathrm{C}_1, \cdots, \mathrm{C}_{N+2K+3}, \mathrm{W}_1, \cdots, \mathrm{W}_{10}, \\
& \mathrm{WS}_1, \cdots, \mathrm{WS}_L,
\end{aligned}
$$

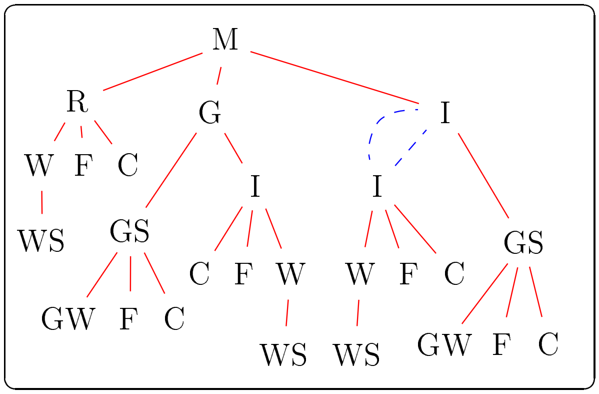


Figure 3: The generic tree of decomposition for geometry. Dashed lines indicates possible recursion. (One level of recursion is shown.)

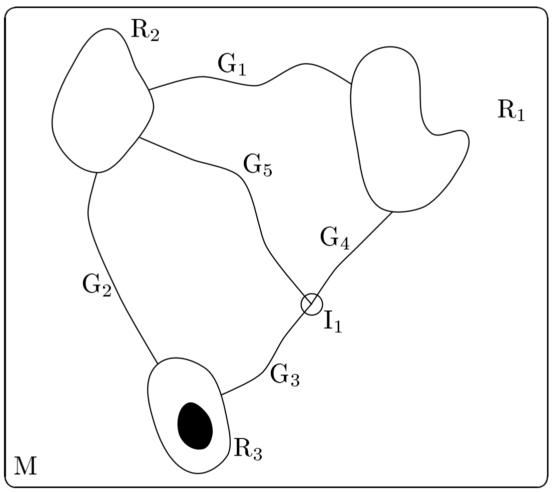


Figure 4: A mine

$\mathrm{GW}_1, \cdots, \mathrm{GW}_{2K}$

},

where K is the number of gallery sections, N the number of intersections, and L the number of wall sections. The $N{+}2K{+}3$ ceilings and floors are 1 for each room, N for the intersections, and $2K$ for the gallery sections.

3.2 Relations Between Attribute-objects

Two relations are represented; both are connection relations. One is qualitative, the other is quantified in order to show how the connection occurs. The graph representing these relations is shown in Figure 5. All connection relations are bidirectional, *i.e.* if an attribute-object is connected to a second one, then the second is also connected to the first. But the quantification (if there is one) is not to be taken as is in both directions. It must be reversed.

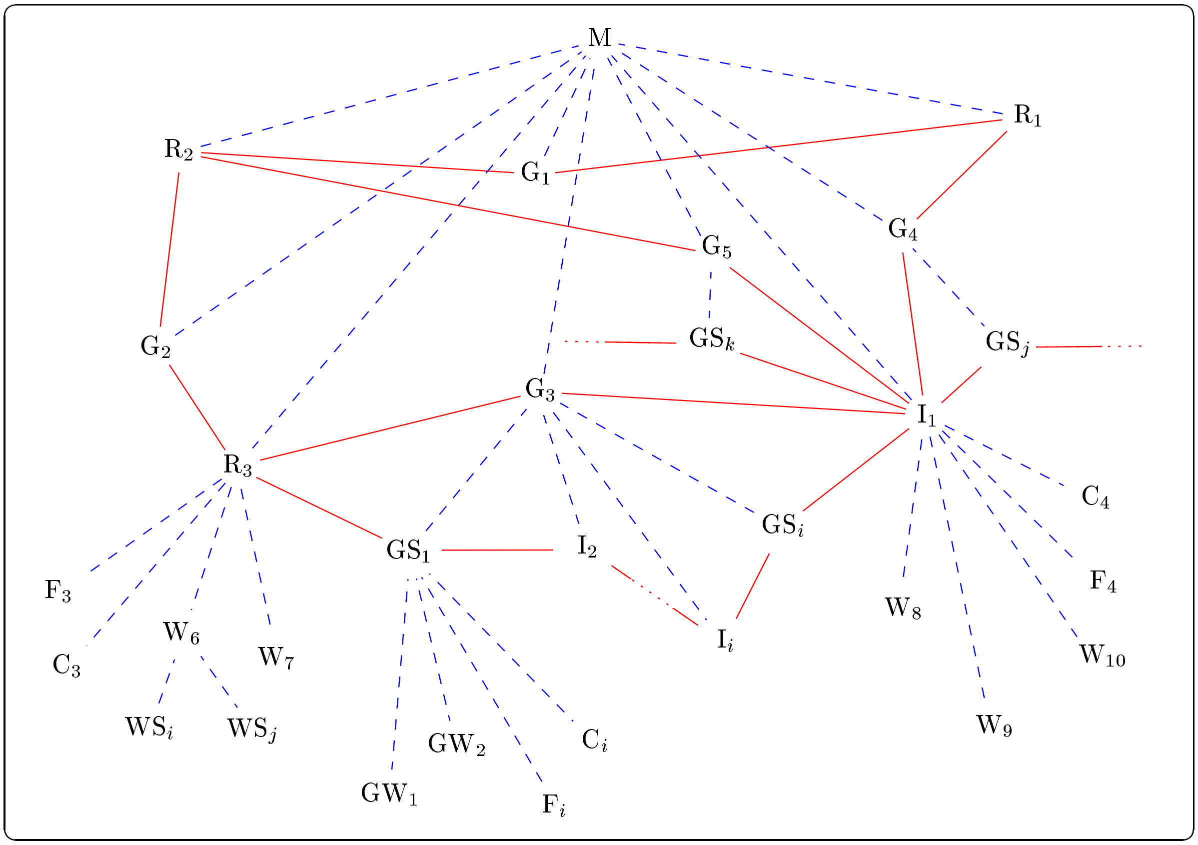


Figure 5: The mine geometric decomposition tree ($\Theta(\Omega_0)$) and relations graph between geometric attribute-objects ($\Gamma(\Omega_0)$). Decomposition is drawn with dashed lines, connection with continuous lines. All decompositions and connections are not shown.

As has already been said, the first connection relation is qualitative. It is noted c. It is applied to galleries, intersections, and rooms. This relation can be used to create a road-book to go from one room to an other one, for example.

For the second relation, some reference systems must be defined for various attribute-objects.

Two reference systems are associated to each gallery section. Their origin is on the section's axis (at a null height) at both extremities. The z axis is tangent to the section axis and points to the other extremity (*i.e.* through increasing curvilinear abscissa), the x axis is horizontal while the y axis is vertical, and the three form a direct orthogonal reference system[5] (Figure 6). The reference systems are arbitrarily numbered 1 and 2.

Reference systems are also associated to rooms and intersections. Each possesses one reference system by gallery connected to it. Each reference system corresponds to the one

[5]In fact, both reference systems can be "slided" to all gallery section axis points, and particularly to both extremities. A reference system "slided" to the other extremity corresponds to the reference system of this extremity if it is rotated by π around its y axis. This will be useful to calculate the orientation of a gallery section in the immediate intersection neighborhood.

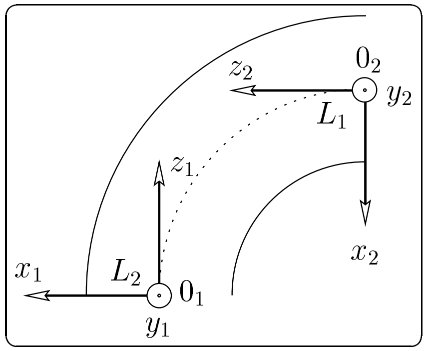


Figure 6: Reference systems associated to a gallery section

of the gallery section connected at this point, rotated by π around its y axis. In other words, this reference system corresponds to the one of the other gallery section extremity slided to this extremity. This way, the reference systems are oriented to the intersection's (or room's) interior. Figure 7 shows reference systems associated to an intersection. The

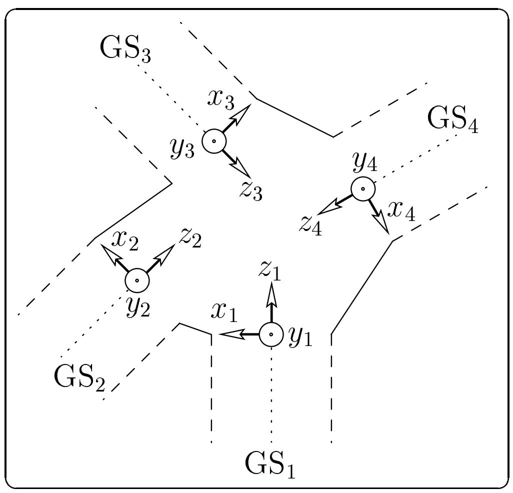


Figure 7: Reference systems associated to an intersection

reference systems of a room or intersection are numbered from 1 to n, where n is the number of galleries connected to the room or intersection. The numbering order is arbitrary. For the rooms and intersections' quantification (Section 3.3.5 for the intersections), the reference system number 1 is used as a reference to position the $n-1$ other reference systems. The same reference system is also used to position wall sections (Section 3.3.3), obstacles (Section 8.3), and lighting (Section 8.4).

All reference systems have their origin at the floor level or, more precisely, at a null height.

The quantitative connection relation indicates the way gallery sections are attached to intersections and rooms. This relation is called *attachment* and noted a. An attachment between a gallery section and an intersection or room is specified as two reference systems, one being a gallery section's reference system and the other an intersection or room's reference system. Let us note that the relative pose of the two attribute-objects is implicit because both reference systems correspond when one is rotated by π around its y axis.

The graph $\Gamma(\Omega_0)$ representing relations between geometric attribute-objects is thus the following set:

$$
\begin{aligned}
\Gamma(\Omega_0) = \{ & \\
& (\mathrm{R}_1, \mathrm{G}_1), (\mathrm{R}_1, \mathrm{G}_4), (\mathrm{R}_2, \mathrm{G}_1), (\mathrm{R}_2, \mathrm{G}_2), (\mathrm{R}_2, \mathrm{G}_5), (\mathrm{R}_3, \mathrm{G}_2), (\mathrm{R}_3, \mathrm{G}_3), \\
& (\mathrm{I}_1, \mathrm{G}_3), (\mathrm{I}_1, \mathrm{G}_4), (\mathrm{I}_1, \mathrm{G}_5), \\
& (\mathrm{I}_1, \mathrm{GS}_i), (\mathrm{I}_1, \mathrm{GS}_j), (\mathrm{I}_1, \mathrm{GS}_k), \\
& (\mathrm{R}_3, \mathrm{GS}_1), \cdots, (\mathrm{GS}_K, \mathrm{R}_1), \\
& (\mathrm{GS}_1, \mathrm{I}_2), (\mathrm{I}_2, \mathrm{GS}_2), \cdots, (\mathrm{GS}_i, \mathrm{I}_i), \cdots, (\mathrm{I}_N, \mathrm{GS}_K) \\
\}. &
\end{aligned}
$$

The function F indicates which relations are represented by each arc:

$$
\mathrm{F} \colon \Gamma(\Omega_0) \to \mathcal{P}(R) \quad (1)
$$

$$
\left.\begin{array}{r}(\mathrm{R}_1, \mathrm{G}_1), (\mathrm{R}_1, \mathrm{G}_4), (\mathrm{R}_2, \mathrm{G}_1), (\mathrm{R}_2, \mathrm{G}_2), (\mathrm{R}_2, \mathrm{G}_5), (\mathrm{R}_3, \mathrm{G}_2), (\mathrm{R}_3, \mathrm{G}_3), \\ (\mathrm{I}_1, \mathrm{G}_3), (\mathrm{I}_1, \mathrm{G}_4), (\mathrm{I}_1, \mathrm{G}_5)\end{array}\right\} \mapsto \{c\}
$$

$$
(\mathrm{I}_1, \mathrm{GS}_i), (\mathrm{I}_1, \mathrm{GS}_j), (\mathrm{I}_1, \mathrm{GS}_k), (\mathrm{GS}_1, \mathrm{I}_2), (\mathrm{I}_2, \mathrm{GS}_2), \cdots, (\mathrm{GS}_i, \mathrm{I}_i), \cdots, (\mathrm{I}_N, \mathrm{GS}_K) \Big\} \mapsto \{a\},
$$

where $R = \{c, a\}$ is the set of labels representing the relations.

3.3 The Attribute-objects' Quantification

The attribute-objects of Ω_0 will be quantified as follows. The mine is quantified by its bounding box. Galleries are quantified by their length, gallery sections by their length, width, height, and parameters describing their axis. Thus, several gallery section types may exist: straight, circular, or clothoidal. Wall sections are quantified by a parametric equation given in the reference system number 1 of the "father" attribute-object which describes their shape. Ceilings and floors are not quantified (we will only associate them surface roughness and rock type). Their shape is determined by the walls since they lie above or beneath the wall sections. They are defined as the flatter surfaces that fits to wall sections. Gallery walls are also not quantified.

Let us formalize some applications of quantification for some of the attribute-objects of the geometric attribute.

3.3.1 Quantification of the Attribute-object "Mine" (M)

We have to write an application that represents the mine's bounding box, *i.e.* its extension under two directions (the mine is supposed to be planar). Let us note that the application is not used to measure the information (here a bounding box) but to represent it.

The application's domain is made of an entity: $\mathtt{BB_M}$. The application takes its values in $\mathbb{R}^2$. The application's graph associates to $\mathtt{BB_M}$ the mine's bounding box, *i.e.* a point in $\mathbb{R}^2$:

$$\begin{aligned} \mathrm{Q_0(M)}\colon \{\mathtt{BB_M}\} &\to \mathbb{R}^2 \\ \mathtt{BB_M} &\mapsto \mathbf{bl}, \end{aligned}$$

where **bl** is the mine's bounding box. This is a constant application associating **bl** to each entry value.

3.3.2 Quantification of the Attribute-object "Gallery" (G_i)

Galleries are quantified by their length. We thus need a constant application giving the gallery's length l.

3.3.3 Quantification of the Attribute-object "Wall Section" (WS_i)

A wall section of a room or an intersection is quantified by a 2D curve with a height at each point. This is represented by two parametric equations whose parameter is the curvilinear abscissa. To each of its values corresponds a 2D point and a height. The 2D curve's equation is given in the room or intersection's reference system number 1. The application of quantification is thus written as follows:

$$\begin{aligned} \mathrm{Q_0(WS}_i)\colon [0\,;L] &\to \mathbb{R}^2 \times \mathbb{R} \\ s &\mapsto (c(s), h(s)), \end{aligned}$$

where s is the parameter, L the wall section's length, $c(s)$ the 2D curve's parametric equation, and $h(s)$ the height at each point. Typically, $c(s)$ and $h(s)$ are polynomial functions.

3.3.4 Quantification of the Attribute-object "Gallery Section" (GS_i)

The gallery sections can be of different types: straight, circular, or clothoidal[6]. For simplification purpose, they all have the same denomination (GS) in the following discussion even though they should probably be identified by their own names. Gallery sections have two reference systems (Section 3.2). So we have to quantify the sections in these two systems. In practice, it however is sufficient to give the quantification in one reference system (typically reference system number 1), the other one being easily obtained. This reference system change will be described for each gallery section type. Let us look at these different types.

The Straight Gallery Sections. They are quantified by their width, height, and length. So the quantification application is constant and gives these three quantities.

In this case, the change from one reference system to the other is straightforward since the gallery section has the same quantification in both reference systems.

[6]It is probable that straight and circular gallery sections are sufficient to (approximately) represent a mine gallery. But, to generate vehicles' paths, it is interesting to use clothoids as they allow a smooth transition from straight to circular paths.

The Circular Gallery Sections. They are quantified by their width, height, length, and radius. The parametric equation of the gallery section's axis is:

$$x = -R(\cos(s/R) - 1)$$
$$z = R\sin(s/R)$$

where (x, z) represents the point position, s the curvilinear abscissa, and R the radius. Figure 8 shows two curves depending on the sign of R. The information represented in the

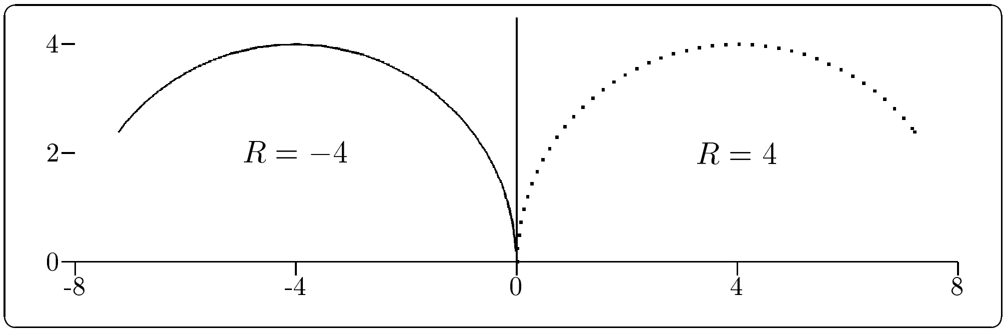


Figure 8: Circular gallery sections for two radius values

quantification is thus sufficient to completely characterize the gallery section. The application is constant and gives the four parameters.

To get the quantification in the other reference system, it is sufficient to change R to $-R$. Indeed, the curve is the same except that it turns in the other sense.

The Clothoidal Gallery Sections. A clothoid is a curve whose curvature varies linearly with curvilinear abscissa. The curvature is thus written $\kappa(s) = -\kappa_0 - ks$ where $-\kappa_0$ is the initial curvature, k the curvature variation constant, and s the curvilinear abscissa. The curve's parametric equation is (Appendix A):

$$x(s) = \mathrm{sgn}(k)\sqrt{\frac{\pi}{|k|}}\left[\cos\left(\frac{\kappa_0^2}{2|k|}\right)\mathrm{FresnelS}\left(\frac{|k|s+\kappa_0}{\sqrt{|k|\pi}}\right) - \sin\left(\frac{\kappa_0^2}{2|k|}\right)\mathrm{FresnelC}\left(\frac{|k|s+\kappa_0}{\sqrt{|k|\pi}}\right)\right]$$
$$z(s) = \sqrt{\frac{\pi}{|k|}}\left[\cos\left(\frac{\kappa_0^2}{2|k|}\right)\mathrm{FresnelC}\left(\frac{|k|s+\kappa_0}{\sqrt{|k|\pi}}\right) + \sin\left(\frac{\kappa_0^2}{2|k|}\right)\mathrm{FresnelS}\left(\frac{|k|s+\kappa_0}{\sqrt{|k|\pi}}\right)\right].$$

The curve is a clothoid or "Cornu spiral". Curves of this type allow a smooth link (*i.e.* without curvature discontinuity) between straight and circular curves ([FSLC95]). Figure 9 shows two clothoids for two values of k.

As for the previous gallery section types, the quantification is a constant application giving all the parameters (the width, height, length, curvature variation constant, and initial curvature).

If, in the first reference system, the clothoid has as curvature $\kappa(s) = -\kappa_0 - ks$, then, the same curve in the second reference system has as curvature $\kappa'(s) = -\kappa_0' - k's$ with $\kappa_0' = -\kappa_0 - kl$ and $k' = k$ where l is the clothoid's length (Appendix B).

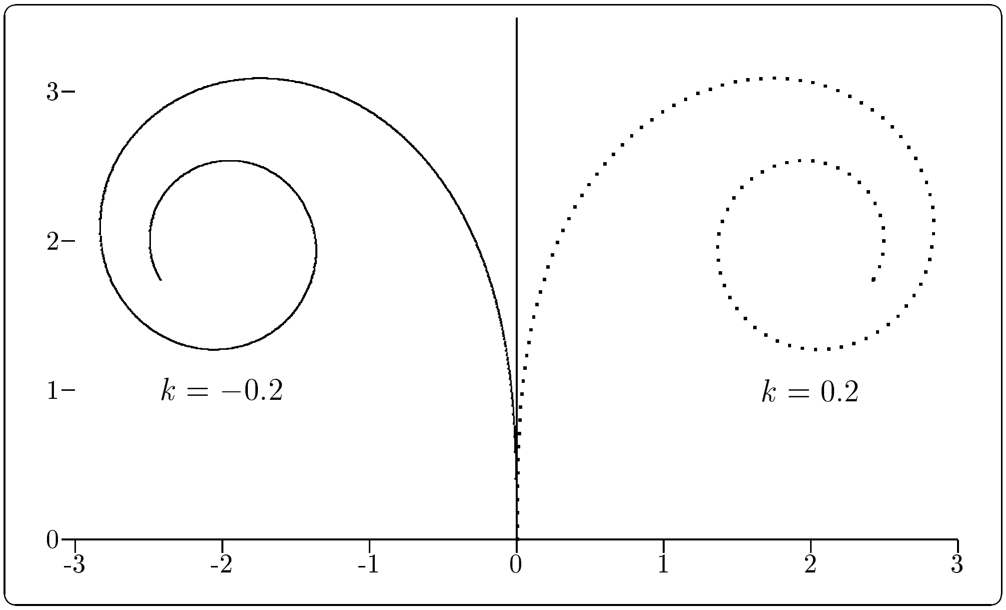


Figure 9: Clothoidal gallery sections for two values of k and with $\kappa_0 = 0$

3.3.5 Quantification of the Attribute-object "Intersection" (I_i)

Intersections (and rooms) are quantified by the transformations (translations and rotations) between the reference system number 1 and the $n-1$ others. Figure 10 shows a pair of such reference systems. This is the transformation between this two reference systems (and all

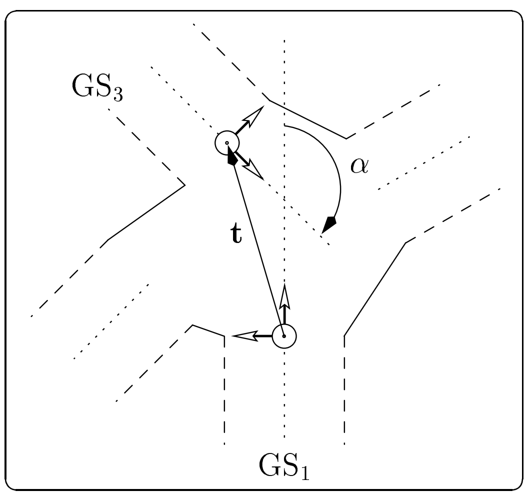


Figure 10: The quantification of an intersection

other such pairs) which is given. The transformation consists in a translation followed by a rotation. The translation vector and the rotation angle are shown in Figure 10.

The application of quantification for the intersections (and rooms) is constant and gives $n-1$ translations and angles, which are respectively elements of $\mathbb{R}^2$ and $\mathbb{R}$.

3.4 The Quantification of Relations

The attachment is the only relation to quantify. The quantification of this relation gives the identification number of two reference systems: the first [respectively the second] is the head [respectively tail] attribute-object's system of reference of the arc representing an instance of the relation. The application of quantification of this relation is thus constant and associates to an entity $\mathtt{a}$ an element of $\mathbb{N} \times \mathbb{N}$:

$$a \in \mathcal{F}(\{\mathtt{a}\}, \mathbb{N} \times \mathbb{N}).$$

The labeling function (1) of relations graph (Section 3.2) thus becomes:

$$F: \Gamma(\Omega_0) \to \mathcal{P}(R \times \mathcal{F}(\{\mathtt{a}\}, \mathbb{N} \times \mathbb{N}) \cup \{\emptyset\}).$$

4 The Roughness

Roughness is represented by three values: the first one gives a fractal dimension while the others give a density and an average deviation of bumps on walls. This attribute is thus represented in a 3D space. Since each gallery or wall section has a roughness, we have to define as many "roughness" attribute-objects as necessary to represent all values present in the mine. These attribute-objects could constitute the finer level of an arborescence of average values. Values could be grouped (averaged) to give rough values to galleries, rooms, and intersections. This is not done in the current implementation of the model because has no immediate interest for our application (it is not interesting to associate roughness values to galleries, rooms, and intersections because this is not used to establish a qualitative road-map). Each attribute-object is quantified by the corresponding value of fractal dimension and bump density and average deviation.

There is no interesting relation to represent between attribute-objects of this attribute (except, perhaps, the decomposition relation described previously).

5 The Rock

We have to represent the different rock types present in the mine. To do this, a classical geological taxonomy is used. Such a taxonomy has a rock hierarchical decomposition similar to the one in GNOME. Each type of rock or family of rock corresponds to an attribute-object of this attribute.

6 The Obstacles

The obstacles are geometric elements placed in different mine's places (in the gallery sections, intersections, and rooms). There is a certain number of elementary attribute-objects corresponding to different sizes and forms. Typically, parallelepipeds (to represent boulders) and cones (to represent stone heaps) have been retained. Parallelepipeds have a direct reference system such that the origin is at the inferior face's center, the x axis follows little horizontal dimension, the y axis follows the large one, and z axis is vertical. For cones, the system of reference is the same except that x and y axis are not uniquely determined (the cone's axis is vertical).

Attribute-objects (obstacles) of a higher level, and thus more complex, could be build by grouping elementary attribute-objects (obstacles).

Parallelepipeds are quantified by their three dimensions (width, length, and height) while cones are quantified by their height and base radius (the axis is supposed to be perpendicular to the base).

There is no relation between this attribute-objects.

7 The Lighting

In a mine, there are various lighting types. First, we have lighting given by bulbs at strategic or dangerous places (intersections, turns, *etc.*) in the mine. This lighting is represented by point lights. Next, there is an ambient lighting given by a large number of lights situated on rooms ceiling where there is a frequent activity, *e.g.* in workshops or resting rooms. This lighting is represented by a uniform directional light coming from the ceiling. Finally, the last type is given by vehicles' headlight and is represented by spotlights[7]. A hierarchical decomposition could be done guided by sources' intensity.

The "point lights" and "directional lights" attribute-objects are quantified by their intensity (in watts) while the "spotlights" attribute-objects are quantified by their intensity (in watts), cut off angle (in radians), and (exponential) drop off rate (a value of 0 means there is no attenuation while a value of 1 means a maximum attenuation).

Spotlights can be grouped to create an other attribute-object: a *vehicle lighting.* This attribute-object is quantified by the total intensity of vehicle's lighting (*i.e.* the sum of intensities of spotlights composing this attribute-object).

There is a relation for the lighting attribute: it characterizes the disposition of spotlights composing an attribute-object "vehicle lighting." These attribute-objects have a direct reference system such z axis is parallel to the vehicle displacement (generally, this axis is parallel to the spotlights' axis), x axis is horizontal and perpendicular to the vehicle displacement, and y axis is vertical bottom-up and also perpendicular to the vehicle displacement. The origin is at the spotlights' barycentre. The relation thus gives a translation which indicates positions of the different spotlights' focuses with respect to the origin and a rotation which indicates spotlights axis' orientation relative to the reference system's z axis.

8 The Inter-attribute Relations

We have now to link the different attributes. This is done in following sections.

[7]Vehicles are still not represented in the mine but the one carrying the camera is at least present.

8.1 Roughness and Geometry

A first link exists between geometric attribute and "roughness" attribute. Indeed, we have to associate a roughness to each geometric mine part. Only lowest level geometric attribute-objects, *i.e.* gallery walls, wall sections, ceilings, and floors, are considered. All roughness attribute-objects are concerned. There is thus a correspondence C which associates to each gallery wall, wall section, ceiling, and floor a roughness attribute-object:

$$\begin{aligned}
&C = (\Gamma, A, B) \text{ with} \\
&A = \{\mathrm{GW}_1, \cdots, \mathrm{GW}_{2K}, \mathrm{WS}_1, \cdots, \mathrm{WS}_L, \mathrm{F}_1, \cdots, \mathrm{F}_{N+3}, \mathrm{C}_1, \cdots, \mathrm{C}_{N+3}\}, \\
&B = \Omega_1, \\
&\Gamma = \{(a, b) \in A \times B \mid c(a, b)\}
\end{aligned}$$

where $c(a, b)$ indicates that attribute-object a corresponds to attribute-object's roughness and Ω_1 is the set of roughness attribute-objects (this attribute is supposed to be number 1). This relation is qualitative.

8.2 Rock and Geometry

This relation is of same type as the one linking roughness with geometry (Section 8.1) but associates a rock to each gallery wall, wall section, ceiling, and floor.

8.3 Obstacles and Geometry

The obstacles' pose have to be specified in the mine, more precisely in gallery sections, rooms, and intersections. Thus, the obstacles are positioned in these geometric attribute-objects.

Placing an obstacle in a gallery section is done by giving the position of its reference system origin in the section (*i.e.* a curvilinear abscissa and a signed distance from the section's axis) and the orientation of z axis of the reference system (*i.e.* an angle from the section's axis tangent to z axis positive in trigonometric sense). This must be done in only one gallery section's reference system, since the pose in the other system can easily be found (Section 3.3.4).

The pose of an obstacle in a room or intersection is the obstacle origin position in the intersection or room's reference system number 1 (x and z coordinates, the obstacle being on the floor) and its orientation (an angle from room or intersection's z axis to obstacle's z axis, positive in trigonometric sense).

Thus, there are two different relations representing obstacles' pose, each quantified by triplets of reals. However, they can be represented by only one relation because the signification of the reals is the same in both cases: the first two represent a position and the third an angle.

8.4 Lighting and Geometry

As for obstacles, we have to position lightings in the mine or, more precisely, in rooms, gallery sections, and intersections. The relation depends on the lighting type and, as for obstacles, the position in the mine.

Point lights can be found in rooms, intersections, and gallery sections. In the latest, they are positioned with a curvilinear abscissa along the section's axis, a perpendicular signed distance from this axis, and a height from floor. Normally, positioning must be performed in both reference systems, but it is sufficient to do it in only the first one (Section 3.3.4). In rooms and intersections, lightings are positioned according to the three dimensions of the reference system number 1. Thus, there are two relations for the point lights, which can be represented by only one relation quantified with three real numbers.

Directional lights can be present in rooms. It is sufficient to associate such lights to rooms with a correspondence.

Spotlights (and more generally vehicle lightings) can be present in gallery sections, rooms, and intersections. The difference with point lights is that spotlights can move in the mine because they are supported by vehicles. Thus, the quantification of the positioning relation for these attribute-objects will vary. But this only allows displacements in one gallery section, intersection, *or* room. Thus, in addition to a variable quantification, this relation will "move" too, *i.e.* the geometric attribute-object associated to spotlights (or vehicle lightings) will vary too. But changes are not unrestricted. Indeed, vehicles can only move from a geometric attribute-object to another one that must be connected and, thus, spotlights (or vehicle lightings) displacements are constrained by the same connections. When a vehicle, and thus its lighting, leaves a first geometric attribute-object to enter a second one, the arc representing the positioning of the lighting in the first geometric attribute-object is destroyed and another arc is created to position the lighting in the second one.

The positioning relation of a spotlight (or vehicle lighting) is quantified by the position (three reals representing curvilinear coordinates in gallery sections or coordinates in rooms and intersections) of the lighting's origin and an orientation (three angles around the lighting's reference system's axes).

8.5 Roughness and Obstacles

We have to give a roughness value to obstacles as was done with geometric attribute-objects (Section 8.1). This is done with a correspondence.

8.6 Rock and Obstacles

A last correspondence associates a rock to each obstacle.

9 Conclusion

An underground mine model has been described. The mine was represented with various attributes: geometry, surface roughness, rock type, obstacles, and lighting. The decomposition tree and relations between attribute-objects of each attribute have been described. Finally, in order to link the attributes, inter-attribute relations have been introduced.

The purpose of this model is twofold. First, it represents an illustration of the generic object model (GNOME) described in [LHC96]. As such, all the formalism of GNOME is applied to the mine model. Second, the model represents a basic tool for the study of navigational problems. Although this problem is not addressed in this report, it is clear that the model provides all the information required by an autonomous vehicle.

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A The Clothoids' Parametric Equation

The planar curve's curvature is:

$$\kappa = \lim_{\Delta s \to 0} \frac{\Delta \tau}{\Delta s} = \frac{\mathrm{d}\tau}{\mathrm{d}s},$$

where s is the curvilinear abscissa along the curve and τ the angle between the curve's tangent and the abscissa axis. This angle is thus given by:

$$\tan \tau = \frac{\dot{y}}{\dot{x}},$$

where $\dot{y} = \frac{\mathrm{d}y}{\mathrm{d}s}$ and $\dot{x} = \frac{\mathrm{d}x}{\mathrm{d}s}$. Thus we have:

$$\begin{aligned} \dot{x} &= \lambda \cos \tau, \\ \dot{y} &= \lambda \sin \tau. \end{aligned}$$

λ is a parameter, which can be, *e.g.*, a speed of displacement along the curve, and which we will take equal to 1. Hence:

$$x(s) = \int_0^s \cos(\tau(u))\,\mathrm{d}u + x_0$$
$$y(s) = \int_0^s \sin(\tau(u))\,\mathrm{d}u + y_0.$$

For a clothoid, we have $\kappa(s) = -\kappa_0 - ks$[8]. This gives $\tau = -\kappa_0 s - \frac{1}{2}ks^2 + \tau_0$ and:

$$\begin{aligned} x(s) = \sqrt{\frac{\pi}{k}} \Bigg[&+\cos\left(\frac{\kappa_0^2 + 2k\tau_0}{2k}\right) \mathrm{FresnelC}\left(\frac{ks+\kappa_0}{\sqrt{k\pi}}\right) \\ &+\sin\left(\frac{\kappa_0^2 + 2k\tau_0}{2k}\right) \mathrm{FresnelS}\left(\frac{ks+\kappa_0}{\sqrt{k\pi}}\right)\Bigg] + x_0 \\ y(s) = \sqrt{\frac{\pi}{k}} \Bigg[&-\cos\left(\frac{\kappa_0^2 + 2k\tau_0}{2k}\right) \mathrm{FresnelS}\left(\frac{ks+\kappa_0}{\sqrt{k\pi}}\right) \\ &+\sin\left(\frac{\kappa_0^2 + 2k\tau_0}{2k}\right) \mathrm{FresnelC}\left(\frac{ks+\kappa_0}{\sqrt{k\pi}}\right)\Bigg] + y_0, \end{aligned}$$

for a positive k. In preceding equations, $\mathrm{FresnelC}(u)$ and $\mathrm{FresnelS}(u)$ are respectively Fresnel cosine and sine and their expression is:

$$\mathrm{FresnelC}(u) = \int_0^u \cos\frac{\pi t^2}{2}\mathrm{d}t$$
$$\mathrm{FresnelS}(u) = \int_0^u \sin\frac{\pi t^2}{2}\mathrm{d}t.$$

In order to handle cases where $k < 0$ and the desired configuration in our case, preceding equations have to be modified:

$$x(s) = \mathrm{sgn}(k)\sqrt{\frac{\pi}{|k|}}\left[\cos\left(\frac{\kappa_0^2}{2|k|}\right)\mathrm{FresnelS}\left(\frac{|k|s+\kappa_0}{\sqrt{|k|\pi}}\right) - \sin\left(\frac{\kappa_0^2}{2|k|}\right)\mathrm{FresnelC}\left(\frac{|k|s+\kappa_0}{\sqrt{|k|\pi}}\right)\right]$$
$$z(s) = \sqrt{\frac{\pi}{|k|}}\left[\cos\left(\frac{\kappa_0^2}{2|k|}\right)\mathrm{FresnelC}\left(\frac{|k|s+\kappa_0}{\sqrt{|k|\pi}}\right) + \sin\left(\frac{\kappa_0^2}{2|k|}\right)\mathrm{FresnelS}\left(\frac{|k|s+\kappa_0}{\sqrt{|k|\pi}}\right)\right].$$

Note the change from $y(s)$ to $z(s)$ due to chosen reference systems for gallery sections (Section 3.2). We have taken x_0 and y_0 equal to 0 and τ_0 equal to $\pi/2$.

B Change of Reference System for a Clothoidal Gallery Section

We have to derive the equation of a clothoidal gallery section in one of its reference system given the equation in the other one. Let $\kappa(s) = -\kappa_0 - ks$ be the given equation, in fact

[8]More generally, $\kappa(s) = \kappa_0 + ks$, but introducing minus signs give nicer equations in our case.

the curvature equation, say in first reference system. We are searching for κ_0' and k' of $\kappa'(s) = -\kappa_0' - k's$, the curvature of the *same* curve in the other reference system, say the second one.

First, in the second reference system, the curve turns in the opposite direction with respect to the case in the first one. This implies a sign change for the curvature. We thus have to obtain:

$$\begin{aligned}\kappa'(0) &= -\kappa(L),\\ \kappa'(L) &= -\kappa(0),\end{aligned}$$

where L is the gallery section's length. This gives:

$$\begin{aligned}\kappa_0' &= -\kappa_0 - kL,\\ k' &= k.\end{aligned}$$