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Classifying Structures for Operator Algebras and Dynamical Systems

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Abstracts

Ahmed Al-Rawashdeh (United Arab Emirates University)

Classifications of C^*-algebras using unitary groups

H. Dye showed that an isomorphism between the (discrete) unitary groups in two factors not of type I_n is implemented by a linear (or a conjugate linear) $*$-isomorphism of the factors. We show that if the unitary groups of two simple unital AH-algebras (of SDG) with real rank zero are isomorphic as abstract groups, then their K_0-ordered groups are isomorphic. Also, using Dadarlat-Gong's classification theorem, we prove that such C^*-algebras are isomorphic if and only if their unitary groups are topological isomorphic. In the case of simple, unital purely infinite C^*-algebras, we show that two unital Kirchberg algebras are $*$-isomorphic if and only if their discrete unitary groups are isomorphic. Following Dye's approach, if φ is an isomorphism between the unitary groups of two unital C^*-algebras, it induces a bijective map θ_φ between the sets of projections. For a large class of unital finite C^*-algebras, we show that θ_φ is always an orthoisomorphism. Based on theses results, we prove that if φ is a continuous automorphism of the unitary group of a UHF-algebra A, we show that φ is implemented by a linear or a conjugate linear $*$-automorphism of A.

Hiroshi Ando (Erwin Schrödinger Institute, Vienna)

Ultraproducts, QWEP von Neumann algebras, and the Effros-Maréchal topology

In the nineties, Kirchberg revealed the remarkable connection among C* tensor norms, Lance's weak expectation property (WEP) and Connes' embedding conjecture, which he showed to be equivalent to so-called QWEP conjecture. Haagerup Winsløw then studied the topological properties of the Effros-Maréchal space vN(H) of all von Neumann algebras acting on a Hilbert space H, and showed that the density of the set of II_1 factors on H is equivalent to QWEP conjecture, by showing that a II_1 factor is R^ω-embeddable if and only if M is in the closure of the set of injective factors. In this talk we use recently established results about non-tracial ultraprouct of von Neumann algebras to generalize Haagerup-Winsløw theory and make a futher link among ultraproducts, QWEP von Neumann algebras and the topological properties of vN(H). The key point is the two different ultraproducts in troduced by Ocneanu and Groh-Raynaud. (Joint work with Uffe Haagerup and Carl Winsløw.)

Edwin Beggs (Swansea University)

Line bundles and characteristic classes in noncommutative geometry

The idea of a line bundle in classical geometry is transferred to noncommutative geometry by the idea of a Morita context. From this we can construct $\mathbb{Z}$ and $\mathbb{N}$ graded algebras, the $\mathbb{Z}$ graded algebra being a Hopf-Galois extension. A non-degenerate Hermitian metric gives a star structure on this algebra, and an additional star operation on the line bundle gives a star operation on the $\mathbb{N}$ graded algebra. In this case, we can carry out the associated circle bundle and Thom constructions. Starting with a C^*-algebra as base, and with some positivity assumptions, the associated circle and Thom algebras are also C^*-algebras. We conclude by examining covariant derivatives and Chern classes on line bundles after the method of Kobayashi and Nomizu.

Tomasz Brzezinski (Swansea University)

Quantization and resolution

We discuss recently studied examples of quantum or noncommutative orbifolds and argue that they provide one with an explicit illustration of noncommutative resolution of singularities. This resolution can be understood on several levels: as separation of roots in polynomial equations defining algebraic varieties; finiteness of the global homological dimension; freeness of the module of top forms; change of the C^*-description. Examples include quantum teardrops and weighted real projective planes, and the noncommutative pillow manifold.

Anthony Dooley (University of Bath)

The AT property is not preserved by finite extensions

The Connes-Krieger dictionary gives a relationship between measurable dynamical systems and von Neumann algebras. In particular, the algebra is ITPF1 iff the dynamical system has the AT property. We find an AT system with a finite (two point, in fact) extension which is not AT; this means that there are non-ITPF1 factors which have ITPF1 subfactors of index 2, contradicting a conjecture of Giordano and Skau.

Søren Eilers (University of Copenhagen)

Classification of graph C^*-algebras

Whereas the simple graph C^*-algebras are readily classified by the classification results by Elliott and Kirchberg/Phillips, we do not in general have a complete understanding of when graph algebras may be classified by K-theoretical invariants. Complete solutions to this question have been obtained in cases with few primitive ideals as well as cases where the graphs are finite, but on the boundary of our understanding are a great number of cases of current interest both in classification and internally in the structure theory for C^*-algebras, especially when it comes to the extension problem: If A/I and I are graph C^*-algebras, can the same be said of A? I will attempt a complete overview of the status of these questions.

George Elliott (University of Toronto)

Cutting up a crossed product

What happens if, to the crossed product of a C^*-algebra by a locally compact abelian group, one adjoins one or more spectral projections of the canonical unitary group? (This question was perhaps first raised by Muhly, Putnam, and Xia in the case of a one-parameter automorphism group of an abelian C^*-algebra, and they obtained results concerning the K-groups of the resulting C^*-algebra.)

If one starts with the irrational rotation C^*-algebra, which may be viewed symmetrically as a crossed product in two ways, then doing this once results (as was pointed out by Cuntz) in a C^*-algebra which is still a crossed product, and is what one might call a Putnam algebra. In particular, the odd K-group drops from rank two to rank one. Doing this a second time, with respect to the other canonical unitary generator, again results in a reduction in the rank of the odd K-group, and in fact, as very recently shown, in joint work with Zhuang Niu, results in an AF algebra.

Other cases would appear to be interesting to study. (The case just considered is also a special case of the Muhly, Putnam, Xia construction.)

David Evans (Cardiff University)

Fusion Categories - from Ising to Haagerup

I will review some work on subfactor fusion categories and connections with statistical mechanics and conformal field theory. This will be illustrated with the case of the Ising model through to work with Terry Gannon on the Haagerup subfactor, near group categories and the search for exotic subfactors.

Rolf Gohm (Aberystwyth University)

Synchronizing words and preparability of states

Synchronizing words are a well known topic in the theory of graphs and automata and in symbolic dynamics. They can be reinterpreted as gadgets for the preparation of states on commutative algebras by repeated interactions with another system. This interpretation suggests noncommutative versions which are relevant to the preparation of states in noncommutative algebras and quantum systems.

Joint work with B. Kuemmerer, T. Lang, F. Haag.

Robin Hillier (Lancaster University)

Universal C^*-algebras and K-theory for conformal nets

The aim of this overview talk is to explain how universal C^*-algebras, dynamical systems, cyclic cohomology and $K(K)$-theory find applications in the theory of conformal nets and help towards new descriptions and characterisations of the latter. Conformal nets are the fundamental objects underlying an operator algebraic approach to conformal field theory, and the necessary background knowledge will be provided.

Michel Hilsum (Mathematical Institute of Jussieu)

Orbit equivalence of dynamical systems and Gelfand Fuchs cohomology

Let V be a smooth manifold and G a Lie groups acting smoothly. There are additive maps of the K-theory group of the C^*-crossed product $C(V) \rtimes G$ which are associated to secondary characteristic classes.

By construction, they are obstructions to the existence of a smooth orbit equivalence (if the action is free) between two such dynamical systems.

In this talk we shall address the existence of a topological orbit equivalence, transversely absolutely continuous, in relation with these characteristic mappings.

Eberhard Kirchberg (Humboldt-Universität zu Berlin)

C^*-correspondences related to Dini spaces

Topological characterizations of primitive spectra of separable amenable C^*-algebras were given several years ago in a long preprint with H. Harnisch based on joint results with M. Rødram. The shortest formulation is:

A sober T_0 space X is homeomorphic to the primitive ideal space of a separable amenable C^*-algebra if and only if the lattice $\mathbb{O}(X)$ of open subsets of X is order-isomorphic to a sub-lattice $\mathcal{Y} \subset \mathbb{O}(P)$ of a locally compact Polish space P that is closed under l.u.b. (unions of families of open sets in $\mathcal{Y}$) *and g.l.b.* (interiors of intersections of families of open sets in $\mathcal{Y}$).

Here we call the second countable sober locally compact T_0 spaces X *Dini spaces*, because they are determined by the semi-lattice of its Dini functions.

The principal open question is: Is every Dini space homeomorphic to the primitive spectrum of a separable amenable C^*-algebra?

An answer is even unknown for the Dini space $X_u := \mathrm{Prim}(C^*(SL_2(\mathbb{Z})))$. (It is believable – but still unknown if true or not – that each Dini space X is homeomorphic to an intersection of an open and a closed subset of X_u.)

I report on a new observation that shows that all (abstract) Dini spaces X are natural related to C^*-algebra correspondences, thus are natural parts of the C^*-algebra theory. The main result is the following theorem:

Theorem *Let X a Dini space. There exists a stable separable amenable C^*-algebra $A \cong A \otimes \mathcal{O}_2$ with coherent primitive spectrum $Z := \mathrm{Prim}(A)$ and a *-monomorphism $h\colon A \to \mathcal{M}(A)$ such that h and the l.s.c. action $\Psi := \Psi_h\colon \mathcal{I}(A) \to \mathcal{I}(A)$ of Z on A given by $\Psi_h(J) := h^{-1}(h(A) \cap \mathcal{M}(A, J))$ satisfy:*

(o) The lattice $\mathbb{O}(X)$ is order isomorphic to the image $\Psi(\mathcal{I}(A))$ of Ψ in the lattice $\mathcal{I}(A)$.

(i) $h(A)A = A$ (h is non-degenerate).

(ii) h is unitarily equivalent in $\mathcal{M}(A)$ to its infinite repeat $\delta_\infty \circ h$ ($= h \oplus h \oplus \cdots$).

(iii) $\mathcal{M}(h) \circ h$ is approximately unitary equivalent to h in $\mathcal{M}(A)$.

(iv) $J \subset \Psi(J)$ for all $J \in \mathcal{I}(A)$.

We report on applications, outline the proofs and ideas for further research.

Akitaka Kishimoto (Hokkaido University)

Approximately inner flows, quasi-diagonal flows,and MF flows

Flows on (or strongly continuous one-parameter automorphism groups of) C*-algebras have been a subject of study since 1970's pioneered by S. Sakai, O. Bratteli, D.W. Robinson et al. Among them I have been concerned with classes of flows which allow KMS states at all temperatures (when the C*-algebras are unital) in view of applicability to quantum statistical mechanics and seeking to characterize such a class and the class of AI (or approximately inner) flows, a well-known class satisfying the above requirement if the C*-algebra is unital and finite. This talk is to report on our failed attempts; I will present the definitions of QD (or quasi-diagonal) flows and MF flows and some others and their basic properties obtained during the efforts. Note QD implies MF. While AI flows could be constructed in an explicit or abstract way, examples of QD flows (resp. MF flows) are obtained as AI flows on QD C*-algebras (resp. MF C*-algebras).

Vladimir V. Kisil (Univesity of Leeds)

Classification by symmetries

In Erlangen Programme, F.Klein (greatly influenced by S.Lie) proposed to classify various geometries (euclidean, affine, conformal, projective, etc.) by the respective groups of their symmetries. This approach can be used for classifications of individual operators or operators algebras as well. In a sense this is an extension of the Erlangen programme to "non-commutative geometry". This task prompts us to revisit and rethink the traditional geometry and theory of functions as a background of operator theory.

Claus Köstler (University College Cork)

Quantum symmetric states on free product C^*-algebras

We introduce the notion of a quantum symmetric state on the infinite universal free product of a unital C^*-algebra. Our main results are a de Finetti type theorem for quantum symmetric states and a characterization of extreme quantum symmetric states. In other words, our results provide the free probability counterpart of Størmer's work on symmetric states on the infinite minimal tensor product of a unital C^*-algebra. This is joint work with Ken Dykema and John Williams.

Nadia S. Larsen (University of Oslo)

C^*-algebras associated to graphs, path spaces and equilibrium states

Given a directed graph E, the graph algebra is the universal C^*-algebra for a generating family of partial isometries and projections satisfying the Cuntz-Krieger relations. There are other C^*-algebras associated to E, such as the Toeplitz graph algebra. We view graph C^*-algebras as crossed products where the free group on the edge set acts by partial homeomorphisms on a space of boundary paths of the graph. For a function on the edge set that induces a time evolution on a graph C^*-algebra, equilibrium (or KMS) states therefore can be characterised using a general theorem of Exel and Laca. Guided by their work on KMS states for Toeplitz-Cuntz-Krieger type algebras associated to infinite matrices, we describe in case of graph C^*-algebras the convex sets of KMS states of finite type and of KMS states of infinite type whose associated measures are supported on recurrent infinite paths. This is joint work with Toke M. Carlsen (NTNU, Norway).

Xin Li (University of Münster)

Semigroup C^*-algebras and K-theory for certain crossed products

We compute K-theory for semigroup C^*-algebras. Our methods are actually more general and lead to K-theory computations for crossed products attached to totally disconnected dynamical systems. As a concrete example, we discuss graph C^*-algebras. This is joint work with J. Cuntz and S. Echterhoff, and also M. Norling.

Martin Mathieu (Queen's University Belfast)

Derivations and local multipliers of C^*-algebras

We discuss the interrelation between derivations and local multipliers of C^*-algebras and report on some recent advances on a problem posed by Gert Pedersen back in 1978. These rely on the sheaf theory for C^*-algebras developed jointly with Pere Ara (Barcelona).

Hiroki Matui (Chiba University)

Continuous orbit equivalence of one-sided shifts of finite type

We classify irreducible one-sided shifts of finite type up to continuous orbit equivalence. Equivalently, for two simple Cuntz-Krieger algebras, we determine when there exists an isomorphism between them which preserves the canonical Cartan subalgebras. This is joint work with Kengo Matsumoto.

Francesc Perera (Universitat Autònoma de Barcelona)

Geometric structure of dimension functions of certain continuous fields

For a continuous field of C*-algebras A, we give a criterion to ensure that the stable rank of A is one. In the particular case of a trivial field this leads to a characterization, completing accomplishments by Nagisa, Osaka and Phillips. Further, for certain continuous fields of C*-algebras, we study when the Cuntz semigroup satisfies the Riesz interpolation property, and we also analyse the structure of its functionals. As an application, we obtain an answer to a conjecture posed by Blackadar and Handelman in a variety of situations. This is joint work with Ramon Antoine, Joan Bosa, and Henning Petzka (UAB).

Mathew Pugh (Cardiff University)

Graph algebras for nimrep graphs associated to $SU(3)$ modular invariant partition functions

The modular invariant partition functions for $SU(2)$ and $SU(3)$ conformal field theories have been classified. The $SU(2)$ theory is closely related to the preprojective algebras of Coxeter-Dynkin quivers. The analogous finite dimensional superpotential algebras, which we call almost Calabi-Yau algebras, associated to the $SU(3)$ invariants will be discussed.

Jean Renault (University of Orléans)

Amenability of some groupoid extensions

Given a locally compact groupoid G, a locally compact group H and a continuous groupoid homomorphism c from G to H, I shall give conditions on H and c which imply the amenability of G. They improve a recent result of J. Spielberg on gauge actions. As in Spielberg, the main ideas can be found in the work of C. K. Ng and of J. Quigg on coactions but are expressed here at the groupoid level. Another tool is the use of Borel amenability. Our study is motivated by and applied to topological higher rank graph algebras. This is joint work with D. Williams.

Yasuhiko Sato (Kyoto University)

Approximately unitarily equivalent morphisms of UHF absorbing C^*-algebras

M. Rørdam showed that two morphisms of a Kirchberg algebra with UCT are approximately unitarily equivalent, if they have the same invariants the KL-group. In that same period, E. Kirchberg obtained an abstract proof of an asymptotical version of this result based on Connes-Higson's E-theory.

On the other hand, for stably finite cases, H. Lin showed analogous results of Rørdam's theorem by using the condition of tracial rank zero. In the present work, we show an alternative proof of H. Lin's theorem in a similar way to Kirchberg's strategy.

Aidan Sims (University of Wollongong)

Equilibrium states on C^*-algebras of higher-rank graphs

KMS states model thermal equilibrium states in C^*-algebraic dynamical systems that represent quantum systems. Even when a given dynamical system is not physical, its KMS states seem to contain interesting information. In 1984, Enomoto, Fujii and Watatani studied KMS states for the gauge actions on Cuntz-Krieger algebras $\mathcal{O}_A$. They showed that there is a unique such state, and its values on the initial projections of generators are determined by the Perron-Frobenius eigenvector of A. More recently, its become apparent that the gauge action on the Toeplitz extensions of a Cuntz-Krieger algebra provides a richer supply of KMS states, and typically exhibits a phase-change at the critical inverse temperature at which the state factors through the Cuntz-Krieger quotient. I will discuss the KMS states on the Toeplitz algebras of finite higher-rank graphs and on their Cuntz-Krieger quotients. This is joint work with Astrid an Huef, Marcelo Laca and Iain Raeburn.

Aaron Tikuisis (University of Aberdeen)

Nuclear dimension and central sequences

A current goal in the classification of C^*-algebras is to firmly establish the correct notion of low-dimension, or regularity, as a refinement of amenability. In concrete terms, this goal amounts to showing that finite nuclear dimension, $\mathcal{Z}$-stability, and regularity in the Cuntz semigroup are equivalent properties (within a class that avoids well-known, obvious obstructions). In this talk, I will focus on arguments to prove $\mathcal{Z}$-stability. It turns out that the central sequence algebra is a useful, and perhaps indispensible, tool in such arguments. $\mathcal{Z}$-stability is equivalent to certain regularity conditions holding in the Cuntz semigroup of the central sequence algebra. We will see how finite nuclear dimension allows us to manipulate this object.

This is largely joint work with Leonel Robert.

Alina Vdovina (University of Newcastle)

Building C^*-algebras and their K-theory

We will show, how one may use groups acting on buildings to construct new C^*-algebras and compute explicitly their K-theory.

Christian Voigt (University of Glasgow)

Fredholm modules for $SU_q(3)/T$ and the Baum-Connes conjecture

Spectral triples play a basic role in Connes' approach to noncommutative geometry. Despite the fact that quantum groups and their homogeneous spaces provide a rich supply of noncommutative spaces, no construction of (equivariant) spectral triples on the full flag manifold K_q/T of the standard q-deformation of a compact simply connected simple Lie group K is known except for the simplest case $K = SU(2)$.

In this talk I will indicate how one can construct nontrivial equivariant Fredholm modules at least for the full flag manifold of $SU_q(3)$, thus providing bounded versions of the desired spectral triples in this case. The methods rely on an analytical version of the Bernstein-Gelfand-Gelfand resolution and techniques originating from bivariant K-theory. I will also explain how this is related to the Baum-Connes conjecture. (Joint work with R. Yuncken.)

Stuart White (University of Glasgow)

Close operator algebras

The set of C^*-subalgebras of $B(H)$ is equipped with a natural metric arising from applying the Hausdorff metric on subsets of $B(H)$ to the unit balls of 2-operator algebras. In 1972, Kadison and Kastler conjectured that sufficiently close algebras should be isomorphic. In this talk I'll survey some recent results regarding the structure and methods for obtaining isomorphisms between close C^*-algebras and close von Neumann algebras.

John D. Maitland Wright (University of Aberdeen)

Monotone complete C^*-algebras and generic dynamics

Monotone complete C^*-algebras (MCAs) are a natural generalisation of von Neumann algebras. But MCAs are far more numerous and much less well-understood; many mysteries remain to be resolved. Generic dynamics is intimately connected to MCAs. This talk will give a general introduction to both these topics and describe recent joint work with Kazuyuki Saito on generic dynamics and classification of MCAs.

Joachim Zacharias (University of Glasgow)

Constructions of spectral triples on C^*-algebras

A basic problem in noncommutative geometry is the construction and existence of spectral triples on given C^*-algebras with various regularity properties. We present some constructions of spectral triples starting from given ones in the spirit of permanence properties, e.g. on extensions and crossed products.