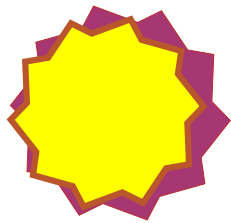


# Statistical Tables

Department of Mathematics

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*Not to be  
removed from the  
examination room*

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# 1 Binomial distribution

This distribution gives the probabilities of the various numbers of successes in  $n$  independent success/fail trials, the probability of success in each trial being  $p$ .

The probability of  $k$  successes is given by  $P(X = k) = {}^nC_k p^k (1-p)^{n-k}$  for  $k = 0, 1, 2, \dots, n$ .

The mean of the distribution is  $np$  and the variance is  $np(1-p)$ .

The tables on this and subsequent pages give, for different values of  $n$ ,  $p$  and  $r$ , the probabilities

$$P(X \geq r) = \sum_{k=r}^n P(X = k) = \sum_{k=r}^n {}^nC_k p^k (1-p)^{n-k}.$$

Probabilities smaller than 0.00005 have been omitted.

		Value of $p$									
		0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.10
$n = 5$	$r$										
	0	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	1	0.0490	0.0961	0.1413	0.1846	0.2262	0.2661	0.3043	0.3409	0.3760	0.4095
	2	0.0010	0.0038	0.0085	0.0148	0.0226	0.0319	0.0425	0.0544	0.0674	0.0815
	3		0.0001	0.0003	0.0006	0.0012	0.0020	0.0031	0.0045	0.0063	0.0086
4						0.0001	0.0001	0.0002	0.0003	0.0005	

		Value of $p$									
		0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.10
$n = 10$	$r$										
	0	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	1	0.0956	0.1829	0.2626	0.3352	0.4013	0.4614	0.5160	0.5656	0.6106	0.6513
	2	0.0043	0.0162	0.0345	0.0582	0.0861	0.1176	0.1517	0.1879	0.2254	0.2639
	3	0.0001	0.0009	0.0028	0.0062	0.0115	0.0188	0.0283	0.0401	0.0540	0.0702
	4			0.0001	0.0004	0.0010	0.0020	0.0036	0.0058	0.0088	0.0128
	5					0.0001	0.0002	0.0003	0.0006	0.0010	0.0016
6									0.0001	0.0001	

		Value of $p$									
		0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.10
$n = 20$	$r$										
	0	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	1	0.1821	0.3324	0.4562	0.5580	0.6415	0.7099	0.7658	0.8113	0.8484	0.8784
	2	0.0169	0.0599	0.1198	0.1897	0.2642	0.3395	0.4131	0.4831	0.5484	0.6083
	3	0.0010	0.0071	0.0210	0.0439	0.0755	0.1150	0.1610	0.2121	0.2666	0.3231
	4		0.0006	0.0027	0.0074	0.0159	0.0290	0.0471	0.0706	0.0993	0.1330
	5			0.0003	0.0010	0.0026	0.0056	0.0107	0.0183	0.0290	0.0432
	6				0.0001	0.0003	0.0009	0.0019	0.0038	0.0068	0.0113
	7						0.0001	0.0003	0.0006	0.0013	0.0024
	8								0.0001	0.0002	0.0004
9									0.0001	0.0001	

		Value of $p$									
		0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.10
$n = 50$	$r$										
	0	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	1	0.3950	0.6358	0.7819	0.8701	0.9231	0.9547	0.9734	0.9845	0.9910	0.9948
	2	0.0894	0.2642	0.4447	0.5995	0.7206	0.8100	0.8735	0.9173	0.9468	0.9662
	3	0.0138	0.0784	0.1892	0.3233	0.4595	0.5838	0.6892	0.7740	0.8395	0.8883
	4	0.0016	0.0178	0.0628	0.1391	0.2396	0.3527	0.4673	0.5747	0.6697	0.7497
	5	0.0001	0.0032	0.0168	0.0490	0.1036	0.1794	0.2710	0.3710	0.4723	0.5688
	6		0.0005	0.0037	0.0144	0.0378	0.0776	0.1350	0.2081	0.2928	0.3839
	7			0.0001	0.0007	0.0036	0.0118	0.0289	0.0583	0.1019	0.1596
	8				0.0001	0.0008	0.0032	0.0094	0.0220	0.0438	0.0768
	9					0.0008	0.0027	0.0073	0.0167	0.0328	0.0579
	10						0.0002	0.0007	0.0022	0.0056	0.0125
	11							0.0002	0.0006	0.0017	0.0043
	12								0.0001	0.0005	0.0013
	13									0.0001	0.0004
14										0.0001	
15											

$r$	Value of $p$									
	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.10
0	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
1	0.6340	0.8674	0.9524	0.9831	0.9941	0.9979	0.9993	0.9998	0.9999	1.0000
2	0.2642	0.5967	0.8054	0.9128	0.9629	0.9848	0.9940	0.9977	0.9991	0.9997
3	0.0794	0.3233	0.5802	0.7679	0.8817	0.9434	0.9742	0.9887	0.9952	0.9981
4	0.0184	0.1410	0.3528	0.5705	0.7422	0.8570	0.9256	0.9633	0.9827	0.9922
5	0.0034	0.0508	0.1821	0.3711	0.5640	0.7232	0.8368	0.9097	0.9526	0.9763
6	0.0005	0.0155	0.0808	0.2116	0.3840	0.5593	0.7086	0.8201	0.8955	0.9424
7	0.0001	0.0041	0.0312	0.1064	0.2340	0.3936	0.5557	0.6968	0.8060	0.8828
8		0.0009	0.0106	0.0475	0.1280	0.2517	0.4012	0.5529	0.6872	0.7939
9		0.0002	0.0032	0.0190	0.0631	0.1463	0.2660	0.4074	0.5506	0.6791
10			0.0009	0.0068	0.0282	0.0775	0.1620	0.2780	0.4125	0.5487
11			0.0002	0.0022	0.0115	0.0376	0.0908	0.1757	0.2882	0.4168
12				0.0007	0.0043	0.0168	0.0469	0.1028	0.1876	0.2970
13				0.0002	0.0015	0.0069	0.0224	0.0559	0.1138	0.1982
14					0.0005	0.0026	0.0099	0.0282	0.0645	0.1239
15					0.0001	0.0009	0.0041	0.0133	0.0341	0.0726
16						0.0003	0.0016	0.0058	0.0169	0.0399
17						0.0001	0.0006	0.0024	0.0078	0.0206
18							0.0002	0.0009	0.0034	0.0100
19							0.0001	0.0003	0.0014	0.0046
20								0.0001	0.0005	0.0020
21									0.0002	0.0008
22									0.0001	0.0003
23										0.0001

$r$	Value of $p$									
	0.10	0.15	1/6	0.20	0.25	0.30	1/3	0.35	0.40	0.45
0	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
1	0.4095	0.5563	0.5981	0.6723	0.7627	0.8319	0.8683	0.8840	0.9222	0.9497
2	0.0815	0.1648	0.1962	0.2627	0.3672	0.4718	0.5391	0.5716	0.6630	0.7438
3	0.0086	0.0266	0.0355	0.0579	0.1035	0.1631	0.2099	0.2352	0.3174	0.4069
4	0.0005	0.0022	0.0033	0.0067	0.0156	0.0308	0.0453	0.0540	0.0870	0.1312
5		0.0001	0.0001	0.0003	0.0010	0.0024	0.0041	0.0053	0.0102	0.0185

$r$	Value of $p$									
	0.10	0.15	1/6	0.20	0.25	0.30	1/3	0.35	0.40	0.45
0	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
1	0.6513	0.8031	0.8385	0.8926	0.9437	0.9718	0.9827	0.9865	0.9940	0.9975
2	0.2639	0.4557	0.5155	0.6242	0.7560	0.8507	0.8960	0.9140	0.9536	0.9767
3	0.0702	0.1798	0.2248	0.3222	0.4744	0.6172	0.7009	0.7384	0.8327	0.9004
4	0.0128	0.0500	0.0697	0.1209	0.2241	0.3504	0.4407	0.4862	0.6177	0.7340
5	0.0016	0.0099	0.0155	0.0328	0.0781	0.1503	0.2131	0.2485	0.3669	0.4956
6	0.0001	0.0014	0.0024	0.0064	0.0197	0.0473	0.0766	0.0949	0.1662	0.2616
7		0.0001	0.0003	0.0009	0.0035	0.0106	0.0197	0.0260	0.0548	0.1020
8				0.0001	0.0004	0.0016	0.0034	0.0048	0.0123	0.0274
9						0.0001	0.0004	0.0005	0.0017	0.0045
10									0.0001	0.0003

$r$	Value of $p$									
	0.10	0.15	1/6	0.20	0.25	0.30	1/3	0.35	0.40	0.45
0	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
1	0.8784	0.9612	0.9739	0.9885	0.9968	0.9992	0.9997	0.9998	1.0000	1.0000
2	0.6083	0.8244	0.8696	0.9308	0.9757	0.9924	0.9967	0.9979	0.9995	0.9999
3	0.3231	0.5951	0.6713	0.7939	0.9087	0.9645	0.9824	0.9879	0.9964	0.9991
4	0.1330	0.3523	0.4335	0.5886	0.7748	0.8929	0.9396	0.9556	0.9840	0.9951
5	0.0432	0.1702	0.2313	0.3704	0.5852	0.7625	0.8485	0.8818	0.9490	0.9811
6	0.0113	0.0673	0.1018	0.1958	0.3828	0.5836	0.7028	0.7546	0.8744	0.9447
7	0.0024	0.0219	0.0371	0.0867	0.2142	0.3920	0.5207	0.5834	0.7500	0.8701
8	0.0004	0.0059	0.0113	0.0321	0.1018	0.2277	0.3385	0.3990	0.5841	0.7480
9	0.0001	0.0013	0.0028	0.0100	0.0409	0.1133	0.1905	0.2376	0.4044	0.5857
10		0.0002	0.0006	0.0026	0.0139	0.0480	0.0919	0.1218	0.2447	0.4086
11			0.0001	0.0006	0.0039	0.0171	0.0376	0.0532	0.1275	0.2493
12				0.0001	0.0009	0.0051	0.0130	0.0196	0.0565	0.1308
13					0.0002	0.0013	0.0037	0.0060	0.0210	0.0580
14						0.0003	0.0009	0.0015	0.0065	0.0214
15							0.0002	0.0003	0.0016	0.0064
16									0.0003	0.0015
17										0.0003

$r$	Value of $p$									
	0.10	0.15	1/6	0.20	0.25	0.30	1/3	0.35	0.40	0.45
0	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
1	0.9948	0.9997	0.9999	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
2	0.9662	0.9971	0.9988	0.9998	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
3	0.8883	0.9858	0.9934	0.9987	0.9999	1.0000	1.0000	1.0000	1.0000	1.0000
4	0.7497	0.9540	0.9762	0.9943	0.9995	1.0000	1.0000	1.0000	1.0000	1.0000
5	0.5688	0.8879	0.9357	0.9815	0.9979	0.9998	1.0000	1.0000	1.0000	1.0000
6	0.3839	0.7806	0.8612	0.9520	0.9930	0.9993	0.9999	0.9999	1.0000	1.0000
7	0.2298	0.6387	0.7494	0.8966	0.9806	0.9975	0.9995	0.9998	1.0000	1.0000
8	0.1221	0.4812	0.6089	0.8096	0.9547	0.9927	0.9983	0.9992	0.9999	1.0000
9	0.0579	0.3319	0.4579	0.6927	0.9084	0.9817	0.9950	0.9975	0.9998	1.0000
10	0.0245	0.2089	0.3170	0.5563	0.8363	0.9598	0.9873	0.9933	0.9992	0.9999
11	0.0094	0.1199	0.2014	0.4164	0.7378	0.9211	0.9716	0.9840	0.9978	0.9998
12	0.0032	0.0628	0.1173	0.2893	0.6184	0.8610	0.9430	0.9658	0.9943	0.9994
13	0.0010	0.0301	0.0627	0.1861	0.4890	0.7771	0.8965	0.9339	0.9867	0.9982
14	0.0003	0.0132	0.0307	0.1106	0.3630	0.6721	0.8285	0.8837	0.9720	0.9955
15	0.0001	0.0053	0.0138	0.0607	0.2519	0.5532	0.7388	0.8122	0.9460	0.9896
16		0.0019	0.0057	0.0308	0.1631	0.4308	0.6310	0.7199	0.9045	0.9780
17		0.0007	0.0022	0.0144	0.0983	0.3161	0.5132	0.6111	0.8439	0.9573
18		0.0002	0.0008	0.0063	0.0551	0.2178	0.3954	0.4940	0.7631	0.9235
19		0.0001	0.0003	0.0025	0.0287	0.1406	0.2874	0.3784	0.6644	0.8727
20			0.0001	0.0009	0.0139	0.0848	0.1964	0.2736	0.5535	0.8026
21				0.0003	0.0063	0.0478	0.1259	0.1861	0.4390	0.7138
22				0.0001	0.0026	0.0251	0.0756	0.1187	0.3299	0.6100
23					0.0010	0.0123	0.0424	0.0710	0.2340	0.4981
24					0.0004	0.0056	0.0222	0.0396	0.1562	0.3866
25					0.0001	0.0024	0.0108	0.0207	0.0978	0.2840
26						0.0009	0.0049	0.0100	0.0573	0.1966
27						0.0003	0.0021	0.0045	0.0314	0.1279
28						0.0001	0.0008	0.0019	0.0160	0.0780
29							0.0003	0.0007	0.0076	0.0444
30							0.0001	0.0003	0.0034	0.0235
31								0.0001	0.0014	0.0116
32									0.0005	0.0053
33									0.0002	0.0022
34									0.0001	0.0009
35										0.0003
36										0.0001

$r$	Value of $p$									
	0.10	0.15	1/6	0.20	0.25	0.30	1/3	0.35	0.40	0.45
0	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
1	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
2	0.9997	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
3	0.9981	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
4	0.9922	0.9999	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
5	0.9763	0.9996	0.9999	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
6	0.9424	0.9984	0.9996	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
7	0.8828	0.9953	0.9987	0.9999	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
8	0.7939	0.9878	0.9962	0.9997	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
9	0.6791	0.9725	0.9905	0.9991	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
10	0.5487	0.9449	0.9787	0.9977	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
11	0.4168	0.9006	0.9573	0.9943	0.9999	1.0000	1.0000	1.0000	1.0000	1.0000
12	0.2970	0.8365	0.9223	0.9874	0.9996	1.0000	1.0000	1.0000	1.0000	1.0000
13	0.1982	0.7527	0.8703	0.9747	0.9990	1.0000	1.0000	1.0000	1.0000	1.0000
14	0.1239	0.6526	0.8000	0.9531	0.9975	0.9999	1.0000	1.0000	1.0000	1.0000
15	0.0726	0.5428	0.7126	0.9196	0.9946	0.9998	1.0000	1.0000	1.0000	1.0000
16	0.0399	0.4317	0.6123	0.8715	0.9889	0.9996	1.0000	1.0000	1.0000	1.0000
17	0.0206	0.3275	0.5058	0.8077	0.9789	0.9990	0.9999	1.0000	1.0000	1.0000
18	0.0100	0.2367	0.4006	0.7288	0.9624	0.9978	0.9998	0.9999	1.0000	1.0000
19	0.0046	0.1628	0.3035	0.6379	0.9370	0.9955	0.9995	0.9999	1.0000	1.0000
20	0.0020	0.1065	0.2197	0.5398	0.9005	0.9911	0.9989	0.9997	1.0000	1.0000
21	0.0008	0.0663	0.1519	0.4405	0.8512	0.9835	0.9976	0.9992	1.0000	1.0000
22	0.0003	0.0393	0.1002	0.3460	0.7886	0.9712	0.9952	0.9983	1.0000	1.0000
23	0.0001	0.0221	0.0631	0.2611	0.7136	0.9521	0.9909	0.9966	0.9999	1.0000
24		0.0119	0.0379	0.1891	0.6289	0.9245	0.9836	0.9934	0.9997	1.0000
25		0.0061	0.0217	0.1314	0.5383	0.8864	0.9719	0.9879	0.9994	1.0000
26		0.0030	0.0119	0.0875	0.4465	0.8369	0.9542	0.9789	0.9988	1.0000
27		0.0014	0.0062	0.0558	0.3583	0.7756	0.9285	0.9649	0.9976	0.9999
28		0.0006	0.0031	0.0342	0.2776	0.7036	0.8934	0.9442	0.9954	0.9998
29		0.0003	0.0015	0.0200	0.2075	0.6232	0.8476	0.9152	0.9916	0.9996
30		0.0001	0.0007	0.0112	0.1495	0.5377	0.7907	0.8764	0.9852	0.9992
31			0.0003	0.0061	0.1038	0.4509	0.7234	0.8270	0.9752	0.9985
32			0.0001	0.0031	0.0693	0.3669	0.6475	0.7669	0.9602	0.9970
33				0.0016	0.0446	0.2893	0.5656	0.6971	0.9385	0.9945
34				0.0007	0.0276	0.2207	0.4812	0.6197	0.9087	0.9902
35				0.0003	0.0164	0.1629	0.3981	0.5376	0.8697	0.9834
36				0.0001	0.0094	0.1161	0.3197	0.4542	0.8205	0.9728
37				0.0001	0.0052	0.0799	0.2489	0.3731	0.7614	0.9571
38					0.0027	0.0530	0.1877	0.2976	0.6932	0.9349
39					0.0014	0.0340	0.1370	0.2301	0.6178	0.9049
40					0.0007	0.0210	0.0966	0.1724	0.5379	0.8657
41					0.0003	0.0125	0.0659	0.1250	0.4567	0.8169
42					0.0001	0.0072	0.0434	0.0877	0.3775	0.7585
43					0.0001	0.0040	0.0276	0.0594	0.3033	0.6913
44						0.0021	0.0169	0.0389	0.2365	0.6172
45						0.0011	0.0100	0.0246	0.1789	0.5387
46						0.0005	0.0057	0.0150	0.1311	0.4587
47						0.0003	0.0031	0.0088	0.0930	0.3804
48						0.0001	0.0017	0.0050	0.0638	0.3069
49						0.0001	0.0009	0.0027	0.0423	0.2404
50							0.0004	0.0015	0.0271	0.1827
51							0.0002	0.0007	0.0168	0.1346
52							0.0001	0.0004	0.0100	0.0960
53								0.0002	0.0058	0.0662
54								0.0001	0.0032	0.0441
55									0.0017	0.0284
56									0.0009	0.0176
57									0.0004	0.0106
58									0.0002	0.0061
59									0.0001	0.0034
60										0.0018
61										0.0009
62										0.0005
63										0.0002
64										0.0001

$n = 100$

Values of  $P(X \geq r)$  where  $X$  has the  $\text{Bin}(n, \frac{1}{2})$  distribution.

		Value of $n$										
		4	5	6	7	8	9	10	11	12	13	14
Value of $r$	0	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	1	0.9375	0.9688	0.9844	0.9922	0.9961	0.9980	0.9990	0.9995	0.9998	0.9999	0.9999
	2	0.6875	0.8125	0.8906	0.9375	0.9648	0.9805	0.9893	0.9941	0.9968	0.9983	0.9991
	3	0.3125	0.5000	0.6562	0.7734	0.8555	0.9102	0.9453	0.9673	0.9807	0.9888	0.9935
	4	0.0625	0.1875	0.3437	0.5000	0.6367	0.7461	0.8281	0.8867	0.9270	0.9539	0.9713
	5		0.0312	0.1094	0.2266	0.3633	0.5000	0.6230	0.7256	0.8062	0.8666	0.9102
	6			0.0156	0.0625	0.1445	0.2539	0.3770	0.5000	0.6128	0.7095	0.7880
	7				0.0078	0.0352	0.0898	0.1719	0.2744	0.3872	0.5000	0.6047
	8					0.0039	0.0195	0.0547	0.1133	0.1938	0.2905	0.3953
	9						0.0020	0.0107	0.0327	0.0730	0.1334	0.2120
	10							0.0010	0.0059	0.0193	0.0461	0.0898
	11								0.0005	0.0032	0.0112	0.0287
	12									0.0002	0.0017	0.0065
	13										0.0001	0.0009
	14											0.0001

		Value of $n$										
		15	16	17	18	19	20	21	22	23	24	25
Value of $r$	0	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	1	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	2	0.9995	0.9997	0.9999	0.9999	0.9999	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	3	0.9963	0.9979	0.9988	0.9993	0.9996	0.9998	0.9999	0.9999	0.9999	1.0000	1.0000
	4	0.9824	0.9894	0.9936	0.9962	0.9978	0.9987	0.9993	0.9996	0.9998	0.9999	0.9999
	5	0.9408	0.9616	0.9755	0.9846	0.9904	0.9941	0.9964	0.9978	0.9987	0.9992	0.9995
	6	0.8491	0.8949	0.9283	0.9519	0.9682	0.9793	0.9867	0.9915	0.9947	0.9967	0.9980
	7	0.6964	0.7728	0.8338	0.8811	0.9165	0.9423	0.9608	0.9738	0.9827	0.9887	0.9927
	8	0.5000	0.5982	0.6855	0.7597	0.8204	0.8684	0.9054	0.9331	0.9534	0.9680	0.9784
	9	0.3036	0.4018	0.5000	0.5927	0.6762	0.7483	0.8083	0.8569	0.8950	0.9242	0.9461
	10	0.1509	0.2272	0.3145	0.4073	0.5000	0.5881	0.6682	0.7383	0.7976	0.8463	0.8852
	11	0.0592	0.1051	0.1662	0.2403	0.3238	0.4119	0.5000	0.5841	0.6612	0.7294	0.7878
	12	0.0176	0.0384	0.0717	0.1189	0.1796	0.2517	0.3318	0.4159	0.5000	0.5806	0.6550
	13	0.0037	0.0106	0.0245	0.0481	0.0835	0.1316	0.1917	0.2617	0.3388	0.4194	0.5000
	14	0.0005	0.0021	0.0064	0.0154	0.0318	0.0577	0.0946	0.1431	0.2024	0.2706	0.3450
	15		0.0003	0.0012	0.0038	0.0096	0.0207	0.0392	0.0669	0.1050	0.1537	0.2122
	16			0.0001	0.0007	0.0022	0.0059	0.0133	0.0262	0.0466	0.0758	0.1148
	17				0.0001	0.0004	0.0013	0.0036	0.0085	0.0173	0.0320	0.0539
	18						0.0002	0.0007	0.0022	0.0053	0.0113	0.0216
	19							0.0001	0.0004	0.0013	0.0033	0.0073
	20								0.0001	0.0002	0.0008	0.0020
	21									0.0001	0.0001	0.0005
22											0.0001	

$n = 50$	13	14	15	16	17	18	19	20	21	22	23
	0.9998	0.9995	0.9987	0.9967	0.9923	0.9836	0.9675	0.9405	0.8987	0.8389	0.7601
	24	25	26	27	28	29	30	31	32	33	34
	0.6641	0.5561	0.4439	0.3359	0.2399	0.1611	0.1013	0.0595	0.0325	0.0164	0.0077
	35	36	37	38							
0.0033	0.0013	0.0005	0.0002								

$n = 100$	32	33	34	35	36	37	38	39	40	41	42
	0.9999	0.9998	0.9996	0.9991	0.9982	0.9967	0.9940	0.9895	0.9824	0.9716	0.9557
	43	44	45	46	47	48	49	50	51	52	53
	0.9334	0.9033	0.8644	0.8159	0.7579	0.6914	0.6178	0.5398	0.4602	0.3822	0.3086
	54	55	56	57	58	59	60	61	62	63	64
	0.2421	0.1841	0.1356	0.0967	0.0666	0.0443	0.0284	0.0176	0.0105	0.0060	0.0033
	65	66	67	68	69						
	0.0018	0.0009	0.0004	0.0002	0.0001						

1. In the tables for  $n = 50$  and  $n = 100$  values above 0.99995 have also been omitted.
2. For large values of  $n$ , provided that  $p$  is neither too large or too small, a Normal approximation may be used in that

$$P(X = k) \cong P\left(k - \frac{1}{2} < Y < k + \frac{1}{2}\right),$$

where  $Y$  has the  $N(np, np(1 - p))$  distribution.

## 2 Binomial coefficients

Values of  ${}^n C_r = \frac{n!}{r!(n-r)!}$ , the number of combinations of  $r$  objects selected from  $n$ .

	$r$													
	0	1	2	3	4	5	6	7	8	9	10	11	12	
1	1	1												
2	1	2	1											
3	1	3	3	1										
4	1	4	6	4	1									
5	1	5	10	10	5	1								
6	1	6	15	20	15	6	1							
7	1	7	21	35	35	21	7	1						
8	1	8	28	56	70	56	28	8	1					
9	1	9	36	84	126	126	84	36	9	1				
10	1	10	45	120	210	252	210	120	45	10	1			
11	1	11	55	165	330	462	462	330	165	55	11	1		
12	1	12	66	220	495	792	924	792	495	220	66	12	1	
13	1	13	78	286	715	1287	1716	1716	1287	715	286	78	13	
14	1	14	91	364	1001	2002	3003	3432	3003	2002	1001	364	91	
15	1	15	105	455	1365	3003	5005	6435	6435	5005	3003	1365	455	
16	1	16	120	560	1820	4368	8008	11440	12870	11440	8008	4368	1820	
17	1	17	136	680	2380	6188	12376	19448	24310	24310	19448	12376	6188	
18	1	18	153	816	3060	8568	18564	31824	43758	48620	43758	31824	18564	
19	1	19	171	969	3876	11628	27132	50388	75582	92378	92378	75582	50388	
20	1	20	190	1140	4845	15504	38760	77520	125970	167960	184756	167960	125970	
21	1	21	210	1330	5985	20349	54264	116280	203490	293930	352716	352716	293930	
22	1	22	231	1540	7315	26334	74613	170544	319770	497420	646646	705432	646646	
23	1	23	253	1771	8855	33649	100947	245157	490314	817190	1144066	1352078	1352078	
24	1	24	276	2024	10626	42504	134596	346104	735471	1307504	1961256	2496144	2704156	
25	1	25	300	2300	12650	53130	177100	480700	1081575	2042975	3268760	4457400	5200300	

Notes:

1.  ${}^n C_r = {}^n C_{n-r}$ ,
2.  ${}^n C_r = {}^{n-1} C_{r-1} + {}^{n-1} C_r$ ,
3.  $\sum_{r=0}^n {}^n C_r = 2^n$ ,
4. For large  $n$ , Stirling's approximation may be useful, where  $n! \cong \sqrt{2\pi n} n^{n+\frac{1}{2}} e^{-n}$ .

### 3 Poisson distribution

This distribution gives the probabilities of the numbers of incidents in a process of random events, if those events occur at rate  $\lambda$ .

The probability of observing  $k$  events is  $P(X = k) = e^{-\lambda} \frac{\lambda^k}{k!}$  for  $k = 0, 1, 2, \dots$

The mean and the variance of the distribution are both equal to  $\lambda$ . The tables on this and subsequent pages give, for different values of  $\lambda$  and  $r$ , the probabilities

$$P(X \geq r) = \sum_{k=r}^{\infty} P(X = k) = \sum_{k=r}^{\infty} e^{-\lambda} \frac{\lambda^k}{k!}.$$

Values smaller than 0.00005 have been omitted.

		Value of $\lambda$									
		0.05	0.10	0.15	0.20	0.25	0.30	0.35	0.40	0.45	0.50
Value of $r$	0	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	1	0.0488	0.0952	0.1393	0.1813	0.2212	0.2592	0.2953	0.3297	0.3624	0.3935
	2	0.0012	0.0047	0.0102	0.0175	0.0265	0.0369	0.0487	0.0616	0.0754	0.0902
	3		0.0002	0.0005	0.0011	0.0022	0.0036	0.0055	0.0079	0.0109	0.0144
	4				0.0001	0.0001	0.0003	0.0005	0.0008	0.0012	0.0018
	5								0.0001	0.0001	0.0002

		Value of $\lambda$									
		0.55	0.60	0.65	0.70	0.75	0.80	0.85	0.90	0.95	1.00
Value of $r$	0	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	1	0.4231	0.4512	0.4780	0.5034	0.5276	0.5507	0.5726	0.5934	0.6133	0.6321
	2	0.1057	0.1219	0.1386	0.1558	0.1734	0.1912	0.2093	0.2275	0.2459	0.2642
	3	0.0185	0.0231	0.0283	0.0341	0.0405	0.0474	0.0549	0.0629	0.0713	0.0803
	4	0.0025	0.0034	0.0044	0.0058	0.0073	0.0091	0.0111	0.0135	0.0161	0.0190
	5	0.0003	0.0004	0.0006	0.0008	0.0011	0.0014	0.0018	0.0023	0.0029	0.0037
	6			0.0001	0.0001	0.0001	0.0002	0.0003	0.0003	0.0005	0.0006
	7								0.0001	0.0001	0.0001

		Value of $\lambda$									
		1.10	1.20	1.30	1.40	1.50	1.60	1.70	1.80	1.90	2.00
Value of $r$	0	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	1	0.6671	0.6988	0.7275	0.7534	0.7769	0.7981	0.8173	0.8347	0.8504	0.8647
	2	0.3010	0.3374	0.3732	0.4082	0.4422	0.4751	0.5068	0.5372	0.5663	0.5940
	3	0.0996	0.1205	0.1429	0.1665	0.1912	0.2166	0.2428	0.2694	0.2963	0.3233
	4	0.0257	0.0338	0.0431	0.0537	0.0656	0.0788	0.0932	0.1087	0.1253	0.1429
	5	0.0054	0.0077	0.0107	0.0143	0.0186	0.0237	0.0296	0.0364	0.0441	0.0527
	6	0.0010	0.0015	0.0022	0.0032	0.0045	0.0060	0.0080	0.0104	0.0132	0.0166
	7	0.0001	0.0003	0.0004	0.0006	0.0009	0.0013	0.0019	0.0026	0.0034	0.0045
	8			0.0001	0.0001	0.0002	0.0003	0.0004	0.0006	0.0008	0.0011
	9							0.0001	0.0001	0.0002	0.0002

		Value of $\lambda$									
		2.10	2.20	2.30	2.40	2.50	2.60	2.70	2.80	2.90	3.00
Value of $r$	0	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
	1	0.8775	0.8892	0.8997	0.9093	0.9179	0.9257	0.9328	0.9392	0.9450	0.9502
	2	0.6204	0.6454	0.6691	0.6916	0.7127	0.7326	0.7513	0.7689	0.7854	0.8009
	3	0.3504	0.3773	0.4040	0.4303	0.4562	0.4816	0.5064	0.5305	0.5540	0.5768
	4	0.1614	0.1806	0.2007	0.2213	0.2424	0.2640	0.2859	0.3081	0.3304	0.3528
	5	0.0621	0.0725	0.0838	0.0959	0.1088	0.1226	0.1371	0.1523	0.1682	0.1847
	6	0.0204	0.0249	0.0300	0.0357	0.0420	0.0490	0.0567	0.0651	0.0742	0.0839
	7	0.0059	0.0075	0.0094	0.0116	0.0142	0.0172	0.0206	0.0244	0.0287	0.0335
	8	0.0015	0.0020	0.0026	0.0033	0.0042	0.0053	0.0066	0.0081	0.0099	0.0119
	9	0.0003	0.0005	0.0006	0.0009	0.0011	0.0015	0.0019	0.0024	0.0031	0.0038
	10	0.0001	0.0001	0.0001	0.0002	0.0003	0.0004	0.0005	0.0007	0.0009	0.0011
	11					0.0001	0.0001	0.0001	0.0002	0.0002	0.0003
	12								0.0001	0.0001	0.0001



Value of $r$	Value of $\lambda$									
	3.50	4.00	4.50	5.00	5.50	6.00	6.50	7.00	7.50	8.00
0	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
1	0.9698	0.9817	0.9889	0.9933	0.9959	0.9975	0.9985	0.9991	0.9994	0.9997
2	0.8641	0.9084	0.9389	0.9596	0.9734	0.9826	0.9887	0.9927	0.9953	0.9970
3	0.6792	0.7619	0.8264	0.8753	0.9116	0.9380	0.9570	0.9704	0.9797	0.9862
4	0.4634	0.5665	0.6577	0.7350	0.7983	0.8488	0.8882	0.9182	0.9409	0.9576
5	0.2746	0.3712	0.4679	0.5595	0.6425	0.7149	0.7763	0.8270	0.8679	0.9004
6	0.1424	0.2149	0.2971	0.3840	0.4711	0.5543	0.6310	0.6993	0.7586	0.8088
7	0.0653	0.1107	0.1689	0.2378	0.3140	0.3937	0.4735	0.5503	0.6218	0.6866
8	0.0267	0.0511	0.0866	0.1334	0.1905	0.2560	0.3272	0.4013	0.4754	0.5470
9	0.0099	0.0214	0.0403	0.0681	0.1056	0.1528	0.2084	0.2709	0.3380	0.4075
10	0.0033	0.0081	0.0171	0.0318	0.0538	0.0839	0.1226	0.1695	0.2236	0.2834
11	0.0010	0.0028	0.0067	0.0137	0.0253	0.0426	0.0668	0.0985	0.1378	0.1841
12	0.0003	0.0009	0.0024	0.0055	0.0110	0.0201	0.0339	0.0533	0.0792	0.1119
13	0.0001	0.0003	0.0008	0.0020	0.0045	0.0088	0.0160	0.0270	0.0427	0.0638
14		0.0001	0.0003	0.0007	0.0017	0.0036	0.0071	0.0128	0.0216	0.0342
15			0.0001	0.0002	0.0006	0.0014	0.0030	0.0057	0.0103	0.0173
16				0.0001	0.0002	0.0005	0.0012	0.0024	0.0046	0.0082
17					0.0001	0.0002	0.0004	0.0010	0.0020	0.0037
18						0.0001	0.0002	0.0004	0.0008	0.0016
19							0.0001	0.0001	0.0003	0.0007
20									0.0001	0.0003
21										0.0001

Value of $r$	Value of $\lambda$									
	8.50	9.00	9.50	10.00	10.50	11.00	11.50	12.00	12.50	13.00
0	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
1	0.9998	0.9999	0.9999	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
2	0.9981	0.9988	0.9992	0.9995	0.9997	0.9998	0.9999	0.9999	0.9999	1.0000
3	0.9907	0.9938	0.9958	0.9972	0.9982	0.9988	0.9992	0.9995	0.9997	0.9998
4	0.9699	0.9788	0.9851	0.9897	0.9929	0.9951	0.9966	0.9977	0.9984	0.9989
5	0.9256	0.9450	0.9597	0.9707	0.9789	0.9849	0.9893	0.9924	0.9947	0.9963
6	0.8504	0.8843	0.9115	0.9329	0.9496	0.9625	0.9723	0.9797	0.9852	0.9893
7	0.7438	0.7932	0.8351	0.8699	0.8984	0.9214	0.9397	0.9542	0.9654	0.9741
8	0.6144	0.6761	0.7313	0.7798	0.8215	0.8568	0.8863	0.9105	0.9302	0.9460
9	0.4769	0.5443	0.6082	0.6672	0.7206	0.7680	0.8094	0.8450	0.8751	0.9002
10	0.3470	0.4126	0.4782	0.5421	0.6029	0.6595	0.7112	0.7576	0.7986	0.8342
11	0.2366	0.2940	0.3547	0.4170	0.4793	0.5401	0.5983	0.6528	0.7029	0.7483
12	0.1513	0.1970	0.2480	0.3032	0.3613	0.4207	0.4802	0.5384	0.5942	0.6468
13	0.0909	0.1242	0.1636	0.2084	0.2580	0.3113	0.3671	0.4240	0.4810	0.5369
14	0.0514	0.0739	0.1019	0.1355	0.1747	0.2187	0.2670	0.3185	0.3722	0.4270
15	0.0274	0.0415	0.0600	0.0835	0.1121	0.1460	0.1847	0.2280	0.2750	0.3249
16	0.0138	0.0220	0.0335	0.0487	0.0683	0.0926	0.1217	0.1556	0.1940	0.2364
17	0.0066	0.0111	0.0177	0.0270	0.0396	0.0559	0.0764	0.1013	0.1307	0.1645
18	0.0030	0.0053	0.0089	0.0143	0.0219	0.0322	0.0458	0.0630	0.0842	0.1095
19	0.0013	0.0024	0.0043	0.0072	0.0115	0.0177	0.0262	0.0374	0.0519	0.0698
20	0.0005	0.0011	0.0020	0.0035	0.0058	0.0093	0.0143	0.0213	0.0306	0.0427
21	0.0002	0.0004	0.0009	0.0016	0.0028	0.0047	0.0075	0.0116	0.0173	0.0250
22	0.0001	0.0002	0.0004	0.0007	0.0013	0.0023	0.0038	0.0061	0.0094	0.0141
23		0.0001	0.0001	0.0003	0.0006	0.0010	0.0018	0.0030	0.0049	0.0076
24			0.0001	0.0001	0.0002	0.0005	0.0008	0.0015	0.0025	0.0040
25					0.0001	0.0002	0.0004	0.0007	0.0012	0.0020
26						0.0001	0.0002	0.0003	0.0006	0.0010
27							0.0001	0.0001	0.0003	0.0005
28								0.0001	0.0001	0.0002
29									0.0001	0.0001

Value of $r$	Value of $\lambda$									
	14.00	15.00	16.00	17.00	18.00	19.00	20.00	21.00	22.00	23.00
0	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
1	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
2	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
3	0.9999	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
4	0.9995	0.9998	0.9999	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
5	0.9982	0.9991	0.9996	0.9998	0.9998	0.9999	1.0000	1.0000	1.0000	1.0000
6	0.9945	0.9972	0.9986	0.9993	0.9997	0.9998	0.9999	1.0000	1.0000	1.0000
7	0.9858	0.9924	0.9960	0.9979	0.9990	0.9995	0.9997	0.9999	0.9999	1.0000
8	0.9684	0.9820	0.9900	0.9946	0.9971	0.9985	0.9992	0.9996	0.9998	0.9999
9	0.9379	0.9626	0.9780	0.9874	0.9929	0.9961	0.9979	0.9989	0.9994	0.9997
10	0.8906	0.9301	0.9567	0.9739	0.9846	0.9911	0.9950	0.9972	0.9985	0.9992
11	0.8243	0.8815	0.9226	0.9509	0.9696	0.9817	0.9892	0.9937	0.9965	0.9980
12	0.7400	0.8152	0.8730	0.9153	0.9451	0.9653	0.9786	0.9871	0.9924	0.9956
13	0.6415	0.7324	0.8069	0.8650	0.9083	0.9394	0.9610	0.9755	0.9849	0.9909
14	0.5356	0.6368	0.7255	0.7991	0.8574	0.9016	0.9339	0.9566	0.9722	0.9826
15	0.4296	0.5343	0.6325	0.7192	0.7919	0.8503	0.8951	0.9284	0.9523	0.9689
16	0.3306	0.4319	0.5333	0.6285	0.7133	0.7852	0.8435	0.8889	0.9231	0.9480
17	0.2441	0.3359	0.4340	0.5323	0.6249	0.7080	0.7789	0.8371	0.8830	0.9179
18	0.1728	0.2511	0.3407	0.4360	0.5314	0.6216	0.7030	0.7730	0.8310	0.8772
19	0.1174	0.1805	0.2577	0.3450	0.4378	0.5305	0.6186	0.6983	0.7675	0.8252
20	0.0765	0.1248	0.1878	0.2637	0.3491	0.4394	0.5297	0.6157	0.6940	0.7623
21	0.0479	0.0830	0.1318	0.1945	0.2693	0.3528	0.4409	0.5290	0.6131	0.6899
22	0.0288	0.0531	0.0892	0.1385	0.2009	0.2745	0.3563	0.4423	0.5284	0.6106
23	0.0167	0.0327	0.0582	0.0953	0.1449	0.2069	0.2794	0.3595	0.4436	0.5277
24	0.0093	0.0195	0.0367	0.0633	0.1011	0.1510	0.2125	0.2840	0.3626	0.4449
25	0.0050	0.0112	0.0223	0.0406	0.0683	0.1067	0.1568	0.2178	0.2883	0.3654
26	0.0026	0.0062	0.0131	0.0252	0.0446	0.0731	0.1122	0.1623	0.2229	0.2923
27	0.0013	0.0033	0.0075	0.0152	0.0282	0.0486	0.0779	0.1174	0.1676	0.2277
28	0.0006	0.0017	0.0041	0.0088	0.0173	0.0313	0.0525	0.0825	0.1225	0.1726
29	0.0003	0.0009	0.0022	0.0050	0.0103	0.0195	0.0343	0.0564	0.0871	0.1274
30	0.0001	0.0004	0.0011	0.0027	0.0059	0.0118	0.0218	0.0374	0.0602	0.0915
31	0.0001	0.0002	0.0006	0.0014	0.0033	0.0070	0.0135	0.0242	0.0405	0.0640
32		0.0001	0.0003	0.0007	0.0018	0.0040	0.0081	0.0152	0.0265	0.0436
33			0.0001	0.0004	0.0010	0.0022	0.0047	0.0093	0.0169	0.0289
34			0.0001	0.0002	0.0005	0.0012	0.0027	0.0055	0.0105	0.0187
35				0.0001	0.0002	0.0006	0.0015	0.0032	0.0064	0.0118
36					0.0001	0.0003	0.0008	0.0018	0.0038	0.0073
37					0.0001	0.0002	0.0004	0.0010	0.0022	0.0044
38						0.0001	0.0002	0.0005	0.0012	0.0026
39							0.0001	0.0003	0.0007	0.0015
40							0.0001	0.0001	0.0004	0.0008
41								0.0001	0.0002	0.0004
42									0.0001	0.0002
43										0.0001
44										0.0001

Value of $r$	Value of $\lambda$									
	24.0	25.0	26.0	27.0	28.0	29.0	30.0	35.0	40.0	50.0
0	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
1	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
2	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
3	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
4	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
5	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
6	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
7	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
8	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
9	0.9998	0.9999	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
10	0.9996	0.9998	0.9999	0.9999	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
11	0.9989	0.9994	0.9997	0.9998	0.9999	1.0000	1.0000	1.0000	1.0000	1.0000
12	0.9975	0.9986	0.9992	0.9996	0.9998	0.9999	0.9999	1.0000	1.0000	1.0000
13	0.9946	0.9969	0.9982	0.9990	0.9994	0.9997	0.9998	1.0000	1.0000	1.0000
14	0.9893	0.9935	0.9962	0.9978	0.9987	0.9993	0.9996	1.0000	1.0000	1.0000
15	0.9802	0.9876	0.9924	0.9954	0.9973	0.9984	0.9991	1.0000	1.0000	1.0000
16	0.9656	0.9777	0.9858	0.9912	0.9946	0.9967	0.9981	0.9999	1.0000	1.0000
17	0.9437	0.9623	0.9752	0.9840	0.9899	0.9937	0.9961	0.9997	1.0000	1.0000
18	0.9129	0.9395	0.9589	0.9726	0.9821	0.9885	0.9927	0.9994	1.0000	1.0000
19	0.8717	0.9080	0.9354	0.9555	0.9700	0.9801	0.9871	0.9988	0.9999	1.0000
20	0.8197	0.8664	0.9032	0.9313	0.9522	0.9674	0.9781	0.9977	0.9998	1.0000

	Value of $\lambda$									
	24.0	25.0	26.0	27.0	28.0	29.0	30.0	35.0	40.0	50.0
21	0.7574	0.8145	0.8613	0.8985	0.9273	0.9489	0.9647	0.9957	0.9996	1.0000
22	0.6861	0.7527	0.8095	0.8564	0.8940	0.9233	0.9456	0.9924	0.9993	1.0000
23	0.6083	0.6825	0.7483	0.8048	0.8517	0.8896	0.9194	0.9872	0.9986	1.0000
24	0.5272	0.6061	0.6791	0.7441	0.8002	0.8471	0.8854	0.9792	0.9974	1.0000
25	0.4460	0.5266	0.6041	0.6758	0.7401	0.7958	0.8428	0.9676	0.9955	1.0000
26	0.3681	0.4471	0.5261	0.6021	0.6728	0.7363	0.7916	0.9514	0.9924	0.9999
27	0.2962	0.3706	0.4481	0.5256	0.6003	0.6699	0.7327	0.9295	0.9877	0.9999
28	0.2323	0.2998	0.3730	0.4491	0.5251	0.5986	0.6671	0.9012	0.9807	0.9997
29	0.1775	0.2366	0.3033	0.3753	0.4500	0.5247	0.5969	0.8657	0.9706	0.9995
30	0.1321	0.1821	0.2407	0.3065	0.3774	0.4508	0.5243	0.8230	0.9568	0.9991
31	0.0958	0.1367	0.1866	0.2447	0.3097	0.3794	0.4516	0.7731	0.9383	0.9984
32	0.0678	0.1001	0.1411	0.1908	0.2485	0.3126	0.3814	0.7167	0.9145	0.9973
33	0.0467	0.0715	0.1042	0.1454	0.1949	0.2521	0.3155	0.6551	0.8847	0.9956
34	0.0314	0.0498	0.0751	0.1082	0.1495	0.1989	0.2556	0.5898	0.8486	0.9930
35	0.0206	0.0338	0.0528	0.0787	0.1121	0.1535	0.2027	0.5225	0.8061	0.9892
36	0.0132	0.0225	0.0363	0.0559	0.0822	0.1159	0.1574	0.4552	0.7576	0.9838
37	0.0082	0.0146	0.0244	0.0388	0.0589	0.0856	0.1196	0.3898	0.7037	0.9762
38	0.0050	0.0092	0.0160	0.0263	0.0413	0.0619	0.0890	0.3279	0.6453	0.9660
39	0.0030	0.0057	0.0103	0.0175	0.0283	0.0438	0.0648	0.2709	0.5840	0.9526
40	0.0017	0.0034	0.0064	0.0113	0.0190	0.0303	0.0463	0.2198	0.5210	0.9354
41	0.0010	0.0020	0.0039	0.0072	0.0125	0.0205	0.0323	0.1751	0.4581	0.9139
42	0.0005	0.0012	0.0024	0.0045	0.0080	0.0136	0.0221	0.1369	0.3967	0.8877
43	0.0003	0.0007	0.0014	0.0027	0.0050	0.0089	0.0148	0.1050	0.3382	0.8565
44	0.0002	0.0004	0.0008	0.0016	0.0031	0.0056	0.0097	0.0791	0.2838	0.8202
45	0.0001	0.0002	0.0004	0.0009	0.0019	0.0035	0.0063	0.0585	0.2343	0.7790
46		0.0001	0.0002	0.0005	0.0011	0.0022	0.0040	0.0425	0.1903	0.7331
47		0.0001	0.0001	0.0003	0.0006	0.0013	0.0025	0.0303	0.1521	0.6833
48			0.0001	0.0002	0.0004	0.0008	0.0015	0.0212	0.1196	0.6303
49				0.0001	0.0002	0.0004	0.0009	0.0146	0.0925	0.5751
50					0.0001	0.0002	0.0005	0.0098	0.0703	0.5188
51					0.0001	0.0001	0.0003	0.0065	0.0526	0.4625
52						0.0001	0.0002	0.0043	0.0387	0.4073
53							0.0001	0.0027	0.0281	0.3542
54							0.0001	0.0017	0.0200	0.3041
55								0.0011	0.0140	0.2577
56								0.0007	0.0097	0.2155
57								0.0004	0.0066	0.1779
58								0.0002	0.0044	0.1449
59								0.0001	0.0029	0.1164
60								0.0001	0.0019	0.0923
61									0.0012	0.0722
62									0.0008	0.0557
63									0.0005	0.0424
64									0.0003	0.0318
65									0.0002	0.0236
66									0.0001	0.0173
67									0.0001	0.0125
68										0.0089
69										0.0062
70										0.0043
71										0.0030
72										0.0020
73										0.0013
74										0.0009
75										0.0006
76										0.0004
77										0.0002
78										0.0001
79										0.0001
80										0.0001

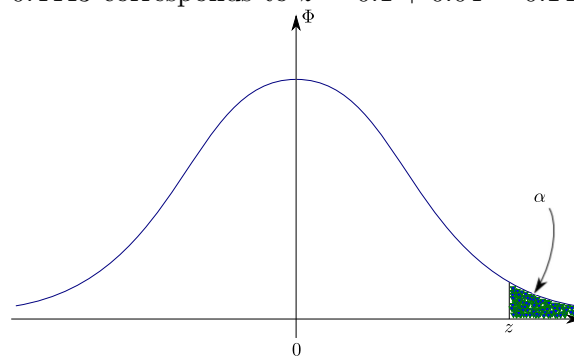
For large  $\lambda$ ,  $P(X = k) \cong P(k - \frac{1}{2} < Y < k + \frac{1}{2})$  where  $Y$  has the  $N(\lambda, \lambda)$  distribution.

## 4 Standard Normal $Z$ distribution

The table shows the relationship between  $z$  and  $\alpha$  in the diagram shown, the curve  $\Phi$  being the probability density function of the standard Normal distribution;  $\alpha$  is the area to the right of  $z$ , and represents the chance that a variable having the standard Normal distribution takes a value greater than  $z$ .

For a value  $x$  from a Normal distribution having mean  $\mu$  and variance  $\sigma^2$ , use  $z = \frac{x - \mu}{\sigma}$ .

In the first table, the values in the body of the table give the values of  $\alpha$  corresponding



$z$	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.4960	0.4920	0.4880	0.484	0.4801	0.4761	0.4721	0.4681	0.4641
0.1	0.4602	0.4562	0.4522	0.4483	0.4443	0.4404	0.4364	0.4325	0.4286	0.4247
0.2	0.4207	0.4168	0.4129	0.4090	0.4052	0.4013	0.3974	0.3936	0.3897	0.3859
0.3	0.3821	0.3783	0.3745	0.3707	0.3669	0.3632	0.3594	0.3557	0.3520	0.3483
0.4	0.3446	0.3409	0.3372	0.3336	0.3300	0.3264	0.3228	0.3192	0.3156	0.3121
0.5	0.3085	0.3050	0.3015	0.2981	0.2946	0.2912	0.2877	0.2843	0.2810	0.2776
0.6	0.2743	0.2709	0.2676	0.2643	0.2611	0.2578	0.2546	0.2514	0.2483	0.2451
0.7	0.2420	0.2389	0.2358	0.2327	0.2296	0.2266	0.2236	0.2206	0.2177	0.2148
0.8	0.2119	0.2090	0.2061	0.2033	0.2005	0.1977	0.1949	0.1922	0.1894	0.1867
0.9	0.1841	0.1814	0.1788	0.1762	0.1736	0.1711	0.1685	0.1660	0.1635	0.1611
1.0	0.1587	0.1562	0.1539	0.1515	0.1492	0.1469	0.1446	0.1423	0.1401	0.1379
1.1	0.1357	0.1335	0.1314	0.1292	0.1271	0.1251	0.1230	0.1210	0.1190	0.1170
1.2	0.1151	0.1131	0.1112	0.1093	0.1075	0.1056	0.1038	0.1020	0.1003	0.0985
1.3	0.0968	0.0951	0.0934	0.0918	0.0901	0.0885	0.0869	0.0853	0.0838	0.0823
1.4	0.0808	0.0793	0.0778	0.0764	0.0749	0.0735	0.0721	0.0708	0.0694	0.0681
1.5	0.0668	0.0655	0.0643	0.0630	0.0618	0.0606	0.0594	0.0582	0.0571	0.0559
1.6	0.0548	0.0537	0.0526	0.0516	0.0505	0.0495	0.0485	0.0475	0.0465	0.0455
1.7	0.0446	0.0436	0.0427	0.0418	0.0409	0.0401	0.0392	0.0384	0.0375	0.0367
1.8	0.0359	0.0351	0.0344	0.0336	0.0329	0.0322	0.0314	0.0307	0.0301	0.0294
1.9	0.0287	0.0281	0.0274	0.0268	0.0262	0.0256	0.0250	0.0244	0.0239	0.0233
2.0	0.0228	0.0222	0.0217	0.0212	0.0207	0.0202	0.0197	0.0192	0.0188	0.0183
2.1	0.0179	0.0174	0.0170	0.0166	0.0162	0.0158	0.0154	0.0150	0.0146	0.0143
2.2	0.0139	0.0136	0.0132	0.0129	0.0125	0.0122	0.0119	0.0116	0.0113	0.0110
2.3	0.0107	0.0104	0.0102	0.0099	0.0096	0.0094	0.0091	0.0089	0.0087	0.0084
2.4	0.0082	0.0080	0.0078	0.0075	0.0073	0.0071	0.0069	0.0068	0.0066	0.0064
2.5	0.0062	0.0060	0.0059	0.0057	0.0055	0.0054	0.0052	0.0051	0.0049	0.0048
2.6	0.0047	0.0045	0.0044	0.0043	0.0041	0.0040	0.0039	0.0038	0.0037	0.0036
2.7	0.0035	0.0034	0.0033	0.0032	0.0031	0.0030	0.0029	0.0028	0.0027	0.0026
2.8	0.0026	0.0025	0.0024	0.0023	0.0023	0.0022	0.0021	0.0021	0.0020	0.0019
2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
3.0	0.0013	0.0013	0.0013	0.0012	0.0012	0.0011	0.0011	0.0011	0.0010	0.0010
3.1	0.0010	0.0009	0.0009	0.0009	0.0008	0.0008	0.0008	0.0008	0.0007	0.0007
3.2	0.0007	0.0007	0.0006	0.0006	0.0006	0.0006	0.0006	0.0005	0.0005	0.0005
3.3	0.0005	0.0005	0.0005	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0003
3.4	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0002
3.5	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002	0.0002
3.6	0.0002	0.0002	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
3.7	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
3.8	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001	0.0001
3.9	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
4.0	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000

$\alpha$	0.2500	0.2000	0.1500	0.1000	0.0500	0.0250	0.0100	0.0050	0.0010	0.0001
$z$	0.6745	0.8416	1.0364	1.2816	1.6449	1.9600	2.3263	2.5758	3.0902	3.7195

## 5 Student's $t$ distribution

The table shows the percentage points of the  $t$ -distribution.

For example, for the  $t$ -distribution on 3 degrees of freedom,  $0.05 = P(T > 2.3534) = P(|T| > 3.1824)$ .

For degrees of freedom  $v > 40$ , interpolation in  $\frac{120}{v}$  is recommended. If we have  $v = 80$ , for example, note that  $\frac{120}{80} = 1.5$  is midway between  $\frac{120}{120} = 1$  and  $\frac{120}{60} = 2$  so that the percentage points for  $v = 80$  are (approximately) mid-way between those for  $v = 60$  and  $v = 120$ .

		Tail area probability										
1-tailed:	25%	20%	15%	10%	5%	2.5%	1%	0.5%	0.25%	0.1%	0.05%	
1	1.000	1.3764	1.9626	3.0777	6.3138	12.7062	31.8205	63.6567	127.3213	318.3088	636.6192	
2	0.8165	1.0607	1.3862	1.8856	2.9200	4.3027	6.9646	9.9248	14.0890	22.3271	31.5991	
3	0.7649	0.9785	1.2498	1.6377	2.3534	3.1824	4.5407	5.8409	7.4533	10.2145	12.9240	
4	0.7407	0.9410	1.1896	1.5332	2.1318	2.7764	3.7469	4.6041	5.5976	7.1732	8.6103	
5	0.7267	0.9195	1.1558	1.4759	2.0150	2.5706	3.3649	4.0321	4.7733	5.8934	6.8688	
6	0.7176	0.9057	1.1342	1.4398	1.9432	2.4469	3.1427	3.7074	4.3168	5.2076	5.9588	
7	0.7111	0.8960	1.1192	1.4149	1.8946	2.3646	2.9980	3.4995	4.0293	4.7853	5.4079	
8	0.7064	0.8889	1.1081	1.3968	1.8595	2.3060	2.8965	3.3554	3.8325	4.5008	5.0413	
9	0.7027	0.8834	1.0997	1.3830	1.8331	2.2622	2.8214	3.2498	3.6897	4.2968	4.7809	
10	0.6998	0.8791	1.0931	1.3722	1.8125	2.2281	2.7638	3.1693	3.5814	4.1437	4.5869	
11	0.6974	0.8755	1.0877	1.3634	1.7959	2.2010	2.7181	3.1058	3.4966	4.0247	4.4370	
12	0.6955	0.8726	1.0832	1.3562	1.7823	2.1788	2.6810	3.0545	3.4284	3.9296	4.3178	
13	0.6938	0.8702	1.0795	1.3502	1.7709	2.1604	2.6503	3.0123	3.3725	3.8520	4.2208	
14	0.6924	0.8681	1.0763	1.3450	1.7613	2.1448	2.6245	2.9768	3.3257	3.7874	4.1405	
15	0.6912	0.8662	1.0735	1.3406	1.7531	2.1314	2.6025	2.9467	3.2860	3.7328	4.0728	
16	0.6901	0.8647	1.0711	1.3368	1.7459	2.1199	2.5835	2.9208	3.2520	3.6862	4.0150	
17	0.6892	0.8633	1.0690	1.3334	1.7396	2.1098	2.5669	2.8982	3.2224	3.6458	3.9651	
18	0.6884	0.8620	1.0672	1.3304	1.7341	2.1009	2.5524	2.8784	3.1966	3.6105	3.9216	
19	0.6876	0.8610	1.0655	1.3277	1.7291	2.0930	2.5395	2.8609	3.1737	3.5794	3.8834	
20	0.6870	0.8600	1.0640	1.3253	1.7247	2.0860	2.5280	2.8453	3.1534	3.5518	3.8495	
21	0.6864	0.8591	1.0627	1.3232	1.7207	2.0796	2.5176	2.8314	3.1352	3.5272	3.8193	
22	0.6858	0.8583	1.0614	1.3212	1.7171	2.0739	2.5083	2.8188	3.1188	3.5050	3.7921	
23	0.6853	0.8575	1.0603	1.3195	1.7139	2.0687	2.4999	2.8073	3.1040	3.4850	3.7676	
24	0.6848	0.8569	1.0593	1.3178	1.7109	2.0639	2.4922	2.7969	3.0905	3.4668	3.7454	
25	0.6844	0.8562	1.0584	1.3163	1.7081	2.0595	2.4851	2.7874	3.0782	3.4502	3.7251	
26	0.6840	0.8557	1.0575	1.3150	1.7056	2.0555	2.4786	2.7787	3.0669	3.4350	3.7066	
27	0.6837	0.8551	1.0567	1.3137	1.7033	2.0518	2.4727	2.7707	3.0565	3.4210	3.6896	
28	0.6834	0.8546	1.0560	1.3125	1.7011	2.0484	2.4671	2.7633	3.0469	3.4082	3.6739	
29	0.6830	0.8542	1.0553	1.3114	1.6991	2.0452	2.4620	2.7564	3.0380	3.3962	3.6594	
30	0.6828	0.8538	1.0547	1.3104	1.6973	2.0423	2.4573	2.7500	3.0298	3.3852	3.6460	
31	0.6825	0.8534	1.0541	1.3095	1.6955	2.0395	2.4528	2.7440	3.0221	3.3749	3.6335	
32	0.6822	0.8530	1.0535	1.3086	1.6939	2.0369	2.4487	2.7385	3.0149	3.3653	3.6218	
33	0.6820	0.8526	1.0530	1.3077	1.6924	2.0345	2.4448	2.7333	3.0082	3.3563	3.6109	
34	0.6818	0.8523	1.0525	1.3070	1.6909	2.0322	2.4411	2.7284	3.0020	3.3479	3.6007	
35	0.6816	0.8520	1.0520	1.3062	1.6896	2.0301	2.4377	2.7238	2.9960	3.3400	3.5911	
36	0.6814	0.8517	1.0516	1.3055	1.6883	2.0281	2.4345	2.7195	2.9905	3.3326	3.5821	
37	0.6812	0.8514	1.0512	1.3049	1.6871	2.0262	2.4314	2.7154	2.9852	3.3256	3.5737	
38	0.6810	0.8512	1.0508	1.3042	1.6860	2.0244	2.4286	2.7116	2.9803	3.3190	3.5657	
39	0.6808	0.8509	1.0504	1.3036	1.6849	2.0227	2.4258	2.7079	2.9756	3.3128	3.5581	
40	0.6807	0.8507	1.0500	1.3031	1.6839	2.0211	2.4233	2.7045	2.9712	3.3069	3.5510	
60	0.6786	0.8477	1.0455	1.2958	1.6706	2.0003	2.3901	2.6603	2.9146	3.2317	3.4602	
120	0.6765	0.8446	1.0409	1.2886	1.6577	1.9799	2.3578	2.6174	2.8599	3.1595	3.3735	
$\infty$	0.6745	0.8416	1.0364	1.2816	1.6449	1.9600	2.3263	2.5758	2.8070	3.0902	3.2905	
2-tailed:	50%	40%	30%	20%	10%	5%	2.0%	1%	0.5%	0.2%	0.1%	

## 6 Chi-squared $\chi^2$ distribution

The table shows the upper percentage points of the *chi-squared* ( $\chi^2$ ) distribution.

For example, for the chi-squared distribution on 3 degrees of freedom,  $0.95 = P(\chi^2 > 0.3518)$ .

	0.999	0.995	0.990	0.980	0.975	0.950	0.900	0.800	0.750	0.700	0.600	0.500
1	0.0000	0.0000	0.0002	0.0006	0.0010	0.0039	0.0158	0.0642	0.1015	0.1485	0.2750	0.4549
2	0.0020	0.0100	0.0201	0.0404	0.0506	0.1026	0.2107	0.4463	0.5754	0.7133	1.0217	1.3863
3	0.0243	0.0717	0.1148	0.1848	0.2158	0.3518	0.5844	1.0052	1.2125	1.4237	1.8692	2.3660
4	0.0908	0.2070	0.2971	0.4294	0.4844	0.7107	1.0636	1.6488	1.9226	2.1947	2.7528	3.3567
5	0.2102	0.4117	0.5543	0.7519	0.8312	1.1455	1.6103	2.3425	2.6746	2.9999	3.6555	4.3515
6	0.3811	0.6757	0.8721	1.1344	1.2373	1.6354	2.2041	3.0701	3.4546	3.8276	4.5702	5.3481
7	0.5985	0.9893	1.2390	1.5643	1.6899	2.1673	2.8331	3.8223	4.2549	4.6713	5.4932	6.3458
8	0.8571	1.3444	1.6465	2.0325	2.1797	2.7326	3.4895	4.5936	5.0706	5.5274	6.4226	7.3441
9	1.1519	1.7349	2.0879	2.5324	2.7004	3.3251	4.1682	5.3801	5.8988	6.3933	7.3570	8.3428
10	1.4787	2.1559	2.5582	3.0591	3.2470	3.9403	4.8652	6.1791	6.7372	7.2672	8.2955	9.3418
11	1.8339	2.6032	3.0535	3.6087	3.8157	4.5748	5.5778	6.9887	7.5841	8.1479	9.2373	10.3410
12	2.2142	3.0738	3.5706	4.1783	4.4038	5.2260	6.3038	7.8073	8.4384	9.0343	10.1820	11.3403
13	2.6172	3.5650	4.1069	4.7654	5.0088	5.8919	7.0415	8.6339	9.2991	9.9257	11.1291	12.3398
14	3.0407	4.0747	4.6604	5.3682	5.6287	6.5706	7.7895	9.4673	10.1653	10.8215	12.0785	13.3393
15	3.4827	4.6009	5.2293	5.9849	6.2621	7.2609	8.5468	10.3070	11.0365	11.7212	13.0297	14.3389
16	3.9416	5.1422	5.8122	6.6142	6.9077	7.9616	9.3122	11.1521	11.9122	12.6243	13.9827	15.3385
17	4.4161	5.6972	6.4078	7.2550	7.5642	8.6718	10.0852	12.0023	12.7919	13.5307	14.9373	16.3382
18	4.9048	6.2648	7.0149	7.9062	8.2307	9.3905	10.8649	12.8570	13.6753	14.4399	15.8932	17.3379
19	5.4068	6.8440	7.6327	8.5670	8.9065	10.1170	11.6509	13.7158	14.5620	15.3517	16.8504	18.3377
20	5.9210	7.4338	8.2604	9.2367	9.5908	10.8508	12.4426	14.5784	15.4518	16.2659	17.8088	19.3374
21	6.4467	8.0337	8.8972	9.9146	10.2829	11.5913	13.2396	15.4446	16.3444	17.1823	18.7683	20.3372
22	6.9830	8.6427	9.5425	10.6000	10.9823	12.3380	14.0415	16.3140	17.2396	18.1007	19.7288	21.3370
23	7.5292	9.2604	10.1957	11.2926	11.6886	13.0905	14.8480	17.1865	18.1373	19.0211	20.6902	22.3369
24	8.0849	9.8862	10.8564	11.9918	12.4012	13.8484	15.6587	18.0618	19.0373	19.9432	21.6525	23.3367
25	8.6493	10.5197	11.5240	12.6973	13.1197	14.6114	16.4734	18.9398	19.9393	20.8670	22.6156	24.3366
26	9.2221	11.1602	12.1981	13.4086	13.8439	15.3792	17.2919	19.8202	20.8434	21.7924	23.5794	25.3365
27	9.8028	11.8076	12.8785	14.1254	14.5734	16.1514	18.1139	20.7030	21.7494	22.7192	24.5440	26.3363
28	10.3909	12.4613	13.5647	14.8475	15.3079	16.9279	18.9392	21.5880	22.6572	23.6475	25.5093	27.3362
29	10.9861	13.1211	14.2565	15.5745	16.0471	17.7084	19.7677	22.4751	23.5666	24.5770	26.4751	28.3361
30	11.5880	13.7867	14.9535	16.3062	16.7908	18.4927	20.5992	23.3641	24.4776	25.5078	27.4416	29.3360
31	12.1963	14.4578	15.6555	17.0423	17.5387	19.2806	21.4336	24.2551	25.3901	26.4397	28.4087	30.3359
32	12.8107	15.1340	16.3622	17.7827	18.2908	20.0719	22.2706	25.1478	26.3041	27.3728	29.3763	31.3359
33	13.4309	15.8153	17.0735	18.5271	19.0467	20.8665	23.1102	26.0422	27.2194	28.3069	30.3444	32.3358
34	14.0567	16.5013	17.7891	19.2754	19.8063	21.6643	23.9523	26.9383	28.1361	29.2421	31.3130	33.3357
35	14.6878	17.1918	18.5089	20.0274	20.5694	22.4650	24.7967	27.8359	29.0540	30.1782	32.2821	34.3356
36	15.3241	17.8867	19.2327	20.7829	21.3359	23.2686	25.6433	28.7350	29.9730	31.1152	33.2517	35.3356
37	15.9653	18.5858	19.9602	21.5419	22.1056	24.0749	26.4921	29.6355	30.8933	32.0532	34.2216	36.3355
38	16.6112	19.2889	20.6914	22.3040	22.8785	24.8839	27.3430	30.5373	31.8146	32.9919	35.1920	37.3355
39	17.2616	19.9959	21.4262	23.0693	23.6543	25.6954	28.1958	31.4405	32.7369	33.9315	36.1628	38.3354
40	17.9164	20.7065	22.1643	23.8376	24.4330	26.5093	29.0505	32.3450	33.6603	34.8719	37.1340	39.3353
50	24.6739	27.9907	29.7067	31.6639	32.3574	34.7643	37.6886	41.4492	42.9421	44.3133	46.8638	49.3349
60	31.7383	35.5345	37.4849	39.6994	40.4817	43.1880	46.4589	50.6406	52.2938	53.8091	56.6200	59.3347
70	39.0364	43.2752	45.4417	47.8934	48.7576	51.7393	55.3289	59.8978	61.6983	63.3460	66.3961	69.3345
80	46.5199	51.1719	53.5401	56.2128	57.1532	60.3915	64.2778	69.2069	71.1445	72.9153	76.1879	79.3343
90	54.1552	59.1963	61.7541	64.6347	65.6466	69.1260	73.2911	78.5584	80.6247	82.5111	85.9925	89.3342
100	61.9179	67.3276	70.0649	73.1422	74.2219	77.9295	82.3581	87.9453	90.1332	92.1289	95.8078	99.3341
110	69.7894	75.5500	78.4583	81.7228	82.8671	86.7916	91.4710	97.3624	99.6660	101.7656	105.6323	109.3341
120	77.7551	83.8516	86.9233	90.3667	91.5726	95.7046	100.6236	106.8056	109.2197	111.4186	115.4645	119.3340

	0.400	0.300	0.250	0.200	0.100	0.050	0.025	0.020	0.010	0.005	0.001
1	0.7083	1.0742	1.3233	1.6424	2.7055	3.8415	5.0239	5.4119	6.6349	7.8794	10.8276
2	1.8326	2.4079	2.7726	3.2189	4.6052	5.9915	7.3778	7.8240	9.2103	10.5966	13.8155
3	2.9462	3.6649	4.1083	4.6416	6.2514	7.8147	9.3484	9.8374	11.3449	12.8382	16.2662
4	4.0446	4.8784	5.3853	5.9886	7.7794	9.4877	11.1433	11.6678	13.2767	14.8603	18.4668
5	5.1319	6.0644	6.6257	7.2893	9.2364	11.0705	12.8325	13.3882	15.0863	16.7496	20.5150
6	6.2108	7.2311	7.8408	8.5581	10.6446	12.5916	14.4494	15.0332	16.8119	18.5476	22.4577
7	7.2832	8.3834	9.0371	9.8032	12.0170	14.0671	16.0128	16.6224	18.4753	20.2777	24.3219
8	8.3505	9.5245	10.2189	11.0301	13.3616	15.5073	17.5345	18.1682	20.0902	21.9550	26.1245
9	9.4136	10.6564	11.3888	12.2421	14.6837	16.9190	19.0228	19.6790	21.6660	23.5894	27.8772
10	10.4732	11.7807	12.5489	13.4420	15.9872	18.3070	20.4832	21.1608	23.2093	25.1882	29.5883
11	11.5298	12.8987	13.7007	14.6314	17.2750	19.6751	21.9200	22.6179	24.7250	26.7568	31.2641
12	12.5838	14.0111	14.8454	15.8120	18.5493	21.0261	23.3367	24.0540	26.2170	28.2995	32.9095
13	13.6356	15.1187	15.9839	16.9848	19.8119	22.3620	24.7356	25.4715	27.6882	29.8195	34.5282
14	14.6853	16.2221	17.1169	18.1508	21.0641	23.6848	26.1189	26.8728	29.1412	31.3193	36.1233
15	15.7332	17.3217	18.2451	19.3107	22.3071	24.9958	27.4884	28.2595	30.5779	32.8013	37.6973
16	16.7795	18.4179	19.3689	20.4651	23.5418	26.2962	28.8454	29.6332	31.9999	34.2672	39.2524
17	17.8244	19.5110	20.4887	21.6146	24.7690	27.5871	30.1910	30.9950	33.4087	35.7185	40.7902
18	18.8679	20.6014	21.6049	22.7595	25.9894	28.8693	31.5264	32.3462	34.8053	37.1565	42.3124
19	19.9102	21.6891	22.7178	23.9004	27.2036	30.1435	32.8523	33.6874	36.1909	38.5823	43.8202
20	20.9514	22.7745	23.8277	25.0375	28.4120	31.4104	34.1696	35.0196	37.5662	39.9968	45.3147
21	21.9915	23.8578	24.9348	26.1711	29.6151	32.6706	35.4789	36.3434	38.9322	41.4011	46.7970
22	23.0307	24.9390	26.0393	27.3015	30.8133	33.9244	36.7807	37.6595	40.2894	42.7957	48.2679
23	24.0689	26.0184	27.1413	28.4288	32.0069	35.1725	38.0756	38.9683	41.6384	44.1813	49.7282
24	25.1063	27.0960	28.2412	29.5533	33.1962	36.4150	39.3641	40.2704	42.9798	45.5585	51.1786
25	26.1430	28.1719	29.3389	30.6752	34.3816	37.6525	40.6465	41.5661	44.3141	46.9279	52.6197
26	27.1789	29.2463	30.4346	31.7946	35.5632	38.8851	41.9232	42.8558	45.6417	48.2899	54.0520
27	28.2141	30.3193	31.5284	32.9117	36.7412	40.1133	43.1945	44.1400	46.9629	49.6449	55.4760
28	29.2486	31.3909	32.6205	34.0266	37.9159	41.3371	44.4608	45.4188	48.2782	50.9934	56.8923
29	30.2825	32.4612	33.7109	35.1394	39.0875	42.5570	45.7223	46.6927	49.5879	52.3356	58.3012
30	31.3159	33.5302	34.7997	36.2502	40.2560	43.7730	46.9792	47.9618	50.8922	53.6720	59.7031
31	32.3486	34.5981	35.8871	37.3591	41.4217	44.9853	48.2319	49.2264	52.1914	55.0027	61.0983
32	33.3809	35.6649	36.9730	38.4663	42.5847	46.1943	49.4804	50.4867	53.4858	56.3281	62.4872
33	34.4126	36.7307	38.0575	39.5718	43.7452	47.3999	50.7251	51.7429	54.7755	57.6484	63.8701
34	35.4438	37.7954	39.1408	40.6756	44.9032	48.6024	51.9660	52.9952	56.0609	58.9639	65.2472
35	36.4746	38.8591	40.2228	41.7780	46.0588	49.8018	53.2033	54.2438	57.3421	60.2748	66.6188
36	37.5049	39.9220	41.3036	42.8788	47.2122	50.9985	54.4373	55.4889	58.6192	61.5812	67.9852
37	38.5348	40.9839	42.3833	43.9782	48.3634	52.1923	55.6680	56.7305	59.8925	62.8833	69.3465
38	39.5643	42.0451	43.4619	45.0763	49.5126	53.3835	56.8955	57.9688	61.1621	64.1814	70.7029
39	40.5935	43.1053	44.5395	46.1730	50.6598	54.5722	58.1201	59.2040	62.4281	65.4756	72.0547
40	41.6222	44.1649	45.6160	47.2685	51.8051	55.7585	59.3417	60.4361	63.6907	66.7660	73.4020
50	51.8916	54.7228	56.3336	58.1638	63.1671	67.5048	71.4202	72.6133	76.1539	79.4900	86.6608
60	62.1348	65.2265	66.9815	68.9721	74.3970	79.0819	83.2977	84.5799	88.3794	91.9517	99.6072
70	72.3583	75.6893	77.5767	79.7146	85.5270	90.5312	95.0232	96.3875	100.4252	104.2149	112.3169
80	82.5663	86.1197	88.1303	90.4053	96.5782	101.8795	106.6286	108.0693	112.3288	116.3211	124.8392
90	92.7614	96.5238	98.6499	101.0537	107.5650	113.1453	118.1359	119.6485	124.1163	128.2989	137.2084
100	102.9459	106.9058	109.1412	111.6667	118.4980	124.3421	129.5612	131.1417	135.8067	140.1695	149.4493
110	113.1214	117.2690	119.6084	122.2495	129.3851	135.4802	140.9166	142.5617	147.4143	151.9485	161.5807
120	123.2890	127.6159	130.0546	132.8063	140.2326	146.5674	152.2114	153.9182	158.9502	163.6482	173.6174

For very large values of degrees of freedom,  $k$ ,  $\sqrt{2\chi^2}$  is approximately Normally distributed with a mean of  $\sqrt{2k} - 1$  and variance 1.

## 7 Fisher's $F$ -distribution, 5% points

The table shows the upper 5% points of the  $F$ -distribution.

For example, for the  $F$ -distribution on 3 and 5 degrees of freedom,  $0.05 = P(F > 5.4095)$ .

		Numerator degrees of freedom									
		1	2	3	4	5	6	7	8	9	10
Denominator degrees of freedom	1	161.4476	199.5000	215.7073	224.5832	230.1619	233.9860	236.7684	238.8827	240.5433	241.8817
	2	18.5128	19.0000	19.1643	19.2468	19.2964	19.3295	19.3532	19.3710	19.3848	19.3959
	3	10.1280	9.5521	9.2766	9.1172	9.0135	8.9406	8.8867	8.8452	8.8123	8.7855
	4	7.7086	6.9443	6.5914	6.3882	6.2561	6.1631	6.0942	6.0410	5.9988	5.9644
	5	6.6079	5.7861	5.4095	5.1922	5.0503	4.9503	4.8759	4.8183	4.7725	4.7351
	6	5.9874	5.1433	4.7571	4.5337	4.3874	4.2839	4.2067	4.1468	4.0990	4.0600
	7	5.5914	4.7374	4.3468	4.1203	3.9715	3.8660	3.7870	3.7257	3.6767	3.6365
	8	5.3177	4.4590	4.0662	3.8379	3.6875	3.5806	3.5005	3.4381	3.3881	3.3472
	9	5.1174	4.2565	3.8625	3.6331	3.4817	3.3738	3.2927	3.2296	3.1789	3.1373
	10	4.9646	4.1028	3.7083	3.4780	3.3258	3.2172	3.1355	3.0717	3.0204	2.9782
	11	4.8443	3.9823	3.5874	3.3567	3.2039	3.0946	3.0123	2.9480	2.8962	2.8536
	12	4.7472	3.8853	3.4903	3.2592	3.1059	2.9961	2.9134	2.8486	2.7964	2.7534
	13	4.6672	3.8056	3.4105	3.1791	3.0254	2.9153	2.8321	2.7669	2.7144	2.6710
	14	4.6001	3.7389	3.3439	3.1122	2.9582	2.8477	2.7642	2.6987	2.6458	2.6022
	15	4.5431	3.6823	3.2874	3.0556	2.9013	2.7905	2.7066	2.6408	2.5876	2.5437
	16	4.4940	3.6337	3.2389	3.0069	2.8524	2.7413	2.6572	2.5911	2.5377	2.4935
	17	4.4513	3.5915	3.1968	2.9647	2.8100	2.6987	2.6143	2.5480	2.4943	2.4499
	18	4.4139	3.5546	3.1599	2.9277	2.7729	2.6613	2.5767	2.5102	2.4563	2.4117
	19	4.3807	3.5219	3.1274	2.8951	2.7401	2.6283	2.5435	2.4768	2.4227	2.3779
	20	4.3512	3.4928	3.0984	2.8661	2.7109	2.5990	2.5140	2.4471	2.3928	2.3479

		Numerator degrees of freedom									
		11	12	13	14	15	16	17	18	19	20
Denominator degrees of freedom	1	242.9835	243.9060	244.6898	245.3640	245.9499	246.4639	246.9184	247.3232	247.6861	248.0131
	2	19.4050	19.4125	19.4189	19.4244	19.4291	19.4333	19.4370	19.4402	19.4431	19.4458
	3	8.7633	8.7446	8.7287	8.7149	8.7029	8.6923	8.6829	8.6745	8.6670	8.6602
	4	5.9358	5.9117	5.8911	5.8733	5.8578	5.8441	5.8320	5.8211	5.8114	5.8025
	5	4.7040	4.6777	4.6552	4.6358	4.6188	4.6038	4.5904	4.5785	4.5678	4.5581
	6	4.0274	3.9999	3.9764	3.9559	3.9381	3.9223	3.9083	3.8957	3.8844	3.8742
	7	3.6030	3.5747	3.5503	3.5292	3.5107	3.4944	3.4799	3.4669	3.4551	3.4445
	8	3.3130	3.2839	3.2590	3.2374	3.2184	3.2016	3.1867	3.1733	3.1613	3.1503
	9	3.1025	3.0729	3.0475	3.0255	3.0061	2.9890	2.9737	2.9600	2.9477	2.9365
	10	2.9430	2.9130	2.8872	2.8647	2.8450	2.8276	2.8120	2.7980	2.7854	2.7740
	11	2.8179	2.7876	2.7614	2.7386	2.7186	2.7009	2.6851	2.6709	2.6581	2.6464
	12	2.7173	2.6866	2.6602	2.6371	2.6169	2.5989	2.5828	2.5684	2.5554	2.5436
	13	2.6347	2.6037	2.5769	2.5536	2.5331	2.5149	2.4987	2.4841	2.4709	2.4589
	14	2.5655	2.5342	2.5073	2.4837	2.4630	2.4446	2.4282	2.4134	2.4000	2.3879
	15	2.5068	2.4753	2.4481	2.4244	2.4034	2.3849	2.3683	2.3533	2.3398	2.3275
	16	2.4564	2.4247	2.3973	2.3733	2.3522	2.3335	2.3167	2.3016	2.2880	2.2756
	17	2.4126	2.3807	2.3531	2.3290	2.3077	2.2888	2.2719	2.2567	2.2429	2.2304
	18	2.3742	2.3421	2.3143	2.2900	2.2686	2.2496	2.2325	2.2172	2.2033	2.1906
	19	2.3402	2.3080	2.2800	2.2556	2.2341	2.2149	2.1977	2.1823	2.1683	2.1555
	20	2.3100	2.2776	2.2495	2.2250	2.2033	2.1840	2.1667	2.1511	2.1370	2.1242



## 8 Standard Statistical Distributions

		p.m.f. $P(X = x)$	p.d.f. $f(x)$	Mean $\mu$	Variance $\sigma^2$	m.g.f.	Distn. of $T = \sum_{i=1}^k X_i$	Notes / special cases
<i>Binomial</i> $n \in \mathbb{Z}^+$	$\text{Bin}(n, p)$ $0 < p < 1$	${}^n C_x p^x (1-p)^{n-x}$ $x = 0, 1, \dots, n$		$np$	$np(1-p)$ $= npq$	$(pe^t + 1 - p)^n$ all $t$	$X_i \sim \text{i.i.d. Bin}(n_i, p)$ $\Rightarrow T \sim \text{Bin}(\sum n_i, p)$	Number of successes in $n$ independent Bernoulli trials, each with probability $p$ of success.
<i>Geometric</i>	$\text{Geom}(p)$ $0 < p < 1$	$p(1-p)^{x-1} = pq^{x-1}$ $x = 1, 2, \dots$		$\frac{1}{p}$	$\frac{1-p}{p^2}$	$\frac{p}{e^{-t} - (1-p)}$ $t < -\ln(1-p)$	$X_i \sim \text{i.i.d. Geom}(p)$ $\Rightarrow T \sim \text{Pasc}(k, p)$	Number of Bernoulli trials up to and including the first success. $\text{Geom}(p) \equiv \text{Pasc}(1, 1-p)$ .
<i>Negative Binomial (Pascal)</i> $r \in \mathbb{Z}^+$	$\text{Negbin}(r, p)$ $\text{Pasc}(r, p)$ $0 < p < 1$	$x^{-1} {}^{x-1} C_{r-1} p^r (1-p)^{x-r}$ $x = r, r+1, \dots$		$\frac{r}{p}$	$\frac{r(1-p)}{p^2}$	$\left[ \frac{p}{e^{-t} - (1-p)} \right]^r$ all $t$	$X_i \sim \text{i.i.d. Pasc}(r_i, p)$ $\Rightarrow T \sim \text{Pasc}(\sum r_i, p)$	Number of Bernoulli trials up to and including the $r$ -th success. Sometimes denoted by $\text{NB}(r, p)$ .
<i>Poisson</i>	$\text{Po}(\beta)$ $\beta > 0$	$\frac{e^{-\beta} \beta^x}{x!}$ $x = 0, 1, 2, \dots$		$\beta$	$\beta$	$e^{\beta(e^t - 1)}$ all $t$	$X_i \sim \text{i.i.d. Po}(\beta_i)$ $\Rightarrow T \sim \text{Po}(\sum \beta_i)$	Used for number of rare of random events where $\beta$ is the number we expect to occur. $\text{Bin}(n, p) \cong \text{Po}(np)$ if $n$ is very large and $p$ very small.
<i>Uniform (Rectangular)</i>	$U(a, b)$ $b > a$		$\frac{1}{b-a}$ $a < x < b$	$\frac{a+b}{2}$	$\frac{(b-a)^2}{12}$	$\frac{e^{tb} - e^{ta}}{t(b-a)}$ all $t$		Continuous "random numbers". Distribution of $T$ rapidly approaches Normality for $X_i \sim \text{i.i.d. } U(a, b)$
<i>Exponential</i>	$\text{Expon}(\beta)$ $\beta > 0$		$\beta e^{-\beta x}$ $x > 0$	$\frac{1}{\beta}$	$\frac{1}{\beta^2}$	$\frac{\beta}{\beta - t}$ $t < \beta$	$X_i \sim \text{i.i.d. Expon}(\beta)$ $\Rightarrow T \sim \text{Ga}(k, \beta)$	Distribution of time to first occurrence of random events at rate $\beta$ per time unit. $\text{Expon}(\beta) \equiv \text{Ga}(1, \beta)$
<i>Gamma</i> $\alpha, \beta > 0$	$\text{Ga}(\alpha, \beta)$ $\gamma(\alpha, \beta)$		$\frac{\beta^\alpha}{\Gamma(\alpha)} x^{\alpha-1} e^{-\beta x}$ $x > 0$	$\frac{\alpha}{\beta}$	$\frac{\alpha}{\beta^2}$	$\left[ \frac{\beta}{\beta - t} \right]^\alpha$ $t < \beta$	$X_i \sim \text{i.i.d. Ga}(\alpha_i, \beta)$ $\Rightarrow T \sim \text{Ga}(\sum \alpha_i, \beta)$	If $\alpha$ is an integer, distribution of time to $\alpha$ -th random event if occurrence at rate $\beta$ per unit time. If $X \sim \text{Ga}(n, \beta)$ , then $2\beta X \sim \chi^2(2n)$ .
<i>Chi-squared</i> $\nu \in \mathbb{Z}^+$	$\chi^2(\nu)$		$\frac{1}{2^{\frac{\nu}{2}} \Gamma(\frac{\nu}{2})} x^{\frac{\nu}{2}-1} e^{-\frac{x}{2}}$ $x > 0$	$\nu$	$2\nu$	$\frac{1}{(1-2t)^{\frac{\nu}{2}}}$ $t < \frac{1}{2}$	$X_i \sim \text{i.i.d. } \chi^2(\nu_i)$ $\Rightarrow T \sim \chi^2(\sum \nu_i)$	$\chi^2(\nu) \cong \text{Ga}(\frac{\nu}{2}, \frac{1}{2})$ . $\nu$ is called the <i>degrees of freedom</i> .
<i>Normal</i> $\sigma > 0$	$N(\mu, \sigma^2)$		$\frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right)$ all real $x$	$\mu$	$\sigma^2$	$\exp\left(\mu t + \frac{1}{2}\sigma^2 t^2\right)$ all $t$	$X_i \sim \text{Normal}$ $T \Rightarrow \sim \text{Normal}$ Of appr. $\mu$ and $\sigma^2$ .	Approximation for Binomial if $n$ large and $p$ moderate; for Poisson if $\beta$ large; for $\text{Ga}(\alpha, \beta)$ if $\alpha$ large. See also Central Limit Theorem

### Central Limit Theorem:

For any independent, identically distributed  $X$ 's, the sum  $T = X_1 + X_2 + \dots + X_k$  is approximately Normally distributed if  $k$  is large.

$k$  does not in general have to be very large; the more symmetric the distribution of the  $X$ 's, the smaller  $k$  needs to be.

For uniformly distributed  $X$ 's, the distribution of  $T$  with  $k = 12$  is almost indistinguishable from the Normal.