Outline of MA34110 – PDEs (2024–25)

- Classification of PDEs: linearity, order, homogenous.
- Superposition principle.
- Simple PDEs.
- **Method of characteristics**
  - Definition of directional derivative.
  - Constant coefficients: $au_x + bu_y = 0$.
  - Variable coefficients: $a(x, y)u_x + b(x, y)u_y = 0$.
  - General case (parameterisation of characteristic curves): $a(x, y)u_x + b(x, y)u_y + c(x, y)u = f(x, y)$.
- **Initial and boundary conditions**
  - Well-posed and ill-posed problems.
- **Classification of second order PDEs.**
  - Classification.
  - Reduction to canonical forms.
- **The wave equation**
  - General solution: $u(x, t) = F(x + ct) + G(x - ct)$.
  - Initial value problem: d'Alembert's formula.
  - Domains of influence and dependence.
  - Semi-bounded string – reflection method.
  - Inhomogeneous wave equation – Duhamel principle.
  - Bounded string – separation of variables.
  - Energy and uniqueness.
- **The heat equation**
  - Maximum principle and uniqueness.
  - Separation of variables.
  - Heat equation Cauchy problem for the heat equation on an infinitely long rod (uses Fourier transforms).
- **Fourier transforms**
  - Definition of the Fourier transform pair (the transform and the inverse transform).
  - Properties of the Fourier transform - existence, linearity.
  - Fourier transform of the derivative.
  - Convolution theorem.
  - Consideration of the Fourier transform in different function spaces.
  - Solution of the heat equation Cauchy problem using the Fourier transform.