Assignment 2

2024–25

Assignment due in via Blackboard or on paper by the end of Friday 8th November.

1. Classify the following equations as parabolic, elliptic or hyperbolic:
   (a) $u_{xx} - u_{xy} + 2u_y + 3u_{yy} - 5u_{yx} + 8u = 0$;
   (b) $9u_{xx} + 6u_{xy} + u_{yy} + u_x = 0$;
   (c) $u_{xx} - 4u_{xy} + 4u_{yy} = 0$.

2. Consider the Cauchy problem

$$\begin{cases} u_{tt} = u_{xx}, \quad x \in \mathbb{R},\ t > 0, \\ u(x,0) = f(x), \quad u_t(x,0) = g(x). \end{cases}$$

   (a) Find the domain of dependence of u at $(x,t) = (2,1)$.
   (b) Let $f(x) = 0$ outside the interval $[-1,2]$ and $g(x) = 0$ outside the interval $[1,6]$. Find the set E of points (x,t) such that $u(x,t)$ must be zero for $(x,t) \in E$.

3. Find the solution $u(x,t)$ of the one-dimensional wave equation on an infinite string

$$\begin{cases} u_{tt} - c^2 u_{xx} = 0, \quad x \in \mathbb{R},\ t > 0, \\ u(x,0) = f(x), \quad u_t(x,0) = g(x). \end{cases}$$

   with
   (a) $f(x) = x$ and $g(x) = \cos(x)$.
   (b) $f(x) = \ln(x^2 + 6)$ and $g(x) = 3x^3$.
   (c) $f(x) = \sin(x^3)$ and $g(x) = \frac{x^2}{x^2+4x+8}$.

4. Using the method of characteristics, solve the equations
   (a) $2u_x + (\cos x)u_y = 0$, $u(0,y) = e^{-y}$,
   (b) $u_x + 2u_y + (2x - y)u = 2x^2 + 3xy - 2y^2$, $u(x,0) = x$ (harder!).