Assignment 1

2024–25

Starred questions (2a&b, 4a&b, 5, 8) due in either via Blackboard or on good old-fashioned paper by 5pm, Friday 25th October.

1. *(ODEs practice)*: Find the general solution of the following ODEs:
   (a) $\frac{\mathrm{d}y}{\mathrm{d}x} - \frac{2y}{x} = 3x^3$;
   (b) $\frac{\mathrm{d}^2y}{\mathrm{d}x^2} - 2\frac{\mathrm{d}y}{\mathrm{d}x} - 3y = 0$;
   (c) $y\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{3}{x^2y}$;
   (d) $\frac{\mathrm{d}y}{\mathrm{d}x} - y = 0$;
   (e) $\frac{\mathrm{d}^2y}{\mathrm{d}x^2} + y = 0$.

2. For each of the following equations, state the order and whether it is nonlinear, linear inhomogeneous, or linear homogeneous; provide reasons.
   (a) $u_x + xu_y = \sin x$; ⋆
   (b) $u_{xx} + uu_x = 0$; ⋆
   (c) $u_x + uu_y = u$;
   (d) $u_x + 2u_y + 3 = x$. [8 marks]

3. Find the general solution $u = u(x, y)$ of the PDE $u_{xy} = 3xy$.

4. (a) ⋆ Find the general solution $u = u(x, t)$ of the PDE
   $$4u_x + 3u_t = 0.$$
   (b) ⋆ Hence find the solution of the PDE $4u_x + 3u_t = 0$ satisfying the initial condition (for $t = 0$): $u(x, 0) = \cos x$. [8 marks]

5. ⋆ Solve the boundary value problem
   $$\begin{cases} x^2yu_x + 3u_y = 0, \\ u(x, 0) = \frac{1}{x}. \end{cases}$$
   [10 marks]

6. Solve the linear equation
   $$(1 + x^2)u_x + u_y = 0.$$
   Hint: you might like to remind yourself of the derivatives of inverse trigonometric functions.

7. Solve the equation
   $$(\sqrt{1 - x^2})u_x + u_y = 0,$$
   with the condition $u(0, y) = y$.

8. ⋆ Using the method of characteristics, solve $u_x + u_y + u = e^{x+2y}$ with $u(x, 0) = 0$. [14 marks]