Sedimenting discs in a two-dimensional foam

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Abstract

The sedimentation of circular discs in a dry two-dimensional, monodisperse foam is studied. This, a variation of the classical Stokes experiment, provides a prototype experiment to study a foam's response. The interaction between two circular particles of equal size and weight is investigated as they fall through the foam under their own weight. Their positions are tracked and the lift and drag force measured in numerical calculations using the Surface Evolver. The initial placements of the discs are varied in each of two different initial configurations, one in which the discs are side by side and the second in which the discs are one above the other. It is shown that discs that are initially side by side rotate as a system during the descent in the foam. In the second scenario, the upper disc falls into the wake of the lower, after which the discs sediment as one with a constant non-zero separation. We present evidence that the foam screens this interaction for specific initial separations between the discs in both configurations. The force between a channel wall and a nearby sedimenting disc is also investigated.

Key words: foam, rheology, Surface Evolver, discs, sedimentation, interaction
PACS:

1 Introduction

Liquid foams are familiar materials used domestically and in industrial processes such as ore-separation and enhanced oil recovery [1–3]. They are characterised as elasto-visco-plastic complex fluids due to their highly non-linear response to applied stresses. At low stresses they can be considered elastic solids, while increasing the applied stress results in plastic events. Plasticity in a foam is described by topological changes T1s, where a neighbour-swapping of bubbles occurs in response to the applied stress. Increasing the applied stress above a foam's yield stress results in viscous liquid-like behaviour [4]. Thus, foams provide a prototype complex fluid with which it is possible to work at a macroscopic bubble scale instead of the usual molecular scale.

We use a variation of the classical Stokes' experiment [5], originally used to measure the viscosity of a fluid through which a sphere is dropped, to describe and understand these elasto-visco-plastic transitions in foam rheology.

Existing work on experiments in which a constant force is applied to a particle in a foam is limited to a single sphere [6]. Other work where foam flow is probed by a fixed sphere uses the variation in drag force on the particle to quantify the foam response [7,8]. This scenario has proved useful in describing foam ageing [9,10].

Two-dimensional foams can be thought of as a monolayer of bubbles squeezed between two glass plates. We choose to probe the foam response by dropping circular obstacles of greater size than the bubbles into a foam channel. Existing work on smaller particles in foam concentrates on the dispersion of particles within the Plateau borders that constitute the liquid network of the foam [11,12]. Two-dimensional experiments using circular obstacles to probe foam response are a simplification of the 3D case but provide a clearer description. The drag force on a circular obstacle due to the foam has been measured through image analysis [13,14] and it was found to increase with obstacle size and decrease with bubble size while the roughness of the obstacle was not important. Confinement in two dimensions means that images of the foam during such experiments provide information on foam deformation fields as well as bubble velocity and pressure fields [15]. Combining such experiments with simulation has proved beneficial in showing that the drag force on a circular obstacle is also inversely correlated with the liquid fraction of the foam [16]. Combining the work of [13] and [16], the drag force on a circular obstacle of diameter d_0 is approximately $\phi^{-\frac{1}{4}} d_0/\sqrt{A_b}$ where A_b is the bubble area in a two dimensional foam and ϕ its effective liquid fraction.

Experiments investigating the flow of foam past different shaped obstacles such as a cambered airfoil [17] and an ellipse [18] has enhanced the understanding of foam response. An inverse lift force was observed for the cambered airfoil when placed in foam flow while the ellipse rotated so that its axis was parallel with the foam flow for every initial placement. This is known to be a feature of elastic fluids [19]. Thus, we aim to answer the question of whether the plasticity of foam is significant in determining the way in which particles sediment within a foam, and can we therefore treat the foam as an elastic liquid? Moreover, does a foam screen the interaction between particles as it does for the effects of topological changes within its structure [20]?

We choose to work in two dimensions for the reasons stated. We use the Surface Evolver [21] to simulate the sedimentation and interaction of two circular discs falling under their own weight. We look at the position of the discs as they descend and analyse the time-varying drag and lift forces on them. We consider the low velocity limit, in which we expect that the dominant contributions to these forces to come from the tensions of the soap films (network force) and the pressures of the bubbles (pressure force) – see figure 1. We aim to understand the conditions under which two objects falling through a foam are mutually attracted or repelled, as has been done for a number of purely viscoelastic fluids [22–25].

2 Method

We simulate disc sedimentation in a 2D dry foam by tracking the motion of two discs commencing from a position near the top of a foam channel [26]. They descend under the actions of three forces, defined in figure 1: (i) gravity, (ii) the resultant tension force F^n due to the network of films pulling each obstacle; (iii) the resultant pressure force F^p due to the pressure of bubbles contacting each obstacle. Note that the films that are in contact with the obstacle are not uniformly distributed around the circumference – they bunch up behind the obstacle, as shown in figure 2 – so that the resultant forces are usually non-zero.

Newton's second law applied to each disc of mass m gives

$$m\frac{d^2\vec{x}(t)}{dt^2} = mg\hat{y} - \lambda\frac{d\vec{x}(t)}{dt} - \vec{F}^p - \vec{F}^n - \vec{F}^\eta, \tag{1}$$

where $\vec{x}(t)$ denotes the position of the disc at time t, g is the acceleration due to gravity, and $\hat{y}$ is the unit vector in the vertical direction. λ is a friction coefficient due to the interaction of the plane faces of the discs with the bounding surfaces and $\vec{F}^\eta$ represents the viscous force on the circumference of the discs.

We assume that the motion is slow, so that we may neglect the acceleration term and the viscous forces. Then the model simplifies to the following evolution equation:

$$\frac{1}{\epsilon}\frac{d\vec{x}(t)}{dt} = mg\hat{y} - \vec{F}^p - \vec{F}^n, \tag{2}$$

where $\epsilon = 1/\lambda$ sets the effective time scale of the motion.

For each disc the resultant network force is the sum over all those films j that touch the disc. Since viscous drag around the disc is neglected, each film

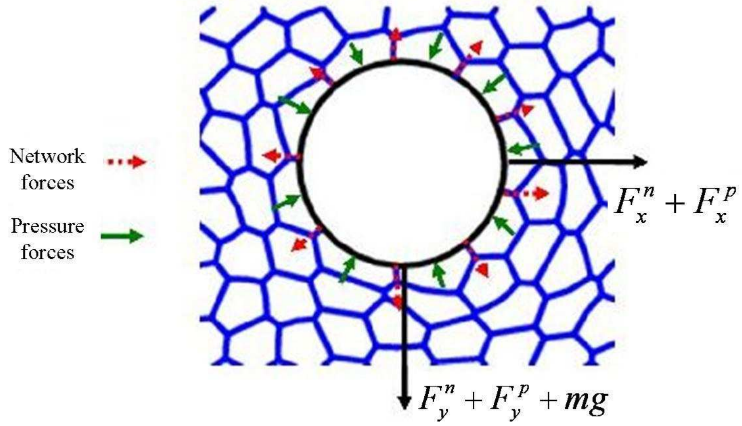

Fig. 1. The positions of the discs evolve under the gravitational, tension and pressure forces shown. Each force is resolved into its horizontal and vertical (the direction in which gravity acts) components.

meets the disc perpendicularly [27] and makes an angle θ_j with the positive y direction. Thus

$$\vec{F}^n = \gamma \sum_{\text{films } j} (\sin\theta_j, \cos\theta_j). \tag{3}$$

The pressure force is a sum over all the bubbles k touching the obstacle

$$\vec{F}^p = \sum_{\text{bubbles } k} p_k l_k (\sin\theta_k, \cos\theta_k) \tag{4}$$

where p_k is the pressure inside the bubble, l_k is the length of the contact line of the bubble with the disc and θ_k is the angle that the inward normal at the midpoint of l_k makes with the positive $y-$direction.

Using the Surface Evolver [21] in a mode in which each film is represented as a circular arc, we perform quasi-static simulations. We use three different foams in a channel of length $L = 1$: the first has $N_1 = 727$ bubbles contained within a channel of width $W_1 = 0.792$. We work with monodisperse foams, so the bubble area is $A_b \approx 1 \times 10^{-3}$ (A_b shrinks slightly in proportion to the disc size, since the total area of the foam and two-disc system is constant). The second foam has $N_2 = 746$ bubbles, channel width $W_2 = 0.805$ and bubble area $A_b \approx 1.1 \times 10^{-3}$. The cut-off length [27,16] for T_1 events is $l_c = 0.002$ for both of these foams, corresponding to a dry foam with liquid fraction $\phi < 0.1\%$. The third foam has $N_3 = 1500$ bubbles in a channel of width $W_3 = 0.432$ and bubble area $A_b \approx 2.9 \times 10^{-4}$. In this case the cut-off length for T_1 events is set to $l_c = 0.001$ so that the effective liquid fraction is consistent with that of the previous two foams. In all three cases the channel is periodic in

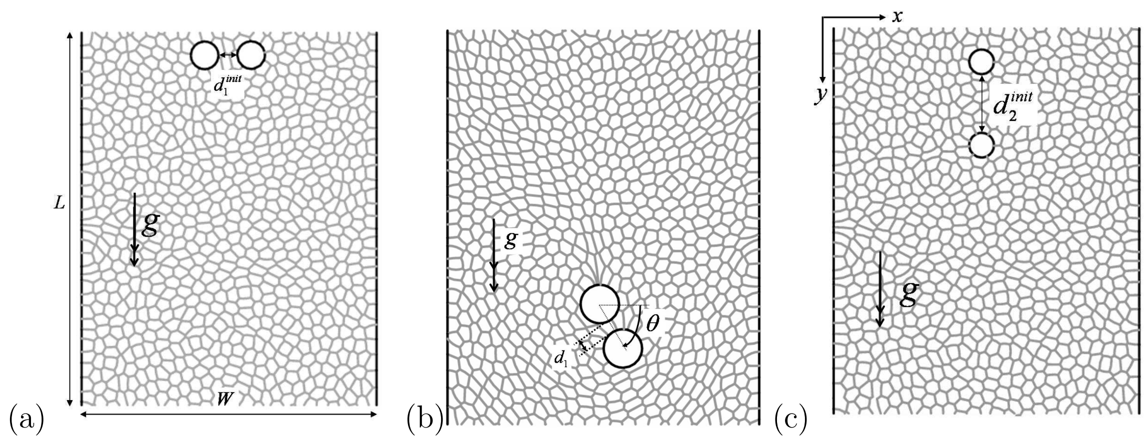

Fig. 2. Two discs sedimenting in a monodisperse foam contained in a channel of width W and length L. (a) Configuration 1, in which the discs start side by side, with a distance d_1^{init} between their centres. (b) If the discs rotate about one another we measure an angle θ between the positive x-direction and the line between the discs' centres. (c) Configuration 2, in which the discs start one above the other, a distance d_2^{init} apart.

the $y-$direction, parallel to the direction of gravitational acceleration. The simulations are stopped before either of the discs return to the top of the foam channel. We set a no-slip condition at the channel wall: the foam films that touch the walls have fixed vertices. The films that are in contact with the discs are free to slip.

We choose dimensionless units such that the line tension γ has value 1 throughout. We keep the disc size and weight fixed throughout our simulations at $4A_b$ and $mg = 10$ respectively. It was ensured that this disc weight was sufficiently large that the discs were not brought to a halt by the foam.

The simulations proceed as follows: a foam containing the two discs in their starting positions is relaxed to equilibrium, using the method described in [16]. The resultant forces on the discs in the x and y directions are calculated and the disc centres moved according to

$$\Delta x = \epsilon(F_x^n + F_x^p) \tag{5}$$
$$\Delta y = \epsilon(F_y^n + F_y^p + mg) \tag{6}$$

where the subscripts denote the x and y components of the forces. The parameter ϵ measures how far the centres move at each iteration ($\epsilon = 5 \times 10^{-4}$ for N_1 and N_2, and $\epsilon = 2 \times 10^{-4}$ for N_3). The foam perimeter is then brought back to a local minimum with the discs fixed. This comprises one iteration, which is repeated until a disc reaches the bottom of the foam channel. The discs' centres are tracked as demonstrated in figure 5. The computational time is dependent upon the number of bubbles: the simulations take about 50 hours for the two smaller foams and more than 120 hours for the large foam.

We first examine the sedimentation of a single disc in the foam to quantify the wall effects and check that the rest of the simulations will be independent of such effects (Section 3.1). We then choose two main initial configurations for our two disc sedimentation simulation, as shown in figure 2. The disc centres are initially separated by a distance d_i^{init}, either horizontally $i = 1$ or vertically $i = 2$.

3 Results

3.1 Single disc falling near a vertical wall

We first ran simulations of one disc and varied the initial placement of this disc at the top of the channel so that the effects of the wall on the motion of the disc could be ascertained in the hope of being able to neglect it when considering the interaction of two discs. We track the disc motion for nine different initial placements, the first being $0.1W$ away from the left wall in increments of $0.1W$, the last being $0.1W$ from the right wall. This is done for the two smaller foams.

It was found that for a fixed obstacle placed in a flow of foam in a similar channel the wall repels the obstacle [27], while sedimenting particles in viscoelastic fluids are attracted to walls [28]. Figure 3 demonstrates the drag and lift forces on a disc as it falls through the foam. There is an initial transient during which the forces rise; they then saturate but fluctuate greatly. The sudden drops in each force occur when a bubble detaches from the back of a disc. We therefore take average values for the forces after the transient, shown as horizontal lines.

Figure 4(a) demonstrates the variation in average drag force on the discs as they fall from different positions along the top of the foam channel. We deduce that a disc's proximity to the walls does not have an effect on the drag force exerted by the foam. Figure 4(b) shows the average lift force on a disc as it descends through the foam. It can be seen that for discs that are released close to either of the walls, there is a small lift force that is in the direction of those walls. For example, a negative lift on the left hand side of the plot demonstrates that the force is to the left and *vice versa*. These forces are considerably smaller and fluctuate less than the drag force. Note however that the fluctuations in the lift force close to the walls are greater, as for the drag.

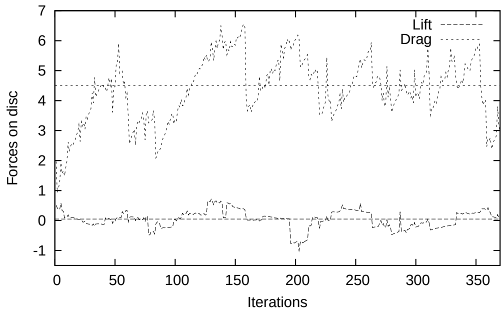

Fig. 3. The variation in drag and lift force on one disc (placed in the centre of the channel) as it descends through the foam. The plots are non-smooth due to the foam structure; jumps in the force appear when T1s occur. Note that a transient stage occurs for roughly the first 100 iterations. We take the average values for the drag and lift force after this transient.

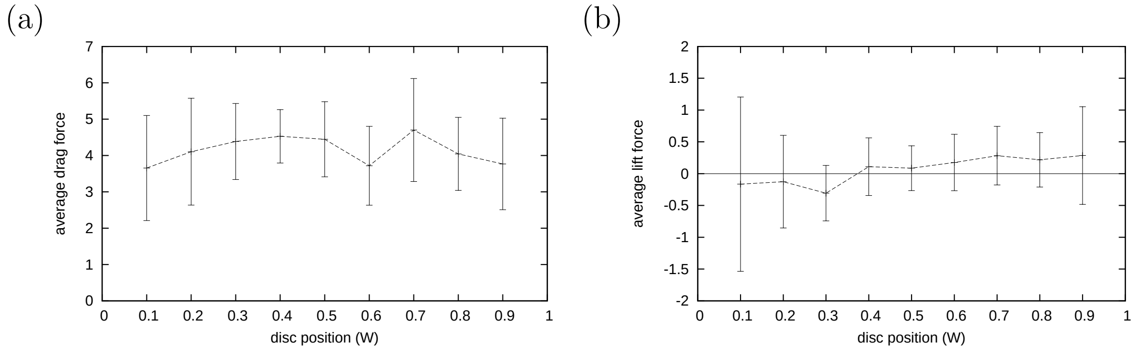

Fig. 4. (a) The variation of the average drag force on the disc as it moves through the foam from different initial positions. The disc is placed in nine positions along the top of the foam channel at equal intervals of $0.1W$. It can be seen that the drag force on the disc does not differ greatly even when the disc is placed close to the walls. (b) The variation of the average lift force on the disc for different initial placements where the positive direction of the force is to the right. The lift is negative when the disc falls from $0.1W$ (close to the left wall) and positive when falling from $0.9W$ (close to the right wall) therefore an attractive force on the disc from the walls exists. Note also the increase in the fluctuation for the values of the average lift as we approach the walls of the channel.

The effect, although small, appears robust with respect to different foams and the discs that are initially close enough to the walls are attracted.

3.2 Two discs in configuration 1

We investigate the interaction of two discs placed side by side within the centre of the foam channel, where we can neglect wall effects. For our simulations, we work in the region $0.3W$ to $0.7W$ of the foam channel where the wall effects have been shown to be negligible. The initial separation between the discs is

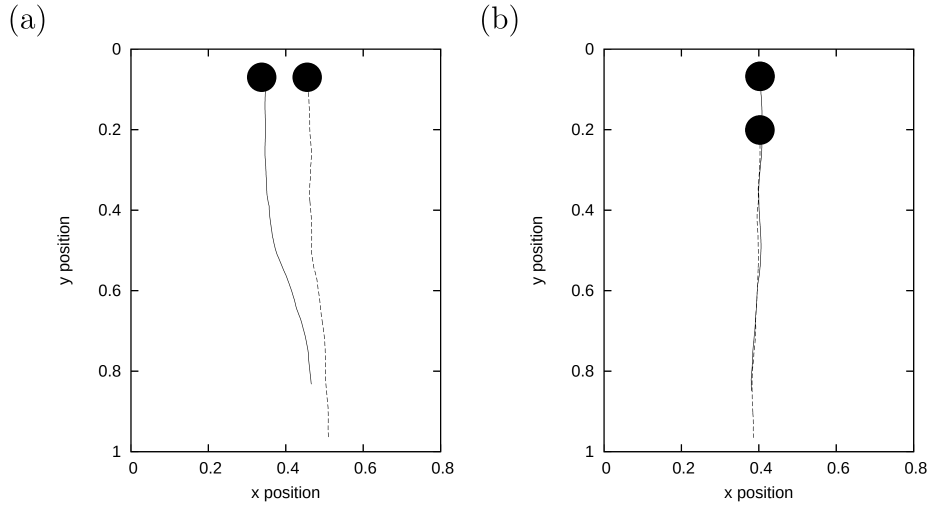

Fig. 5. Tracking the motion of the discs' centres in two typical simulations. Left: Configuration 1, with $d_1 = 0.08$. Here, both discs move a short distance to the right, and the disc initially on the left advances more slowly and moves behind the right-hand disc. Right: Configuration 2, with $d_2 = 0.2$. The discs barely deviate to the sides, but the upper disc moves slightly faster into the lower disc's wake.

varied and we investigate whether the discrete nature of the foam screens [20] the interaction between the discs. This is done for the three foams described. For each simulation, we record at each iteration the disc positions (figure 5) and the drag and lift forces on each one.

3.2.1 Disc Position

It has been shown that in a viscoelastic fluid circular particles in this configuration rotate about one another as they sediment [22–25]. We find the same rotation in foams (figure 5(a)): figure 6(a) shows the variation of the angle between the discs as they descend in the foam. The rotation of the disc system can occur in either a clockwise or an anticlockwise manner. Thus the plasticity of the material doesn't change the sedimenting motion of the particles greatly.

In figure 6(a) it is clearly seen that the discs rotate until they reach a plateau value at $|\theta| = \frac{\pi}{2}$. In this case the discs have rotated from being initially in configuration 1 so that they are finally oriented in configuration 2. The plateau at the positive and negative values for $\frac{\pi}{2}$ demonstrates that once the discs are directly above one another, they stay in this configuration. Notice that there are some simulations which don't reach these plateau values: those in which θ doesn't change dramatically are the ones where the discs were initially more than $3d_b$ apart. Others are those in which the foam was too short for the plateau to be reached.

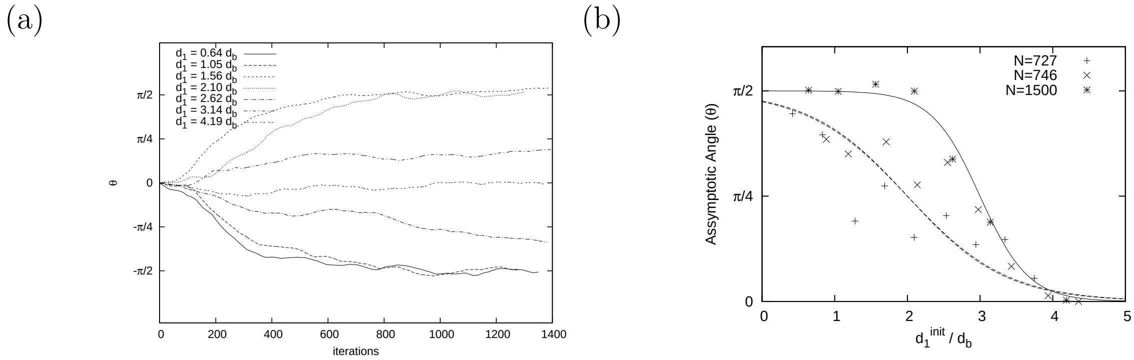

Fig. 6. (a) The angle θ between the discs' centres in configuration 1 with $N_3 = 1500$ for a range of initial separations d_1, demonstrates rotation of the discs as θ increases in either a clockwise and anticlockwise direction. The discs rotate, if close enough into configuration 2 and stay in this configuration. (b) The settling angle of the discs (θ at the bottom of the channel). The data for the large foam and the two smaller foams are fitted to a tanh function (7). It is clear that the foam screens the interaction of the discs if they are initially 3 or more bubble diameters apart and that the lesser slope for the smaller foams is due to the foam being too short for the full rotation to occur.

There is a strong relationship between the initial separation of the discs and the settling angle (the angle between the discs after reaching the bottom of the foam). Discs that are initially far apart rotate less. We look more closely at this trend by fitting the data for the settling angle for the three foams (figure 6b) to the following model:

$$\theta = \frac{\pi}{4} \times (1 + \tanh(\kappa(d_{1c} - d_1))), \tag{7}$$

where $d_{1c} = 3 \pm 1$ and the slope here is $\kappa = N/1000$ which measures the extent to which the plateau has been reached. Thus, if the discs initially have more than three to four bubbles in between them then they don't interact and rotate. When the discs are closer than this then they will rotate until they reach configuration 2 in which they are one above the other.

The variation of disc separation is also important when looking at their motion. In figure 7(a) we see a tendency for the discs to move away from each other as they descend through the foam. We note that the discs that are initially placed closer than the screening value for the separation, stay at a stable separation and in some cases move closer together. It is the discs that are initially further apart (placed further than the screening value) that tend to move further away from each other. These are the discs that don't rotate about each other and therefore don't interact as much. This point is clarified in figure 7(b) where the final separation between the discs is compared with the initial separation. Here we see that in all but three cases, the discs have moved further apart during sedimentation. If we look in more detail at the

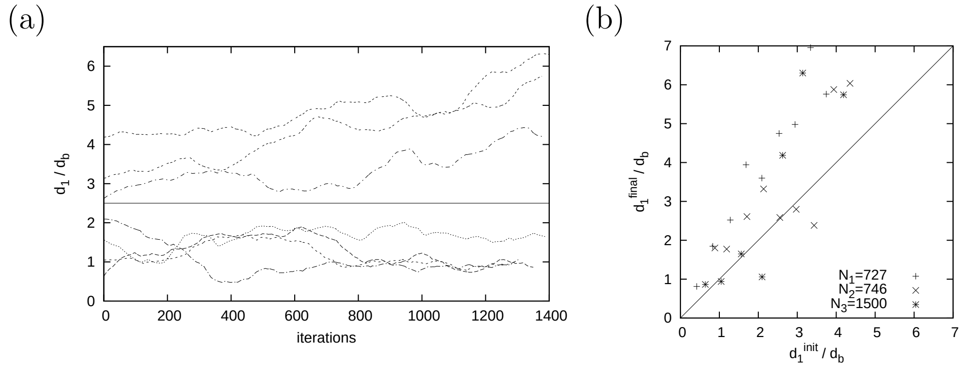

Fig. 7. (a) The separation between disc edges, measured in bubble diameters, as they fall through the foam in configuration 1 with $N_3 = 1500$. There is a general tendency for the discs to move away from each other as they descend through the foam unless they are placed sufficiently close together. When sufficiently close to interact and rotate, the discs remain in close proximity. The horizontal line represents a screening value above which the interaction between the discs is negligible. (b) The difference between final separation and initial separation in terms of bubble diameters d_b. We note that the majority of disc pairs have separated during sedimentation. The effect of this becomes stronger as the initial separation is increased and data appear above the line of unit slope. Thus discs only move closer together after rotation into configuration 2.

plot we can observe two regions that demonstrate different tendencies in the disc-to-disc interaction. The lower left corner of the plot sees the data points close to the line of unit slope, thus demonstrating that the discs have stayed at a steady separation. In some cases the discs have moved slightly further apart. These are the discs that are in the process of rotating about one another. The other cases where discs have moved closer together represent the simulations where the discs have fully rotated into configuration 2. The upper region of the plot however contains data points that lie far to the left of the unit slope, demonstrating that the non-rotating non-interacting discs tend to move away from each other. It was observed in simulations that this increase in separation arose from fluctuations in the vertical displacement rates for the two discs. Differences in the local foam structure around the two discs mean that they don't descend at exactly the same rate through the foam. A lateral fluctuation in their motion was found to be minimal as shown by our study for one disc in section 2. Recall that the lift force on the discs was shown to be negligible within the region of the channel under consideration here.

3.2.2 Forces on the Discs

We look at how the forces on the discs affect this interaction between the discs. Figure 8 shows the drag and lift forces from two different simulations in the $N = 1500$ foam. The first is for two initially close discs that rotate and the

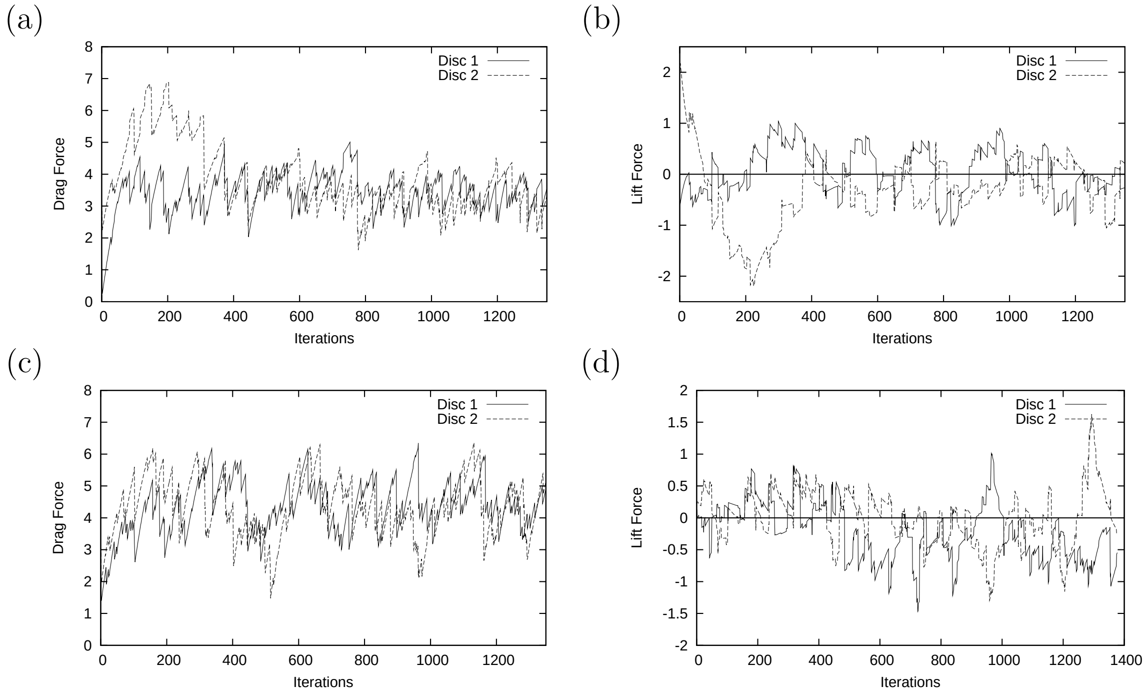

Fig. 8. Fluctuating forces on two discs in configuration 1. (a) drag force on both discs for ${d_1}^{init} = 0.64 d_b$, (b) lift force on the discs when ${d_1}^{init} = 0.64 d_b$. It can be seen that there is an overshoot in the drag for disc 1 in (a) and then an overshoot in the lift on disc 1 in (b). Thus, they interact and rotate about one another. (c) and (d) Same data for two discs that start further apart (${d_1}^{init} = 4.20 d_b$). Here the drag and lift forces are very similar for both discs and follow the same pattern as would be expected on one disc falling in the foam. Thus, the discs don't interact in this case.

second is for two discs that are too far apart to interact. When the discs are initially close together, the drag force is seen to overshoot for one of the discs. This results in slower downward motion of this disc and it is left trailing. An increase in the lift force is seen for this disc at this stage and it is directed so that the disc moves into the wake of the other disc. Thus, the discs begin to rotate so that the resistance to their downward descent is minimized. After rotation has occurred it can be seen that the drag and lift forces on both discs become very similar, at which point the motion of the discs becomes more stable. For the discs that were initially further apart no such overshoots are seen as they don't interact (figure 8(c) and (d)).

3.3 Two discs falling in configuration 2

We consider two discs descending in the foam one above each other, working with the same size discs as before. We vary the initial separation between the discs to interpret how the discs interact when they are oriented in this way (figure 5(b)).

3.3.1 Disc Position

The discs move closer together as they descend in the foam until they have moved so close that only one or two bubbles separate them, after which they move at a constant separation. In contrast to configuration 1, when the discs are initially far apart they will still move closer together, albeit at a slower rate. This is illustrated in figure 9. The wake of the lower disc is a yielded region of the foam and it is this that determines how the discs interact. The data suggest that the wake stretches back roughly 5 bubble diameters from this disc. If the upper disc is initially within this distance from the lower disc then it is able to move closer into the wake until the constant separation of 1 to 2 bubble diameter separation is reached. If the upper disc is initially above the yielded region in the lower discs' wake then the interaction is less apparent. It is possible (but was not observed) for the discs to move closer in this case, depending on the initial structure of the foam. In this case convergence to the plateau value for their separation will take longer and will require a longer foam channel again.

3.3.2 Forces on the discs

The drag force plays an important role in the interaction, while the lift force is assumed to be negligible because the discs are placed at the centre of the foam channel. Figure 10 clarifies the effect that varying the initial separation has on the drag forces on the discs: it shows three regimes in which the interaction between the discs differs. For initially close discs ($d_i^{init} < 2d_b$) the drag force difference between the discs is small but slightly negative, so that the separation increases very slightly as they descend. When the discs are initially separated by a larger distance, $2d_b < d_i^{init} < 6d_b$, the difference between the drag forces increases. The drag on the lower disc is always greater than that on the upper disc so they will move closer together. When the discs are even further apart (more than $6d_b$ initial separation) figure 9 confirms that there is limited interaction between the two discs.

The tendency for the obstacles to move closer together for a particular range of initial separations suggests that the yielded region of the wake of the lower disc extends up to four or five bubble diameters above the disc. Thus, if the initial separation is more than the length of this region the interaction becomes minimal. This is further evidence of the foam screening the interaction between the two objects.

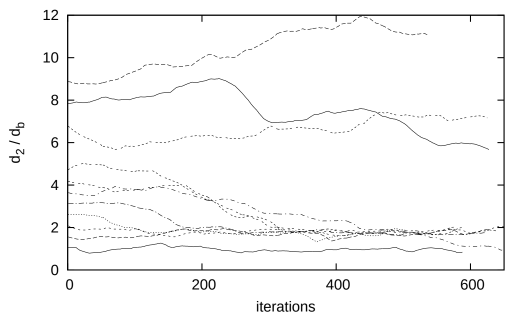

Fig. 9. The separation, measured in bubble diameters, between two discs falling through a foam in configuration 2 with $N_3 = 1500$. It can be seen that for discs initially separated by up to 2 d_b, the discs descend in the foam at a constant separation. If the discs are initially separated by 2 to 5 d_b, then they move closer together until they reach a separation of 1 to 2 d_b, after which the motion is stable. If the initial separation is greater than 6 d_b then the variation in separation is less and they stay far apart. Here the yielded region above the lower disc (the wake) plays an important role in the interaction of the discs.

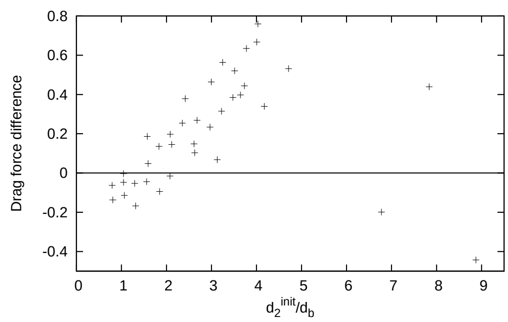

Fig. 10. In configuration 2 the difference between the average drag forces acting on each disc increases with initial separation for all foams studied. This difference is measured by subtracting the average drag force on the upper disc from that on the lower disc.

4 Conclusions

The interaction between a sedimenting disc and a wall was found to be minimal, although a small attractive force exists when the disc was in close proximity to the wall. The main wall effect was to increase the size of the fluctuations in the lift and drag force on the disc. Thus for our simulations of two sedimenting discs, we worked far enough away from the wall so that we could neglect these wall effects.

In the case of two discs sedimenting initially side-by-side, a rotation towards a configuration in which they are one above the other is evident. The rate of rotation is dependent on the initial separation between the discs and it was found that this interaction only occurred if the initial separation between the discs is less than 3 to 4 bubble diameters. When the initial separation was greater than 4 bubble diameters the foam screens the interaction and the motion of each disc is determined by variations in the local structure of the foam.

For the case in which two discs above each other sediment, further evidence of screening was apparent. The initial separation of the discs was again an important parameter that determined their interaction. If the discs were placed less than $4d_b$ apart then they move closer together due to the drag force on the lower disc being greater than that of the upper disc. In this case the upper disc is sedimenting in the yielded region of the foam behind the lower disc, whence it moves into the wake of the lower disc. If the discs move close to a separation of $1 - 2d_b$, the drag force on both is equal and therefore they move at the same rate. However, if the initial separation is increased above $6d_b$ then the drag force on each disc is independent, whence the discs don't interact.

Thus the motion of the discs is stable when their line of centres is parallel to the direction of gravity and separated by one to two bubbles. Although this is reminiscent of elastic fluids, the plasticity of the foam plays an important role: the T_1 events behind the discs as bubbles lose contact change the local structure of the foam and allow the upper disc in the wake to move more quickly. The discrete nature of the foam means that objects don't interact if they are separated by more than $4d_b$ horizontally or $6d_b$ vertically.

It remains to be seen whether these results extend to objects of different dimensions (area, weight) or shape (e.g. ellipses), and to what extent material parameters such as the bubble area dispersity and the liquid fraction of the foam dictate the dynamics of sedimentation.

Inclusion of the viscous forces on the discs may lead to increased rotation of the discs, and simulations that do so are likely to provide a better comparison with experiment.

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