Show me the (shortest) way to go home

Foams, soap films and minimization

Simon Cox

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Consider a bubble…

The force of surface tension causes the *area of the film to be minimized*.

The least area way to enclose a given volume is a sphere.

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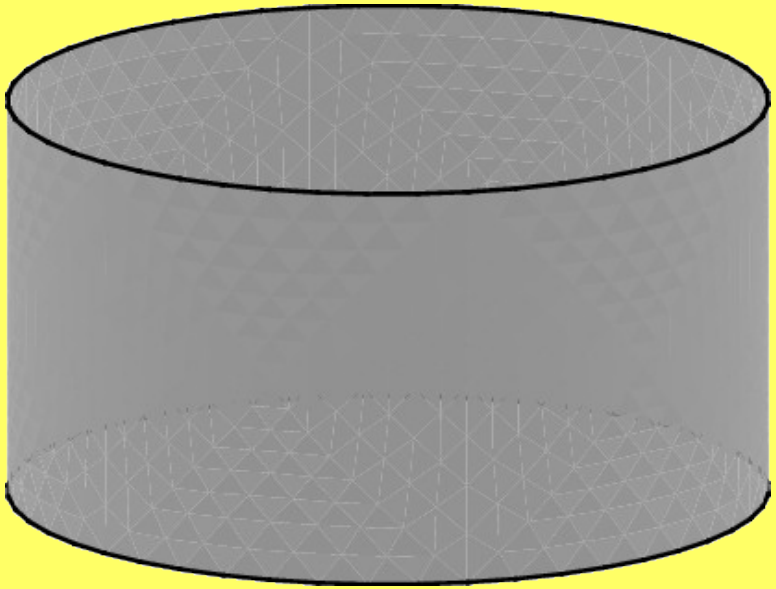
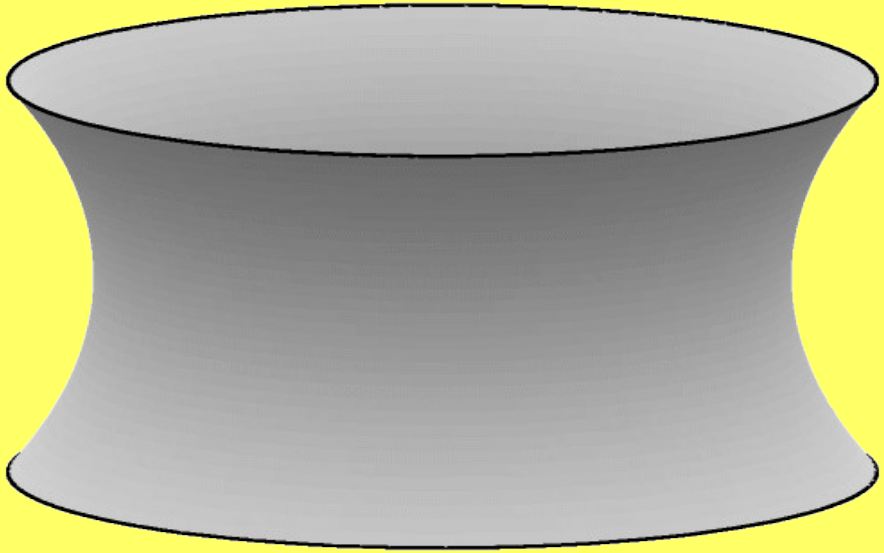
Catenoids

What is the shape of the surface joining two circular rings?





This “catenoid” shape has the least surface area!

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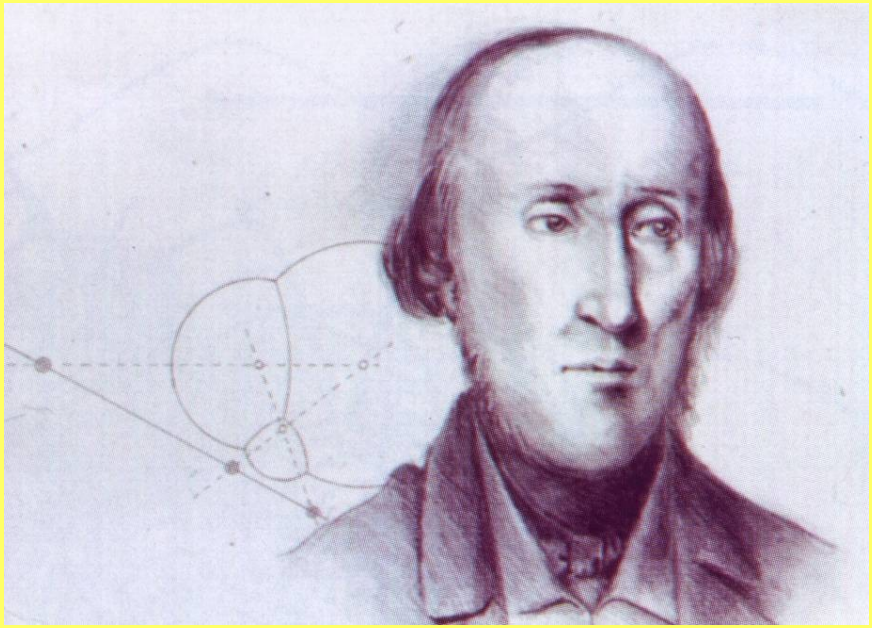
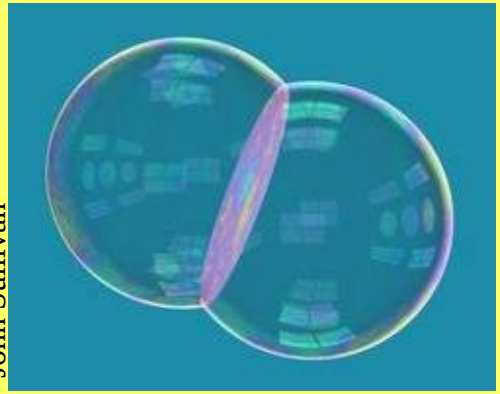
Plateau's Equilibrium Rules



How do two bubbles meet?



John Sullivan

Minimization of area gives geometrical constraints:

- three films meet in a line at 120°
- four lines meet at a point at 109°

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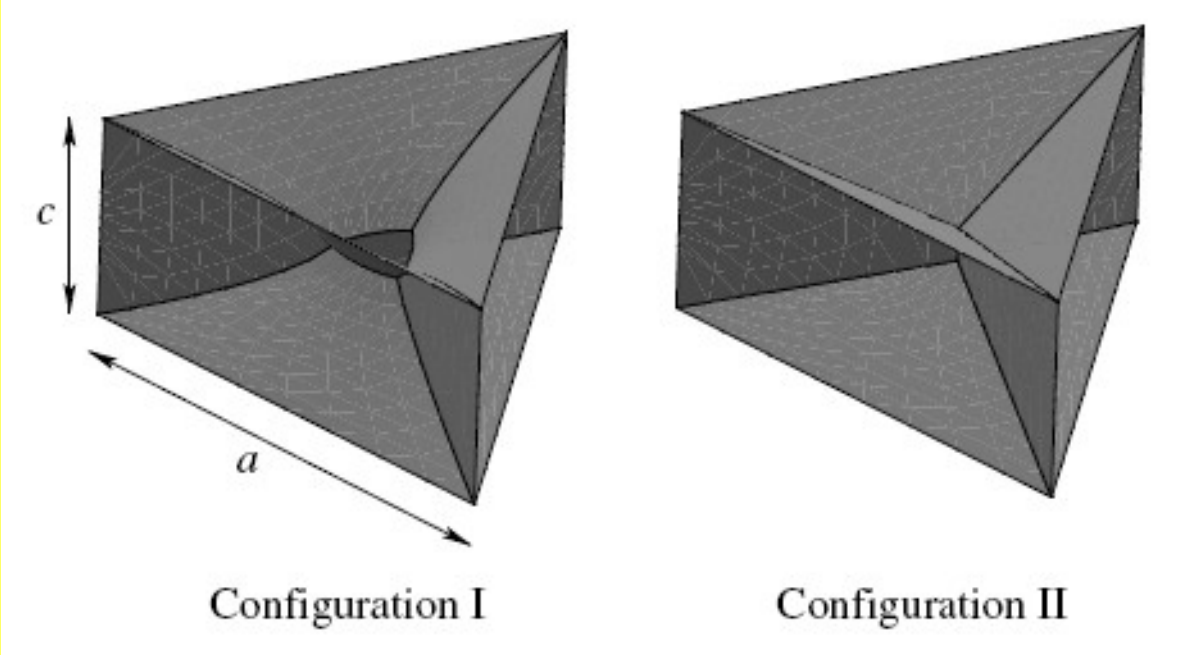
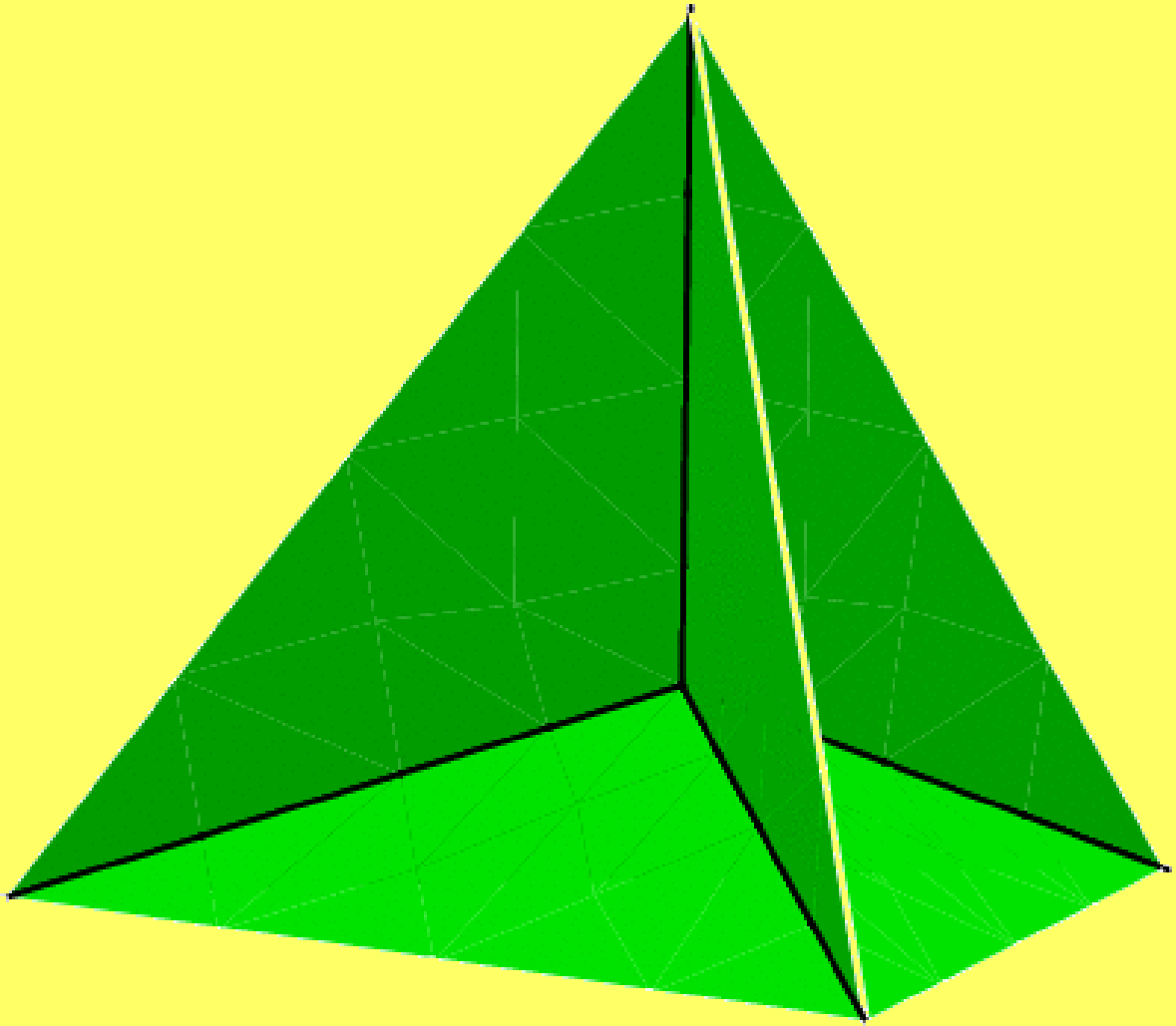
Wire Frames

Plateau used wire frames to observe and formulate his laws. A simple example is the triangular prism, which has two possible (non-trivial) soap film combinations that touch all edges:



Wire Frames



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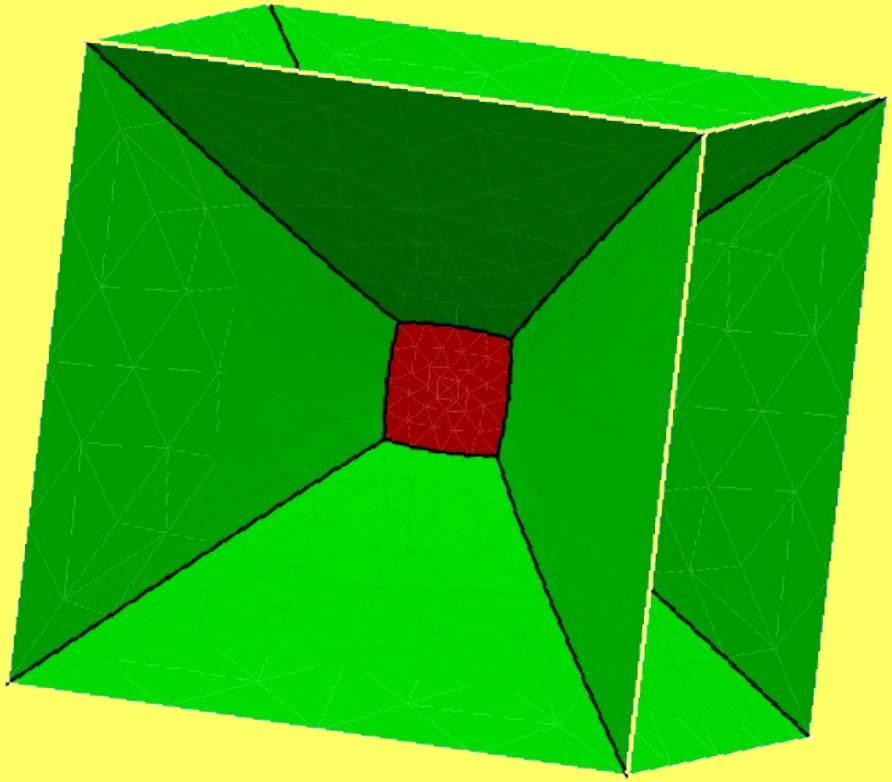
Wire Frames



Each film is flat (planar), but the lines where they meet may not be straight.

Steiner problem

Soap films solve the Steiner problem:

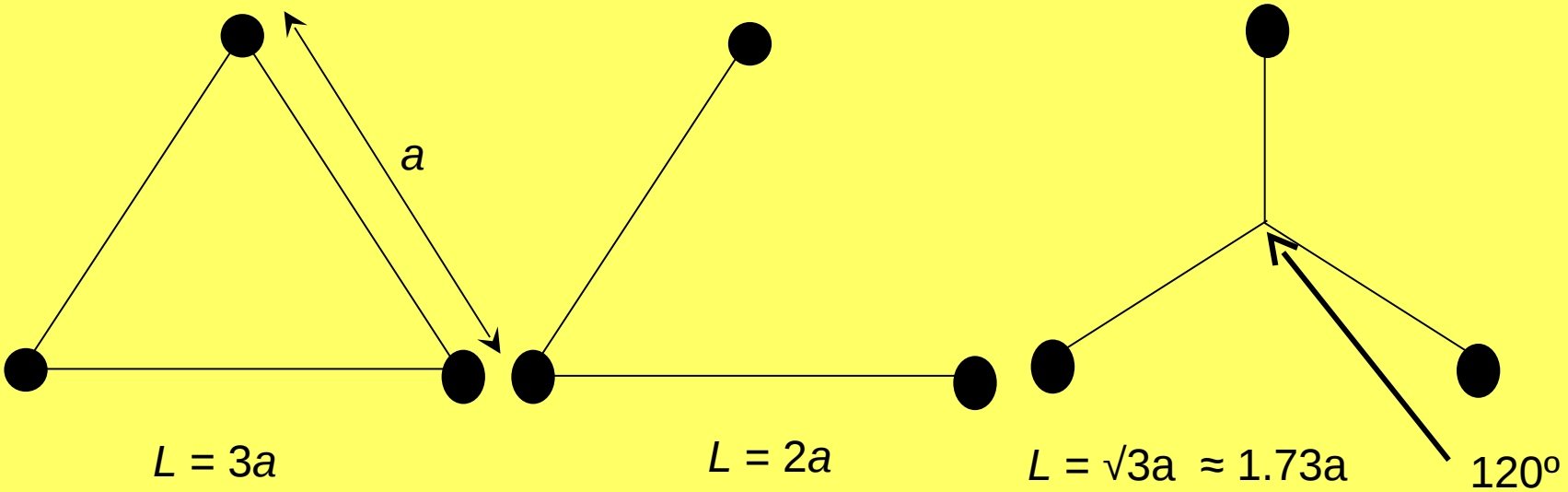
Given *n* cities on a plain (ignoring rivers etc.), what is the arrangement of roads joining them with minimum length?

Geometrical proofs long-winded and complex; experimental demonstration straightforward

Steiner problem

Take three cities at the corners of an equilateral triangle of side-length a.

What is the arrangement of roads connecting them that minimizes the total length of road?



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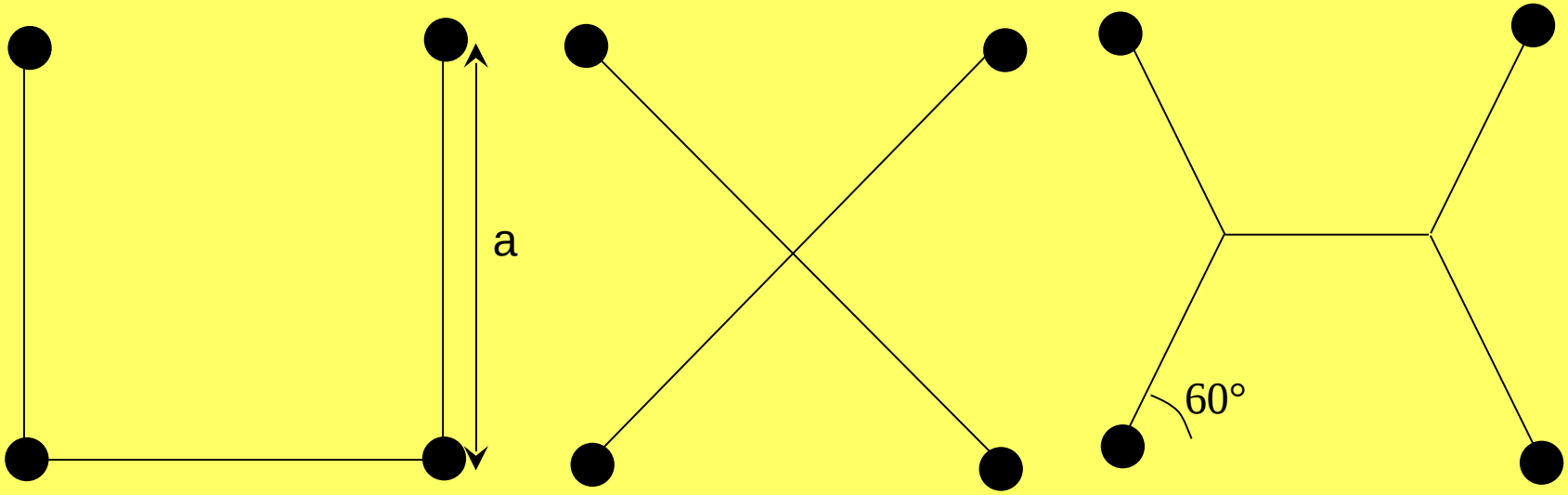
Steiner problem

Four cities at the corners of a square of side a:



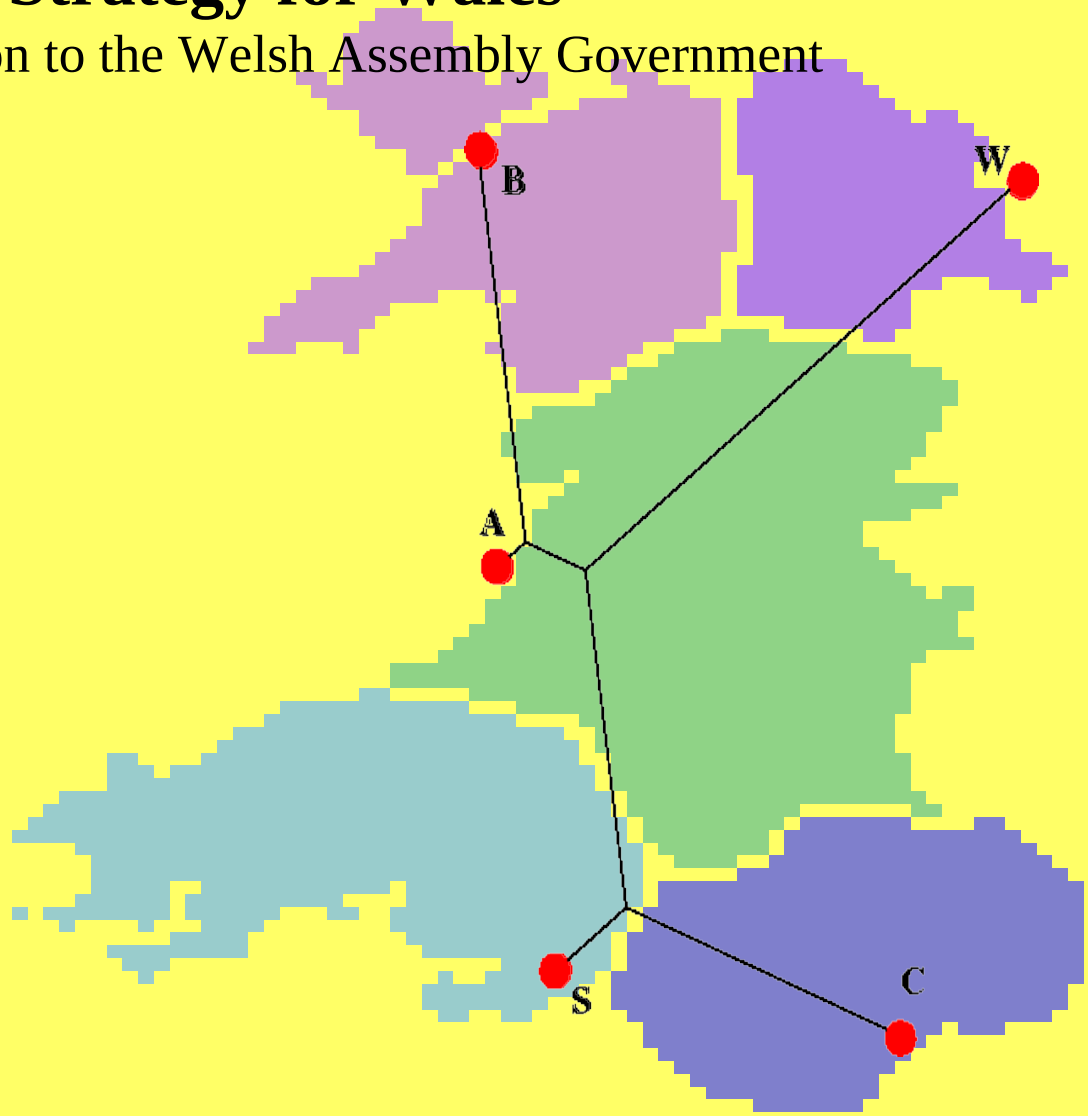
$L = 3a$

$L = 2\sqrt{2}a \approx 2.83a$

$L = (1+\sqrt{3})a \approx 2.73a.$

A Transport Strategy for Wales

A recommendation to the Welsh Assembly Government



Use *Surface Evolver* software to find which is the lowest minimum

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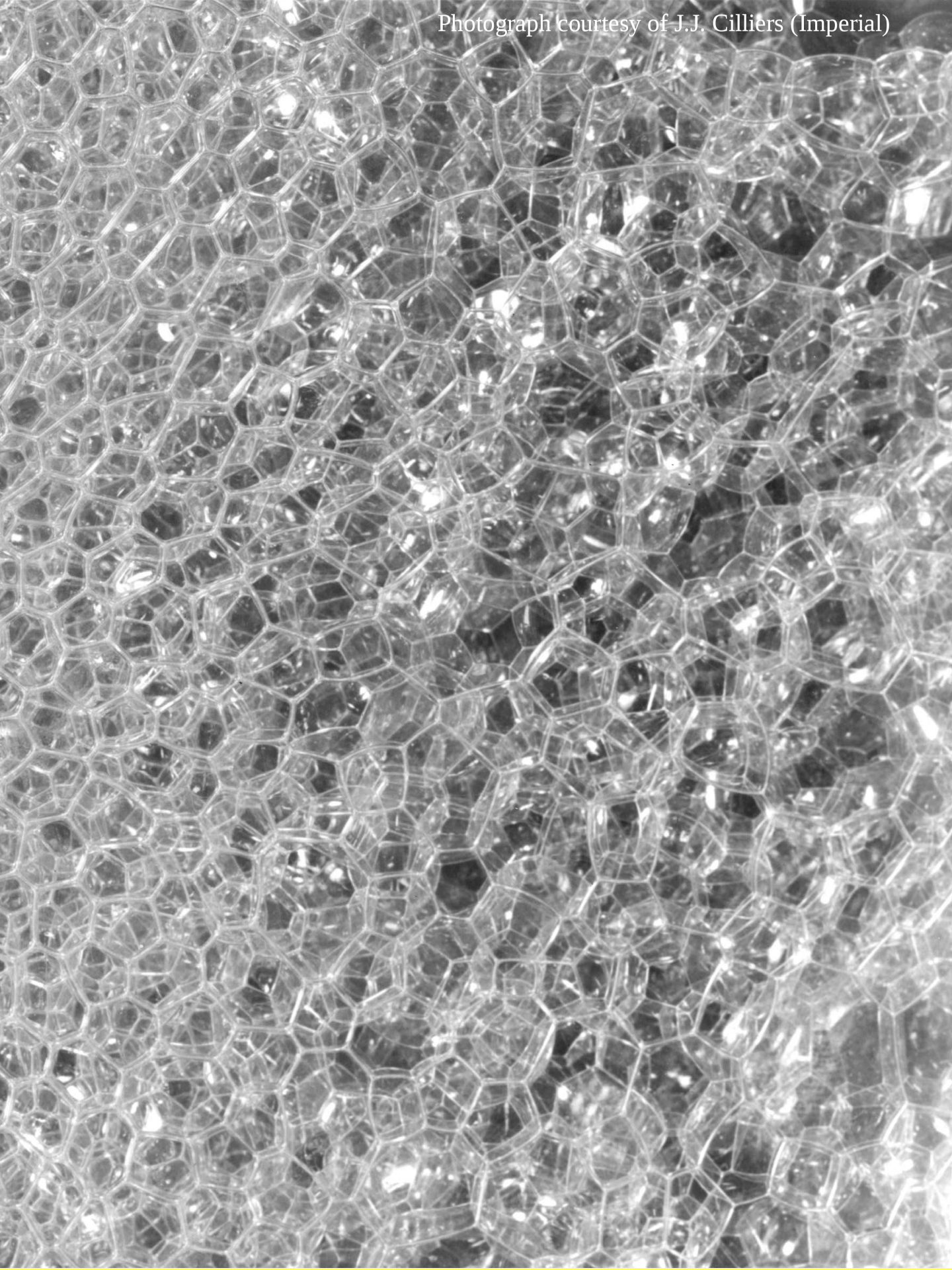
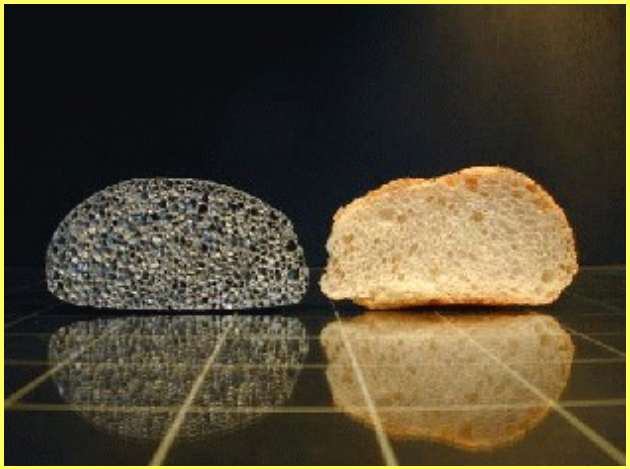


Photograph courtesy of J.J. Cilliers (Imperial)

Why are foams of interest?

Many applications of industrial and domestic importance:

- Oil recovery
- Car manufacture
- (Industrial) cleaning
- Fire-fighting
- Ore separation
- Personal Care
- Food products



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