Drainage induced convection rolls in foams
(revised version)

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Abstract

Forced drainage describes the procedure in which liquid is added to a column of foam at a constant flow rate. For sufficiently high flow rates this results in a convective bubble motion, the symmetry of which depends on the experimental details. We review recent experiments, in particular for the case of convection in a foam column which is tilted away from the vertical. The experimentally-determined dependency of the onset of convection with the tilt angle is well described by a combination of standard drainage theory and simple foam rheology. We also present experimental data for foam convection in a Hele-Shaw cell. The measured foam density and velocity profiles provide an excellent basis upon which to develop a theory of the phenomenon.

Introduction

Swirling bubble motion is a commonly occurring phenomenon in the head of a freshly poured glass of beer. The bubbles in such a foam of high liquid fraction are nearly spherical, making it easier for them to rearrange in some sort of convective motion. Under the controlled laboratory conditions of forced drainage, where soap solution is added to the foam at constant flow rate, the irregular motion in the beer glass is replaced by regular convective rolls. The precise geometry of the rolls depends on the dimensions of the vessel confining the foam and the type of flow input. A theory of this convective

motion requires an understanding of both foam drainage and rheology and is still incomplete for the general case.

In this paper we will describe the key experimental results. Emphasis is given to the case of convective motion in a tube which is tilted from the vertical; this allows for the formulation of a theory which is able to predict the angle of onset for convection as a function of the flow rate of added solution. We also present preliminary results for convection in quasi two-dimensional foams (monolayers of bubbles between two glass plates). Such experiments are particularly accessible to computational image analysis and allow for the determination of local profiles of both liquid fraction and bubble velocity.

Forced and steady drainage

Forced drainage experiments, first performed by Leonard and Lemlich (1965), have proved extremely useful for the understanding of foam drainage (Weaire *et al.* 1993, Koehler *et al.* 1999). In these experiments a column of foam is fed at the top with a supply of surfactant solution as shown in figure 1. The bottom of the column is placed in a pool of the solution. If this supply is constant the result is steady drainage of the solution through the foam due to gravity. Unless the flow rate is very small, the liquid fraction Φ is approximately constant over most of the column, ignoring end effects. It scales with the flow rate as $\Phi \propto Q^{\beta}$, where the exponent β varies between 1/2 and 2/3, depending on the surface viscosity of the liquid that is foamed (Koehler *et al.*, 2000, Saint-Jalmes *et al.* 2004).

Steady drainage experiments were initially thought to be of value for studying wet foams, i.e. foams with a liquid fraction of more than 15%. However, the occurrence of convective rolls, described below, prevents this. Alternative approaches to generating wet foams are micro-gravity experiments (Saint-Jalmes *et al.* 2006) or using bubbles with diameter less than a fraction of a millimetre (van der Net *et al.* 2006), well below the capillary length.

Convection rolls

While for small enough flow rates Q the foam structure is essentially static in the steady state, convective bubble motion sets in at some critical value of Q. This occurs both for foams contained in cylindrical tubes (Hutzler *et al.,* 1998) as well as for foams in rectangular cells (Vera *et al.* 2000). In polydisperse foams the convective motion results in a sorting of the bubbles according to their size: the larger bubbles gather at the top of the foam column (Hutzler *et al.,* 2000). The establishment of a vertical gradient in bubble size eventually suppresses the bubble motion. In a column of monodisperse foam the convective motion continues indefinitely, providing the preferred type of foam for convection studies.

The symmetry of the convective rolls depends on the experimental set-up, i.e. the shape and dimensions of the confining vessel, the position of the flow input, and possibly the bubble size. We distinguish two types of convective roll occurring in cylindrical tubes (typically with a diameter of a few centimetres and length 10-30 centimetres) filled with equal-sized bubbles with radii of a few millimetres.

In a *simple convective roll,* bubbles on one side of the tube move downwards while they move upwards along the opposite side. This appears to be driven mainly by an inhomogeneous flow input. In short columns such a roll can extend over the whole tube, while at moderate flow rates it takes place only within the upper part of the foam, as sketched in figure 2a (Alonso, 2003).

For higher flow rates (and sufficiently long tubes) a *cylindrically symmetric roll* is established, in which all the surface bubbles move downwards with a consequent upward movement of bubbles in the bulk. The onset of this second roll is subject to considerable hysteresis, i.e. it sets in at a higher flow rate than the one at which it stops when the flow rate is decreased.

In the experiments of Vera *et al.* (2000) the average bubble radius is 55μm and the cross section of the confining rectangular cell is 1.3cm by 21.5cm. Here a number of different

bubble motions are found, depending on the flow rate. Irregular swirls at low flow rates are replaced by a single convection roll at higher flow rates. Size sorting again occurs if the flow rate is further increased, and the single roll breaks up into several convection rolls with differing average bubble sizes.

Convective roll in a tilted tube

Recent experiments by Cox *et al.* (2006) explored the effect of tilting a cylindrical column of foam away from the vertical. This has the advantage that only a single convection roll appears in which bubbles on the lower side of the tube move downwards, while bubbles on the opposite side move upwards, as sketched in figure 2b. While the downward motion of the visibly wetter bubbles shows considerable internal shear and bubble rearrangement, the upward motion is essentially a plug flow.

The results are summarized in figure 3. Convective motion for a fixed flow rate occurs once a critical angle θ_c is exceeded. After a linear increase at low angles θ, the velocity v of the upward moving bubbles is roughly constant, at least for low enough flow rates.

This may be conveniently represented by the empirical relationship $v = v_{lim} \tanh((\theta - \theta_c)/\theta_0)$ where the fit parameters v_{lim}, θ_c and θ_0 are flow rate dependent.

The motion of the bubbles in the bulk is not easy to observe visually, due to the opacity of the foam, so that measurements are restricted to the surface. This is different in the case of quasi two-dimensional foams, as will be described below.

The most appealing feature of the experiment is that it is accessible to a theoretical treatment. Foam drainage theory and a stress balance, combined with an empirical rule for the yield stress of a foam, lead to a prediction of the critical tilt angle θ_c for the onset of the motion and its variation with flow rate Q as follows.

The steady-state 2D channel-dominated foam drainage equation (Cox *et al.* 1999), appropriate for the type of surfactant used in these experiments, is transformed to coordinates parallel and perpendicular to the tube axis, and the perpendicular component of flow rate set to zero to give an expression for the variation of liquid fraction across the tube. A stress balance on a small element of foam allows an expression for the variation of stress across the tube to be derived. This is equated to a yield stress, dependent upon liquid fraction, to give a condition for the onset of convection (Cox *et al.* 2006):

$$\theta_c = c \ Q^{-3/4}. \tag{1}$$

Here the constant c contains known parameters such as the value of surface tension and the bubble radius and an unknown geometric constant related to the number of Plateau borders in any cross-section of the tube. The experimental data shown in figure 4 is well described by this power-law, as shown by the solid line.

Experiments with quasi-2D foam

The restriction to an analysis of the motion of surface bubbles only is a hindrance to a complete understanding of the formation of convective rolls. NMR or X-ray tomography, used for example in the study of metal foams (Banhart and Weaire, 2002), can overcome this. Besides being expensive, these techniques have a restricted time resolution, which limits their application to the slow evolution of foam structure, such as in coarsening.

As an alternative to this we have chosen to retreat to two dimensions and study convection rolls of monolayers of bubbles trapped between two vertical glass plates (a Hele-Shaw cell), as shown in figure 5. Since a detailed study is still in progress, we give here only preliminary results.

Information on bubble motion and determination of local liquid fraction is readily obtained using a digital camera and an in-house plugin to the publicly-available image analysis software ImageJ (Abramoff *et al*, 2004).

As in 3D we find that the type of flow input plays a role. With the flow rates available (up to 5 ml/s) and the geometry specified in figure 5, we could not find convective motion for a flow input uniformly distributed through porous fabric laid across the top of the cell.

Introducing liquid at the midpoint of the upper edge of the cell results in steady-state liquid fraction gradients due to a non uniform liquid distribution across the foam. When the flow rate exceeds a critical value (0.06 ml/s) the bubbles start to move at very small velocities, forming two counter-rotating convection rolls. The downward moving bubbles in the centre of the cell are visibly wetter and faster than the upward moving bubbles along the sides of the Hele-Shaw cell. Increasing the input flow does not lead to a change of the flow pattern but does lead to a downward extension of the roll.

Figure 6 shows the results of our image analysis. The arrows represent the time-averaged bubble velocities as a function of position in the Hele-Shaw cell. The shading indicates the wetness of the foam. We have at this stage not attempted to relate this to an actual local liquid fraction, since the geometry of the liquid network is not straightforward to analyse.

The variation of local liquid fraction for the case of a non-convecting 2D foam is readily obtained from numerical solutions of the 2D drainage equation and could be compared with experimental results. Of particular interest is the onset of convection when strong shearing of the foam occurs. This should lead to *dilatancy*, i.e. a local increase in liquid fraction due to shear (Weaire and Hutzler 2003). Since this does not feature in the numerically obtained profiles, the 2D experiments could help to clarify whether dilatancy indeed plays a significant role in bubble convection.

Conclusions and Outlook

Bubble convection in foams presents a variety of challenges for theory. What type of roll is established and how is it maintained? Can one predict its onset and understand the origin of hysteresis? Progress has been made for the simple case of a tilted tube with a

theory that combines drainage and rheology. It is, however, restricted to making a prediction about the onset only and does not explain the variation of bubble velocity with tilt angle. Studying convection in quasi-2D foams is promising in the sense that image analysis provides detailed data which might help to elucidate the role of dilatancy. Nevertheless care must be taken, since experiments and simulations with 2D foams, although often offering useful guidance, may be misleading, particularly in dynamic situations (Weaire *et al.* 2006, Janiaud *et al.* 2006).

General theories of convection have been advanced by Neethling (2006a) and Embley and Grassia (2006). Neethling shows that the shearing of a foam induces an anisotropy in the drainage of liquid and combines Surface Evolver calculations for the sheared structure with numerical solutions of the drainage equation. The computed critical liquid fraction for the onset of the simple convective roll as a function of inverse bubble radius is in accord with experimental data (Neethling 2006b).

Embley and Grassia (2006) present three different mechanisms for the onset of convection, all of which are based on the idea that the instability is due to the loading of the Plateau borders due to the weight of liquid. Again the experimental data (Hutzler *et al.* 1998, Weaire *et al.* 1993) seems well represented by all three approaches.

A full theory in a simple form remains to be developed and tested. In particular, it should be capable of explaining the observed occurrence of (at least) two types of convective roll.

Acknowledgements

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Figures:
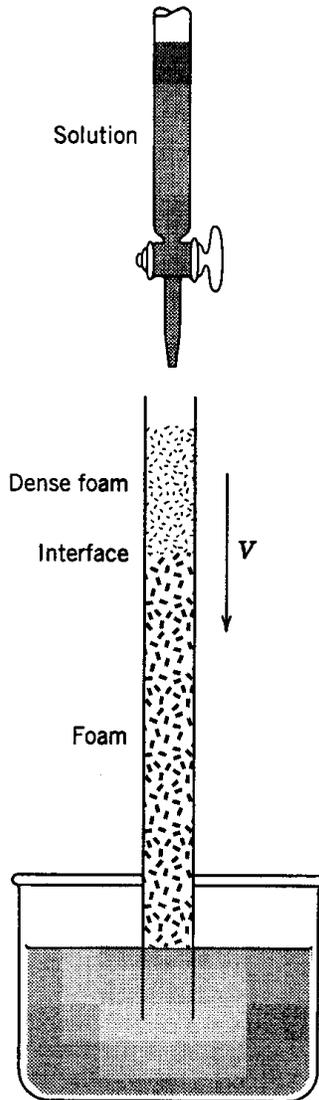
Figure 1: In forced drainage experiments surfactant solution is added to a dry foam at a fixed flow rate. This results in the propagation of a drainage wave through the foam, leaving in its wake a column of foam with a constant liquid fraction. For high enough flow rates convective bubble motion is observed.
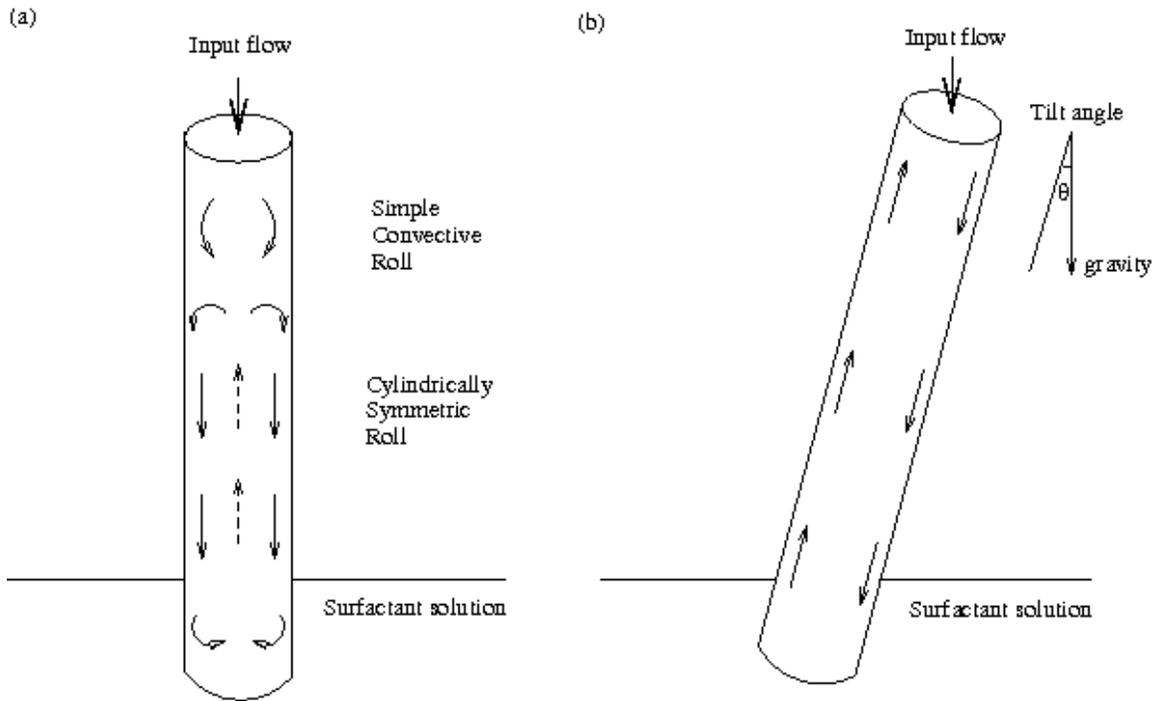
Figure 2: (a) In a vertical cylindrical tube two types of convective rolls may be found. The simple convective roll is dominant in short tubes. In long tubes it only features at the upper part of the tube and gives way to the cylindrically symmetric roll further down the tube. (b) In a tilted tube convection always takes the form of a simple convective roll. The downward moving bubbles are visibly wetter and move with higher velocities (and considerable internal shearing) than the upward moving bubbles.
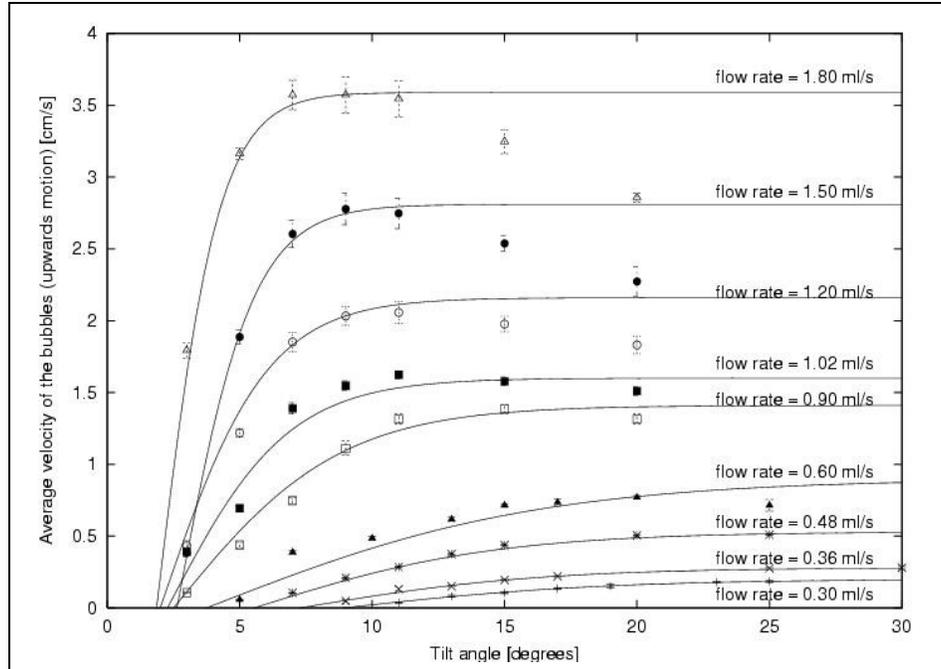
Figure 3: The upward velocity of bubbles in a dry foam varies as the tilt angle θ is increased. The data are fitted to the (empirical) function $v = v_{\text{lim}} \tanh((\theta - \theta_c)/\theta_0)$ and each curve labelled with the fixed flow rate at which the data were taken. For small angles, an increase in θ leads to an increase in velocity. At higher angles we see that for low flow rates the velocity reaches a flow rate dependent plateau, v_{lim}. For higher flow rates, the bubbles achieve a maximum velocity at a certain angle, and then slow down slightly. This slow down is not represented by the empirical fits. (relevant parameters: tube diameter 2.05 cm, tube length 35.5 cm, bubble diameter 1.55 mm, detergent: Fairy liquid, surface tension 28mN/m)
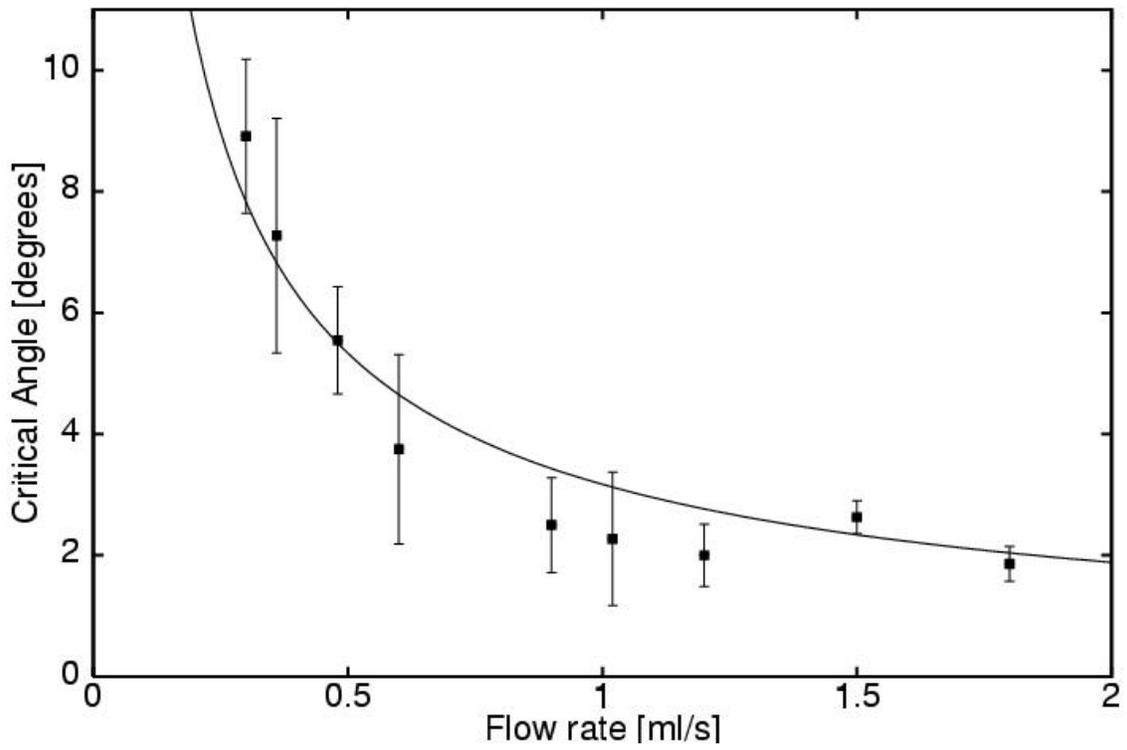
Figure 4: The motion of bubbles in the tilted tube begins, for given flow rate Q, when the angle of tilt is increased beyond a critical angle θ_c. The solid line is a least-squares fit to the theoretical prediction of eqn. (1).
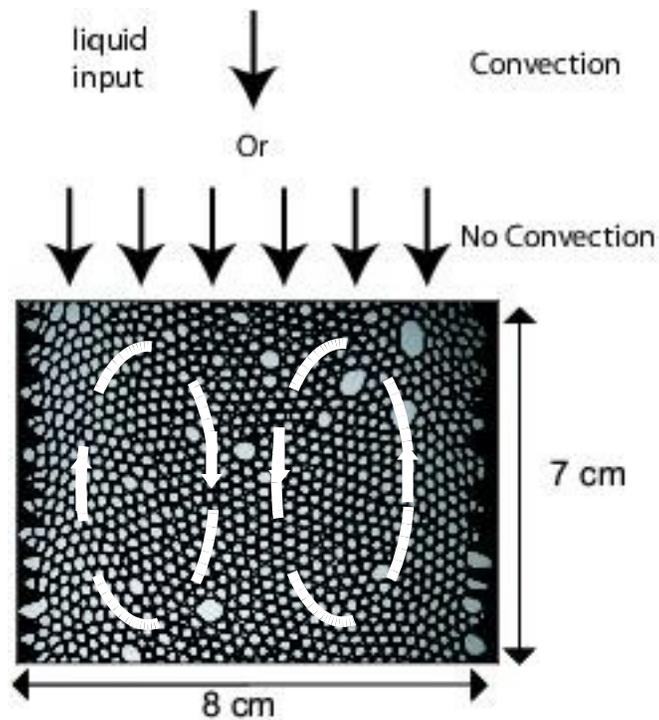
Figure 5: Set-up for the study of convection in a Hele-Shaw cell filled with a monolayer of bubbles (spacing of glass plates: 1.5mm, mean distance between centres of neighbouring bubbles: 2.4 ± 0.2mm). The bottom of the cell (not shown) is submerged in a pool of surfactant solution, the total height of the cell is 20cm. For a uniform input of solution no convection is observed. Input at the midpoint of the upper edge of the cell leads to two counter rotating rolls, as sketched.
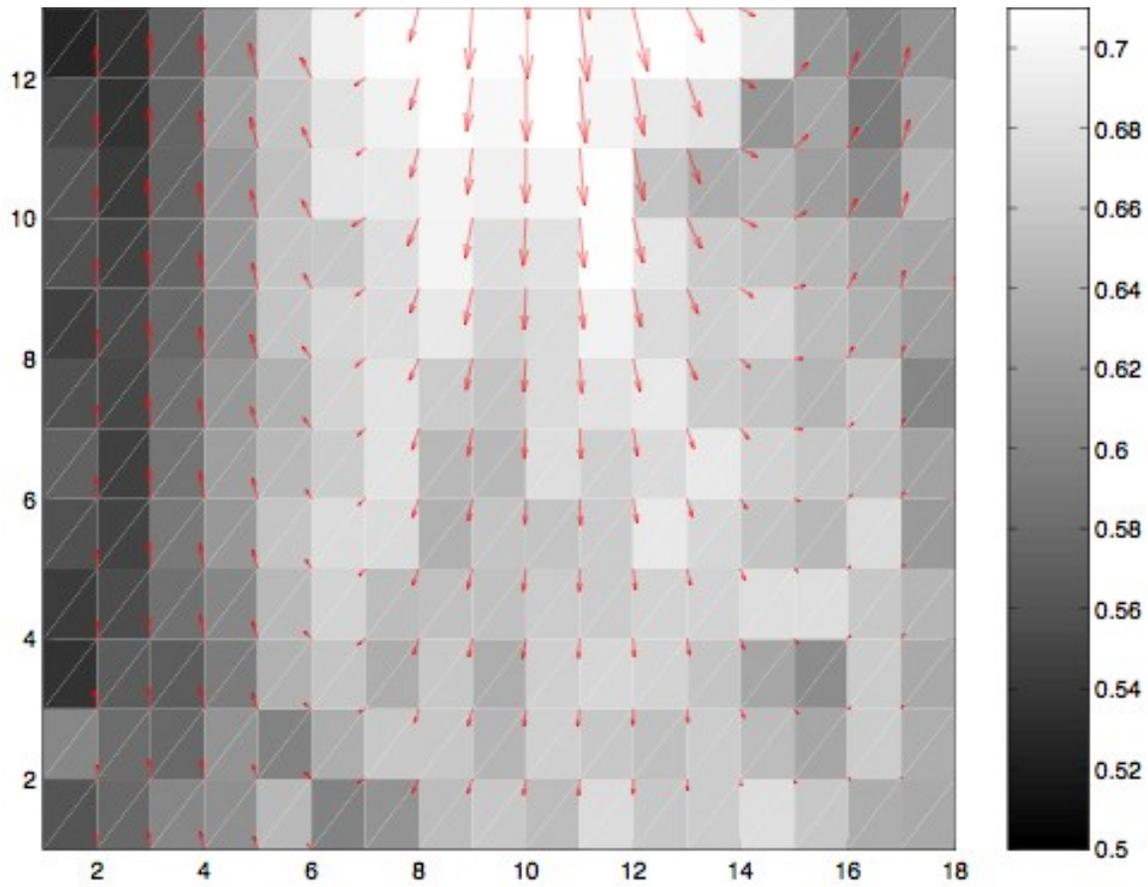
Figure 6: Velocity field (arrows) and density profile (grey levels, in arbitrary units, indicative of the wetness of the foam) obtained for the Hele-Shaw set-up of figure 5 and a point input of surfactant solution. Data was taken once a steady liquid density profile was established. The data is averaged over 20 seconds (spatial average roughly two bubble diameters).