Role of liquid fraction and disorder on macroscopic response

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Dissipative Rheology of Foams, Dublin – january 09th-12th, 2012

Outline

- Role of liquid fraction on :

  - static shear modulus

  - Yield stress and strain

  - Yield drag

  - flow profile of linearly sheared foam

- Role of disorder on :

  - static shear modulus

  - T1 localization and flow profile of linearly sheared foam

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Role of liquid fraction on static shear modulus

Princen & Kiss (1986) $\boxed{G \sim \sigma\, R_{32}^{-1}\, \Phi^{1/3}\, (\Phi - \Phi_c)}$

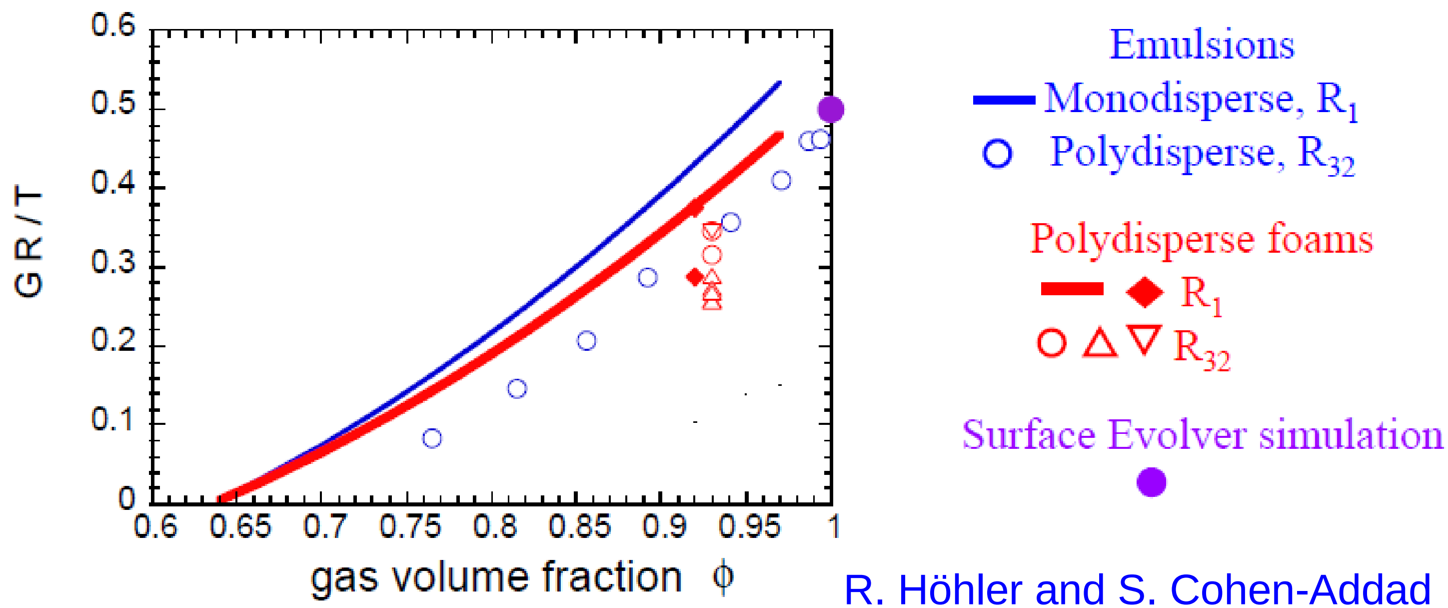
G = shear modulus Φ = gas fraction $R_{32} = \langle R^3 \rangle / \langle R^2 \rangle$ Sauter mean radius

σ = surface tension

$\boxed{G \sim \sigma/R\, \Phi\, (\Phi - \Phi^*)}$

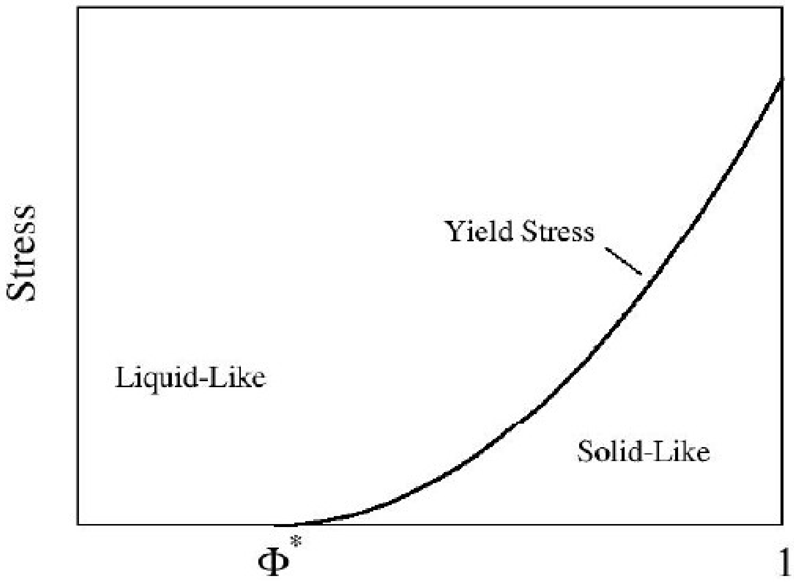
Mason, Bibette & Weitz (1995)

Saint-Jalmes, Durian (1999)

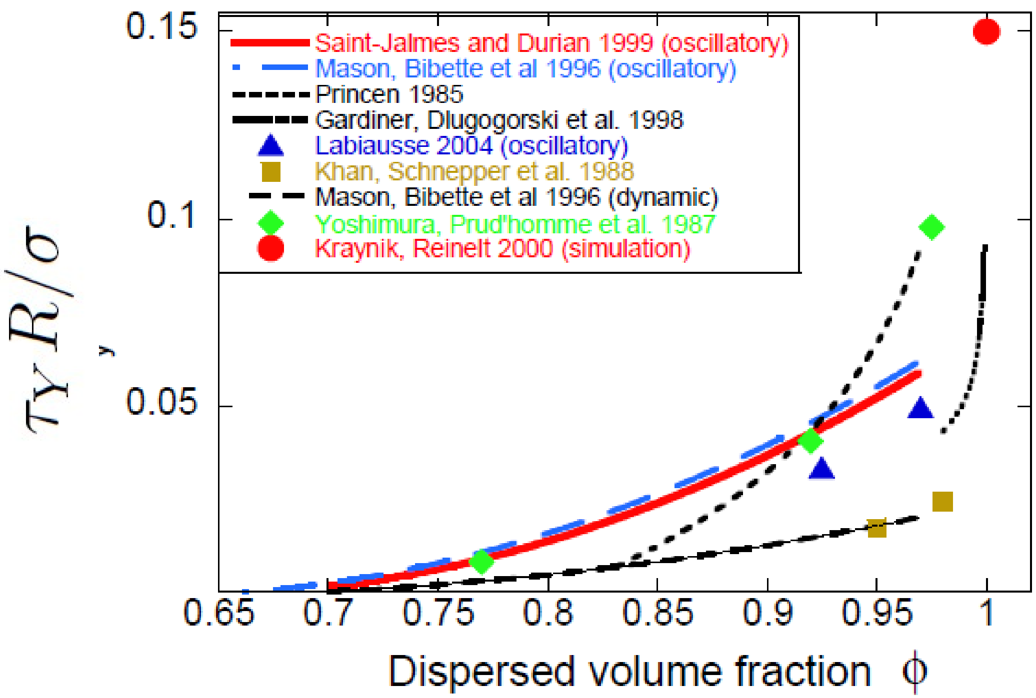


R. Höhler and S. Cohen-Addad
J. Phys. Cond. Matt. (2005)

Role of liquid fraction on Yield stress and strain



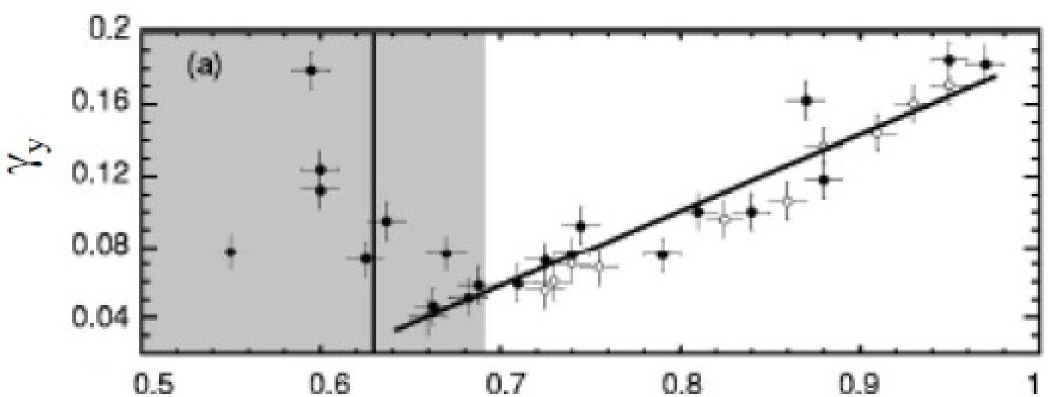
Oscillatory yield stress : $\boxed{\tau_Y \simeq 0.5\frac{\sigma}{R}(\phi-\phi^\star)^2}$



R. Höhler and S. Cohen-Addad (2005)

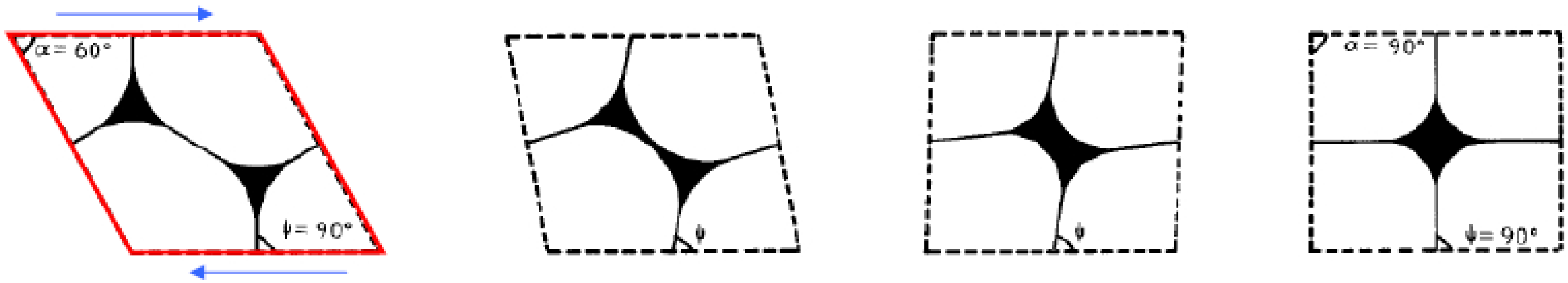
Yield strain : $\boxed{\gamma_Y \simeq 0.5(\phi-\phi^\star)}$

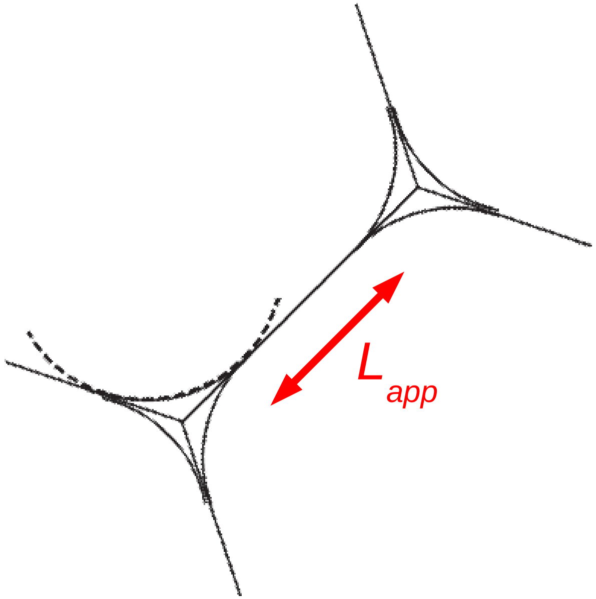
Saint-Jalmes, Durian (1999)

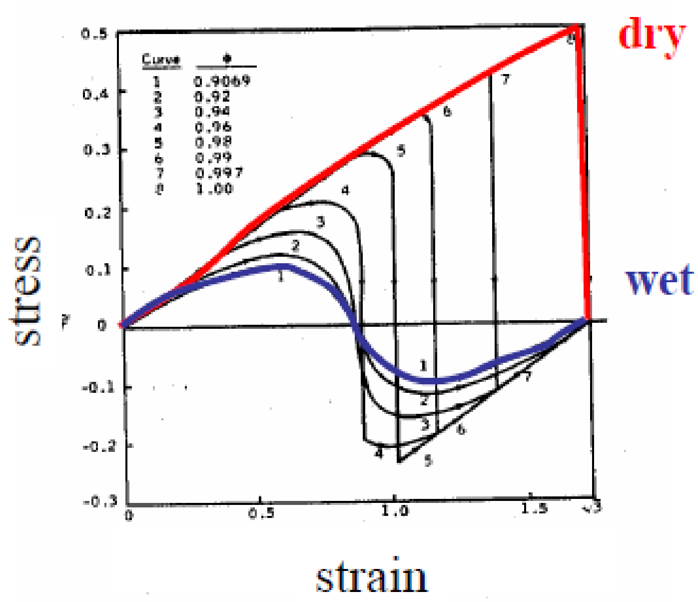


Note: ϕ^* depends on foam disorder !
Use osmotic pressure instead ?
(see R. Höhler's talk)

Why liquid fraction Matters ?







Princen 1983

Role of liquid fraction on Yield drag

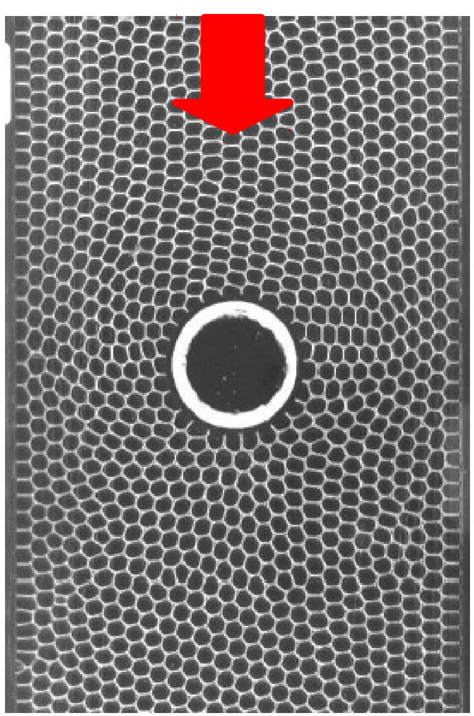
Eur. Phys. J. E **23**, 217–228 (2007)
DOI 10.1140/epje/i2006-10178-9

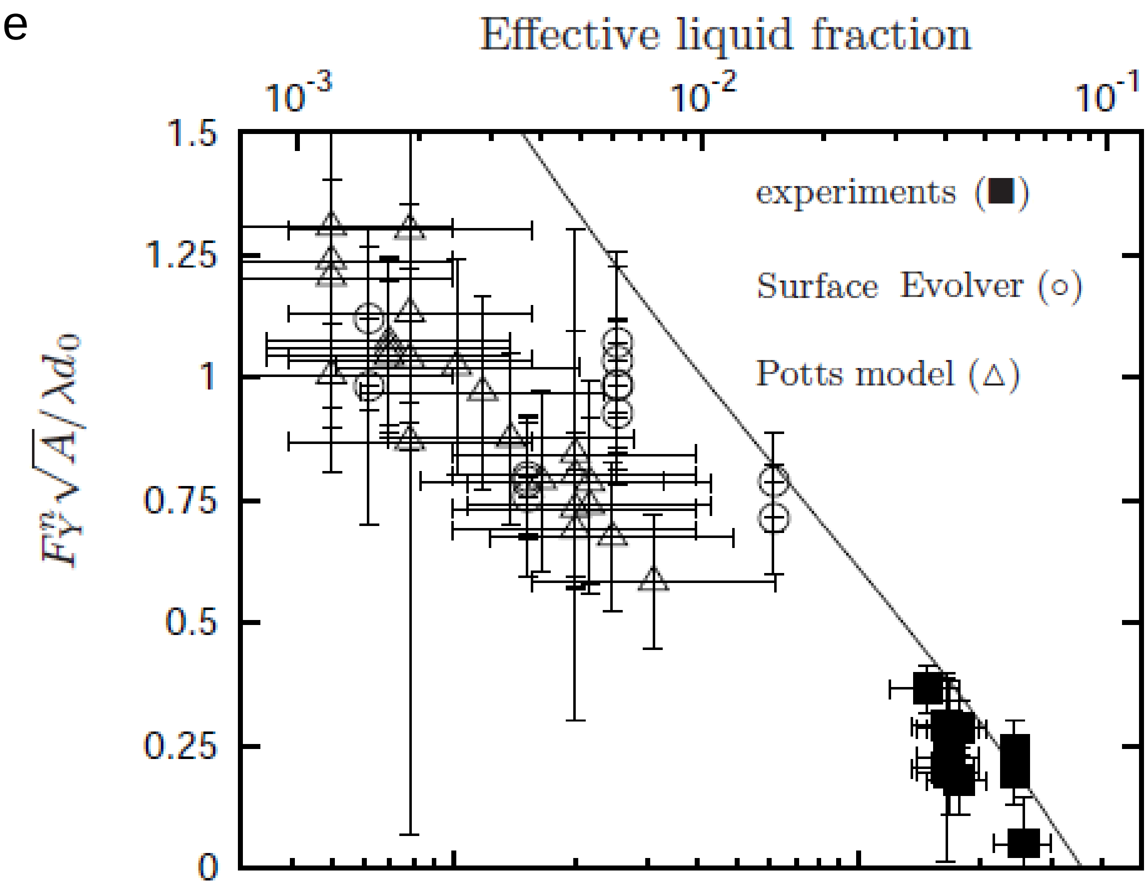
THE EUROPEAN PHYSICAL JOURNAL E

Yield drag in a two-dimensional foam flow around a circular

C. Raufaste[1,a], B. Dollet[1,b], S. Cox[2], Y. Jiang[3], and F. Graner[1]

Yield drag = minimal force required to create a movement of the foam relative to an obstacle

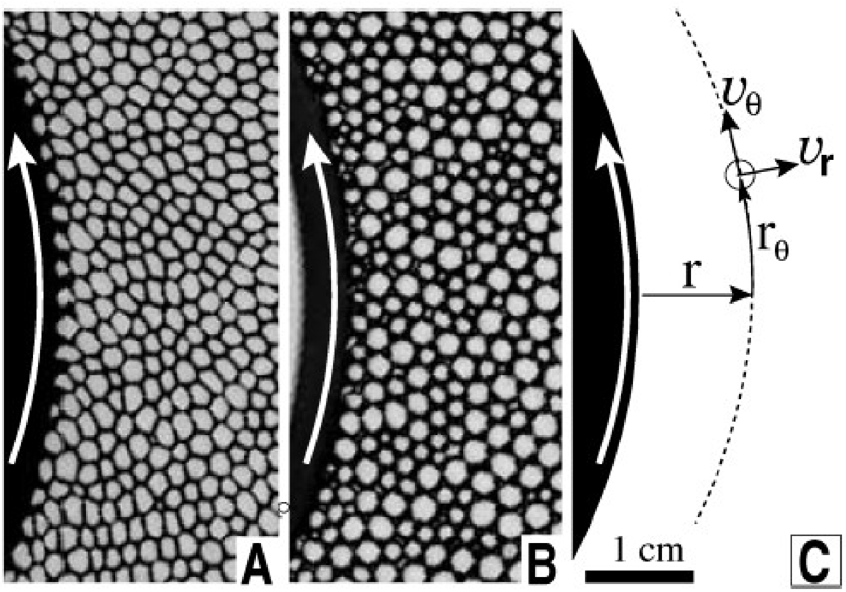




Role of liquid fraction on flow profile

Quasistatic, Couette flow in Hele-Shaw cell

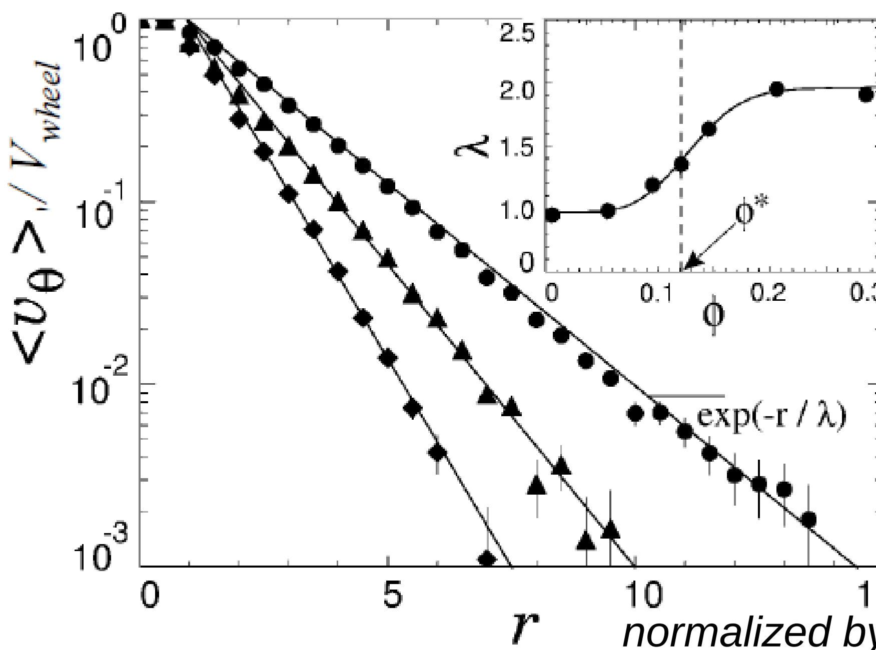
Debregeas, Tabuteau, di Meglio 2001



$$\boxed{\langle v_\theta(r)\rangle \sim \exp(-r/\lambda)}$$

(shear banding)

◆: $\phi = 0.05$; ▲: $\phi = 0.12$; •: $\phi = 0.20$



normalized by mean bubble diameter

Here ϕ is the **liquid** fraction...

Outline

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- Role of disorder on :

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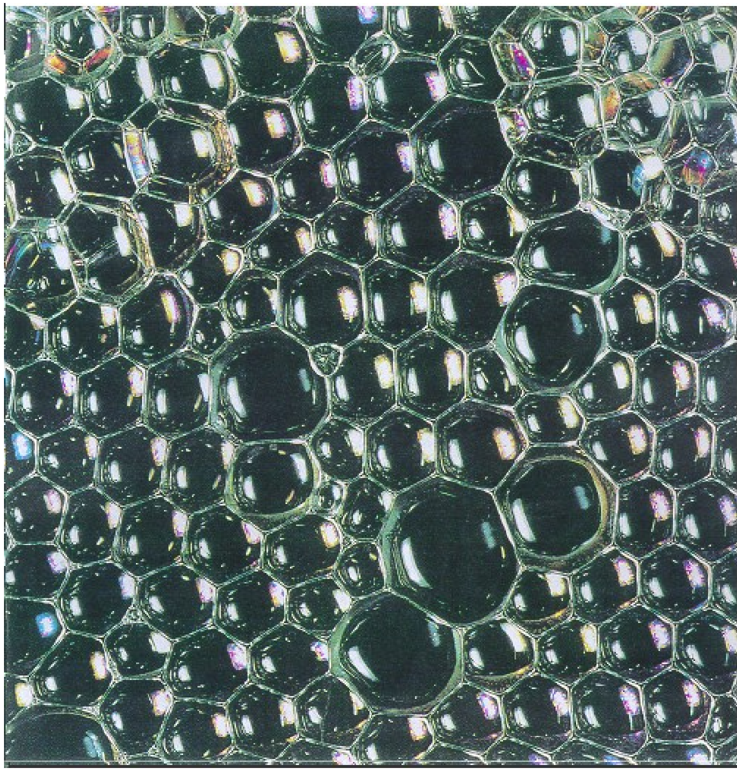
How to quantify disorder(s) ?



2D foams : mean number of sides is fixed : $\langle n \rangle = 6$

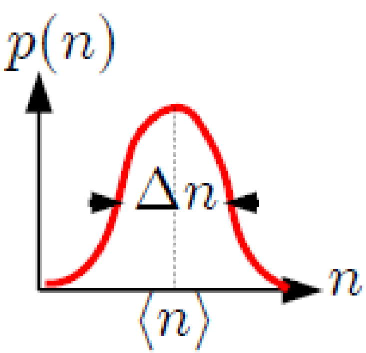
Assumption : Departure from regular (hexagonal) tiling is measured by the second moment of distributions of sides *p*(*n*), areas *p*(*A*), side lengths *p*(*L*),...

→ True for foams with **moderate dispersity** only (exact shape of distribution does not matter)

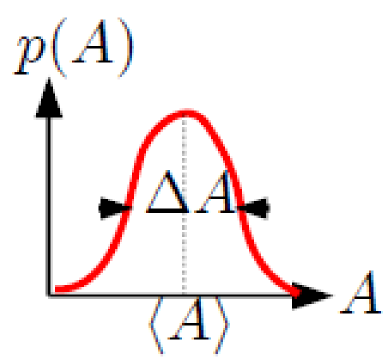
topological disorder : $\mu_2(n) = \sum_n p(n)(n - \langle n \rangle)^2 = \langle n^2 \rangle - \langle n \rangle^2$

geometrical disorders : $\mu_2(A)$ $\mu_2(L)$

To compare different foam samples, use of « normalized » quantities :



$$\frac{\Delta n}{\langle n \rangle} = \frac{\sqrt{\langle n^2 \rangle - \langle n \rangle^2}}{\langle n \rangle}$$

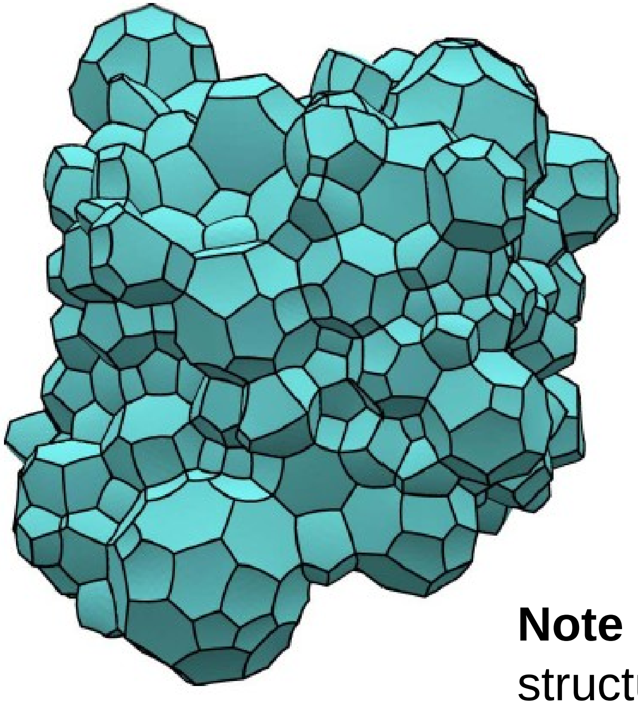


$$\frac{\Delta A}{\langle A \rangle} = \frac{\sqrt{\langle A^2 \rangle - \langle A \rangle^2}}{\langle A \rangle}$$

How to quantify disorder(s) ?

3D foams :

Disorders = second moments of distributions of faces p(f), sides p(n), volumes p(V), areas p(A), side lengths p(L).



topological disorders : $\mu_2(n)$ $\mu_2(f)$

geometrical disorders : $\mu_2(V)$ $\mu_2(A)$ $\mu_2(L)$

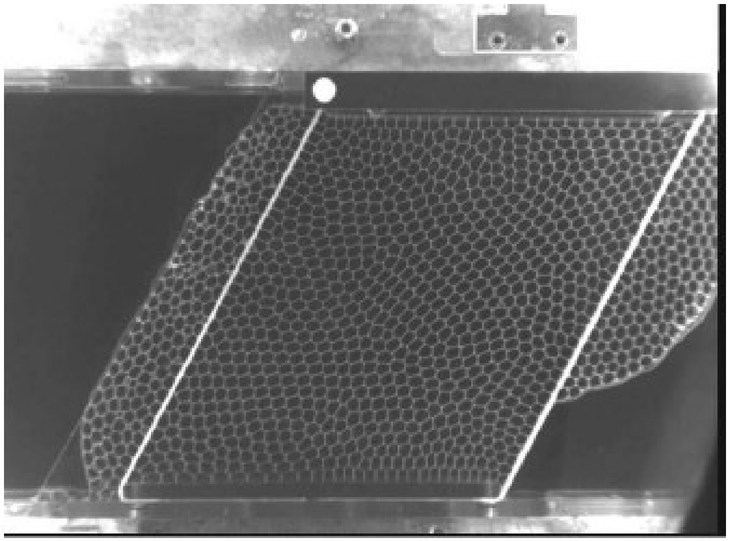
Note : here first moments <*n*> and <*f*> also depend (slightly) on foam structure

$\langle n \rangle \simeq 5$ $\langle f \rangle \simeq 13 - 14$

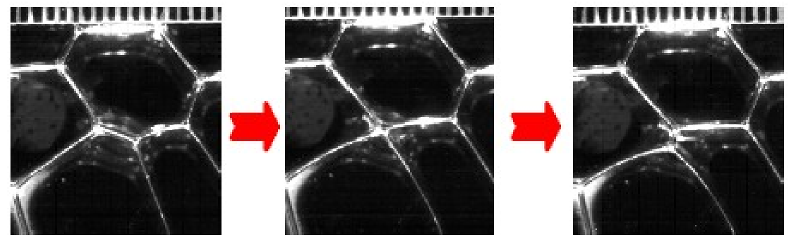
Other measure of geometrical disorder : $p = R_{32}/\langle R^3 \rangle^{\frac{1}{3}} - 1 = \langle R^3 \rangle^{\frac{2}{3}}/\langle R^2 \rangle - 1$

Kraynik, Reinelt 2004

How are related the different measures of disorder ?

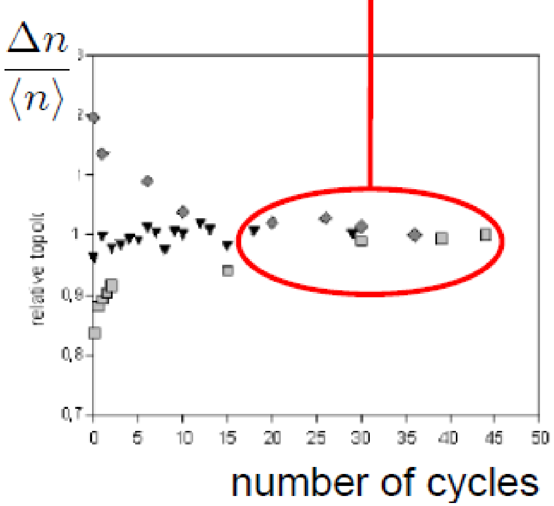


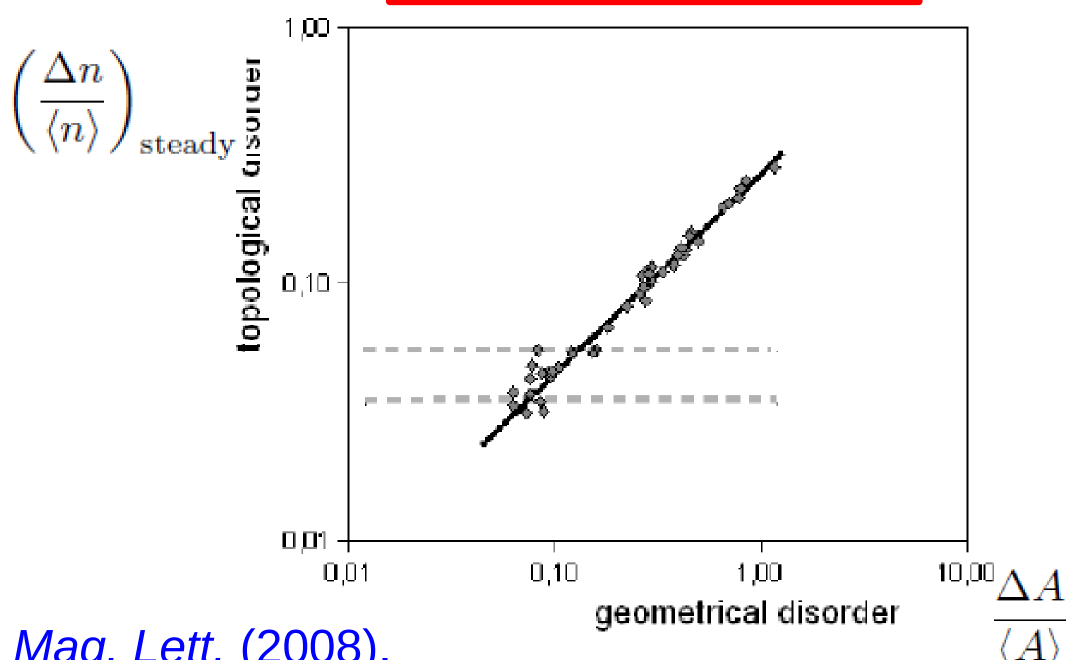
rearrangements in a foam (T1 events):



$$\frac{\Delta n}{\langle n \rangle} = 0.27 \left(\frac{\Delta A}{\langle A \rangle} \right)^{0.8}$$

stationnary macroscopic state



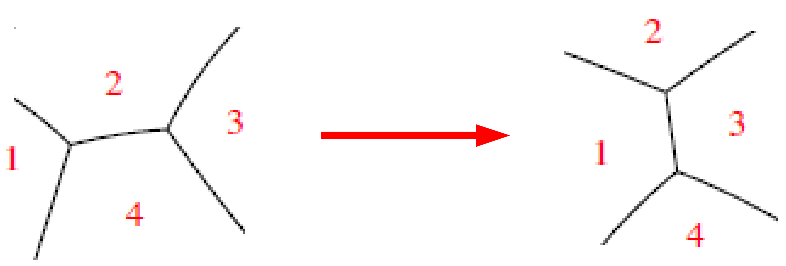


C. Quilliet et al., *Phil. Mag. Lett.* (2008).

Two models for disorder relationship (2D)

Statistical Physics approach

M.Durand *EPL* (2010)
M. Durand, J. Käfer, C. Quilliet, S. Cox, S. Ataei Talebi, F. Graner *PRL* (2011).



Each bubble exchanges sides *n* and curvature κ with rest of foam, such that :

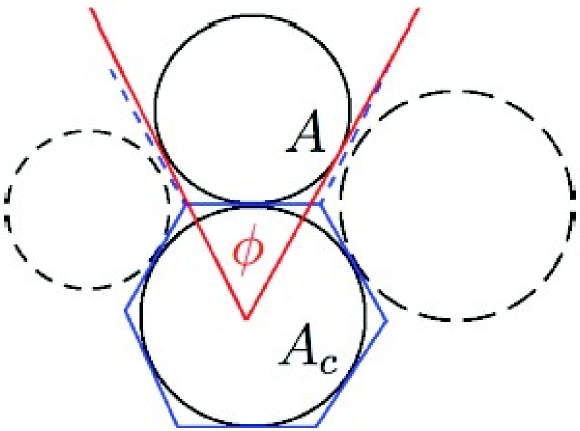
$$n + n_{\text{rest of foam}} = \text{constant} \quad = 6N$$

$$\kappa + \kappa_{\text{rest of foam}} = \text{constant} \quad = 0$$

(large foam)

Granocentric model

M. P. Miklius, S. Hilgenfeldt, *PRL* (2012)



packing of hard discs with steric repulsion

Share common features :

- geometric models (energy does not play explicit role)
- mean field approximation (no correlations)
- no free parameters

See S. Hilgenfeldt and F. Graner's presentation ...

Role of disorder on static shear modulus

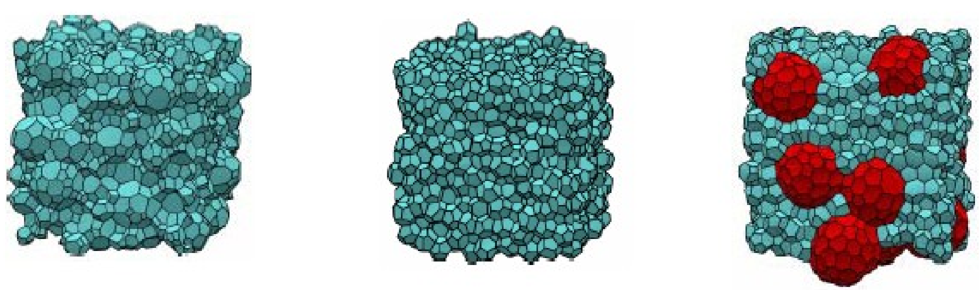
3D foams :

Princen & Kiss (1986) $G \sim \sigma R_{32}^{-1} \Phi^{1/3} (\Phi - \Phi_c)$

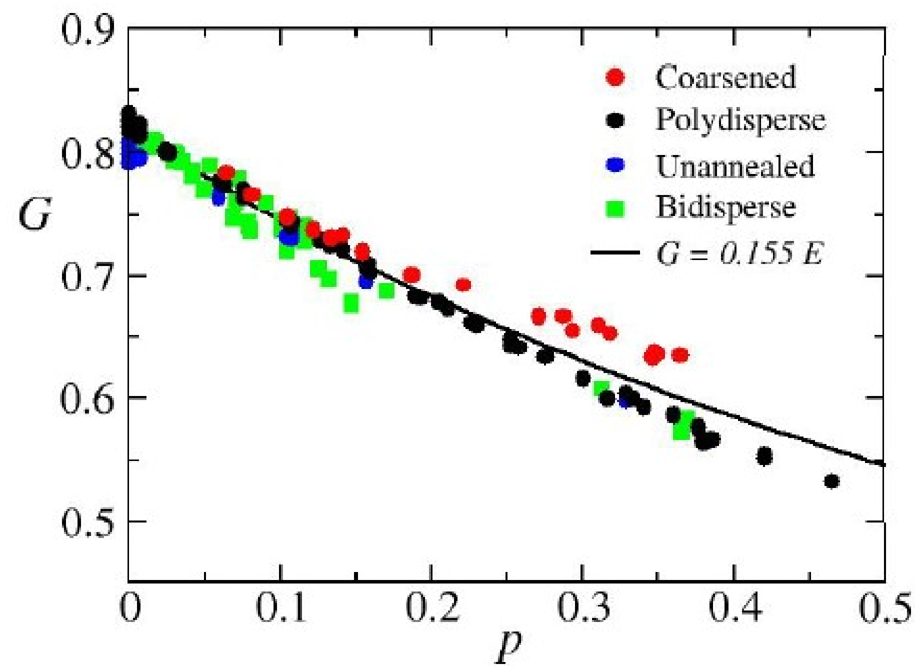
→ "dry" limit (Φ=1) $\boxed{G = 0.51\, \sigma R_{32}^{-1}}$

$R_{32} = \langle R^3 \rangle / \langle R^2 \rangle$ Sauter mean radius

Kraynik, Reinelt 2004



$$p = R_{32}/\langle R^3 \rangle^{\frac{1}{3}} - 1 = \langle R^3 \rangle^{\frac{2}{3}} / \langle R^2 \rangle - 1$$



Role of disorder on static shear modulus

2D foams :

Eur. Phys. J. E **21**, 49–56 (2006)
DOI 10.1140/epje/i2006-10044-x

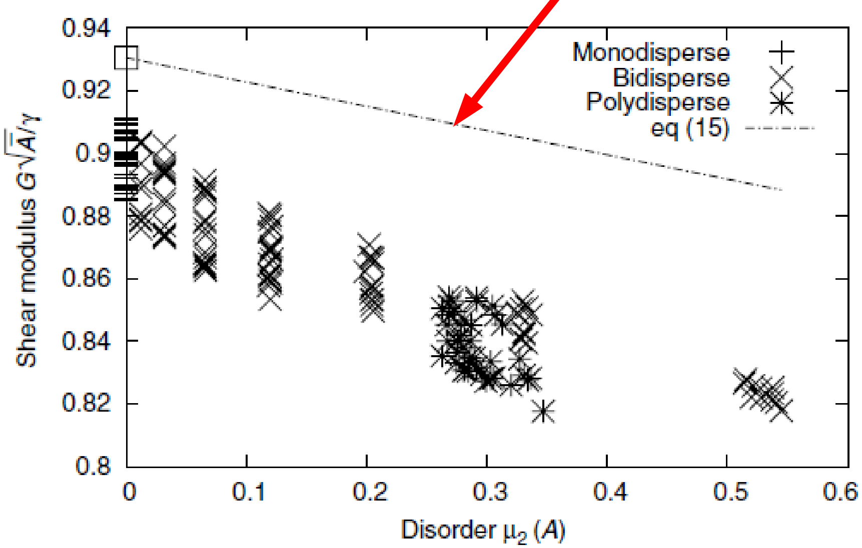
THE EUROPEAN PHYSICAL JOURNAL E

Shear modulus of two-dimensional foams: The effect of area dispersity and disorder

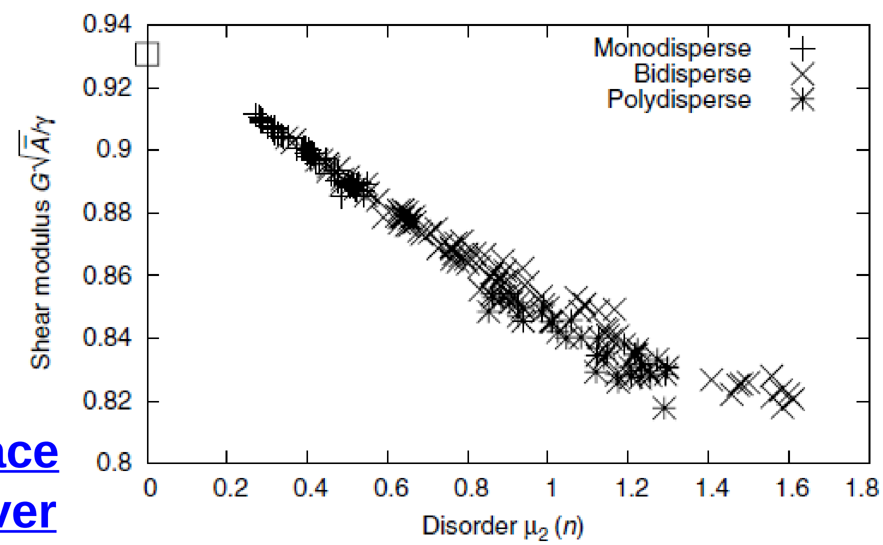
S.J. Cox[a] and E.L. Whittick

$$G \approx \frac{\gamma}{\sqrt{A}} \sqrt[4]{\frac{3}{4}} \left[1 - \frac{1}{12}\mu_2(A)\right]$$

N.P. Kruyt (2006)



Surface Evolver



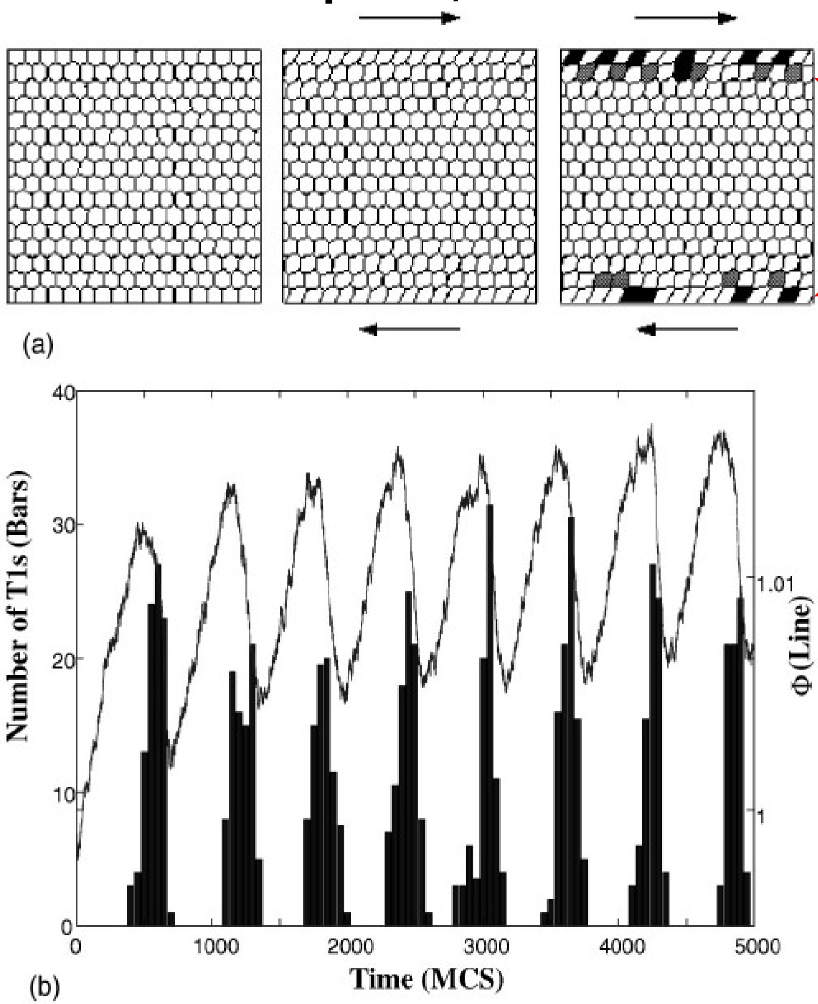
Role of disorder on T1 localization

PHYSICAL REVIEW E VOLUME 59, NUMBER 5 MAY 1999

Hysteresis and avalanches in two-dimensional foam rheology simulations

Yi Jiang,[1,*] Pieter J. Swart,[1] Avadh Saxena,[1] Marius Asipauskas,[2] and James A. Glazier[2]

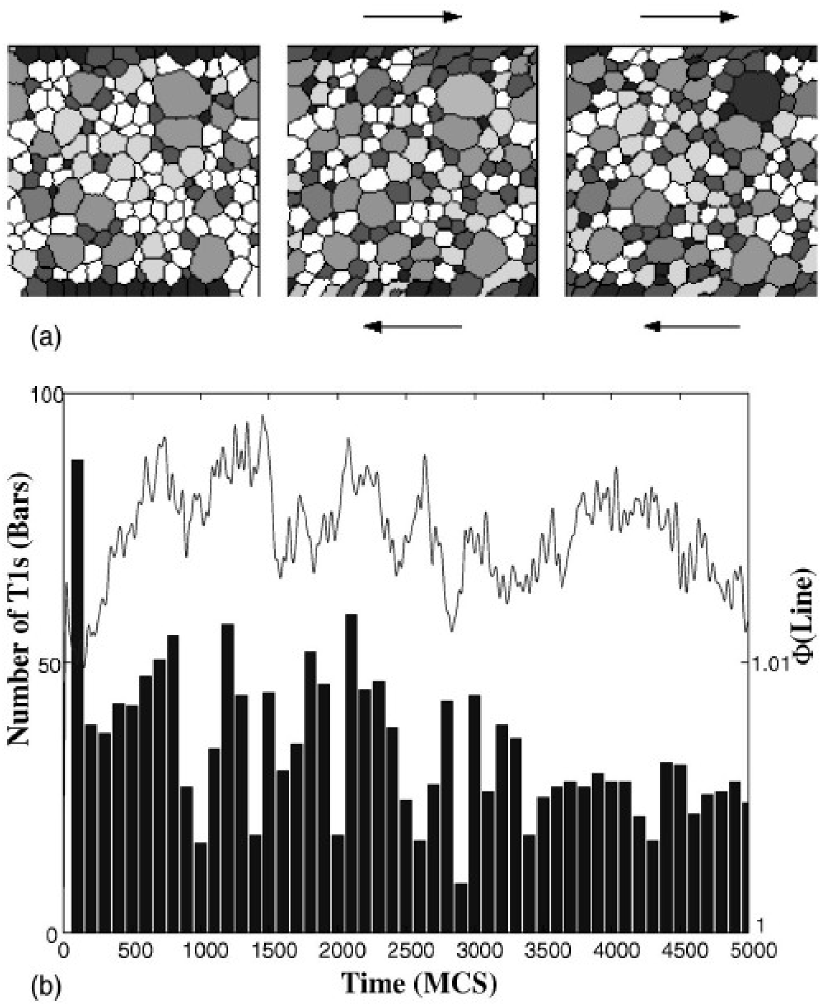
monodisperse, ordered



sliding boundary layers

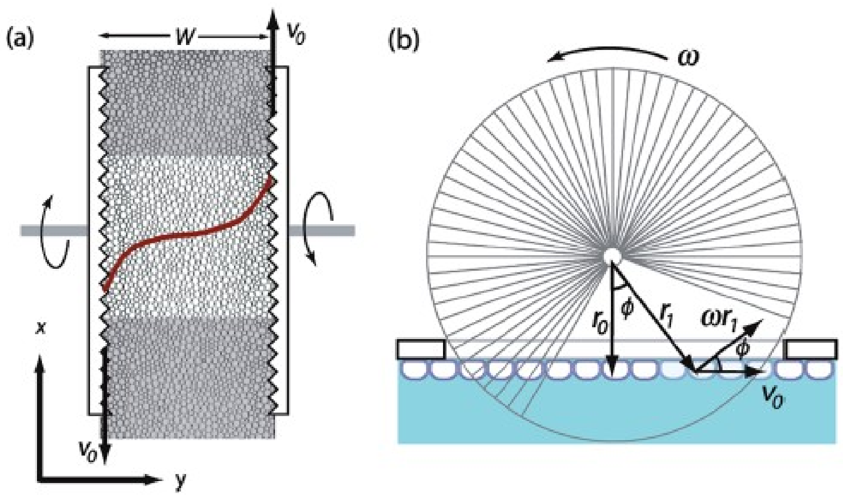
Potts Model

polydisperse



Role of disorder on velocity profile

Linearly-sheared 2D foam



Balance of drag forces between bubble-bubble and bubble-top plate

G. Katgert, M. Möbius, and M. van Hecke (2008)

E. Janiaud, S. Hutzler, and D. Weaire (2006)

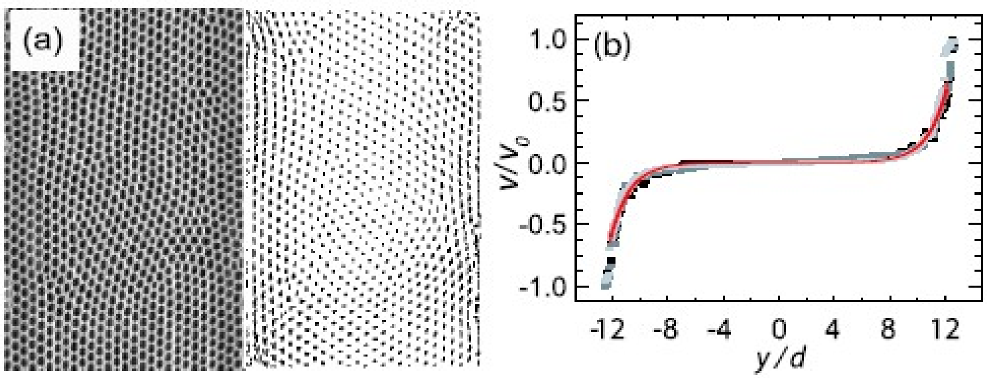
See also Y. Wang, K. Krishan, and M. Dennin (2006)

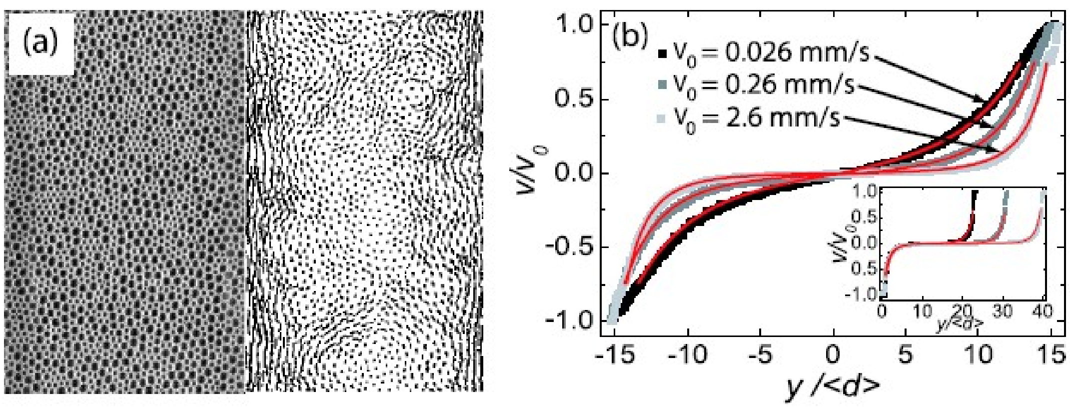
monodisperse

- flow profile independent of shear rate
- localized, sliding boundary layers

bidisperse

- flow profile depends on shear rate
- becomes shear banded as shear rate ↑



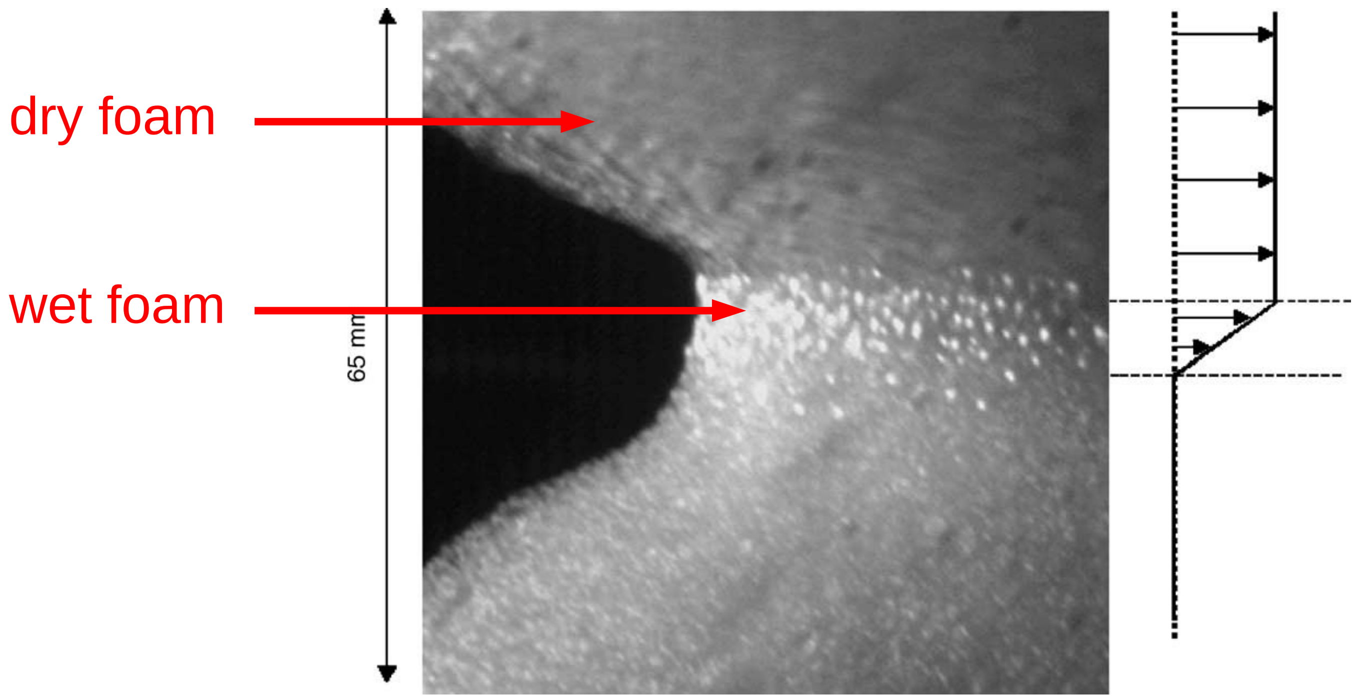


G. Katgert, M. Möbius, and M. van Hecke (2008)

Perspectives – open questions

- Interplay between rheology and drainage

- Interplay between rheology and liquid diffusion



S.P.L. Marze, A. Saint-Jalmes, D. Langevin (2005)

- Models for effect of disorder(s) on mechanical response of a foam.

Statiscal Physics approach

$$\boxed{p_A(n) = \chi(A)^{-1} \exp(-0.28\beta \frac{n(n-6)}{\sqrt{A}} + \mu n)}$$

where effective « temperature » and « chemical potential » are related to the shape of area distribution.

For moderate dispersities :

$$\boxed{\beta^{-1} \simeq 5.06 \frac{\langle A^{1/2}\rangle\langle A^{-1/2}\rangle - 1}{\langle A^{1/2}\rangle}}$$

$$\boxed{\mu' \simeq \frac{1.69}{\langle A^{1/2}\rangle}}$$

correlates geometrical disorder ($p(A)$) and topological disorder ($p(n)$):

$$p(n) = \int_0^\infty p(A) p_A(n) dA$$

Statistical Physics approach

For **moderate** dispersities, *i.e.* $(\Delta A/\langle A\rangle)^2 \ll 4$

$$\boxed{\bar{n}(A) \simeq 3\left(1+\frac{\sqrt{A}}{\langle\sqrt{A}\rangle}\right)}$$

$$\boxed{\frac{\Delta n}{\langle n\rangle} \approx \frac{1}{2^{3/2}}\frac{\Delta A}{\langle A\rangle} \approx 0.35\frac{\Delta A}{\langle A\rangle}}$$