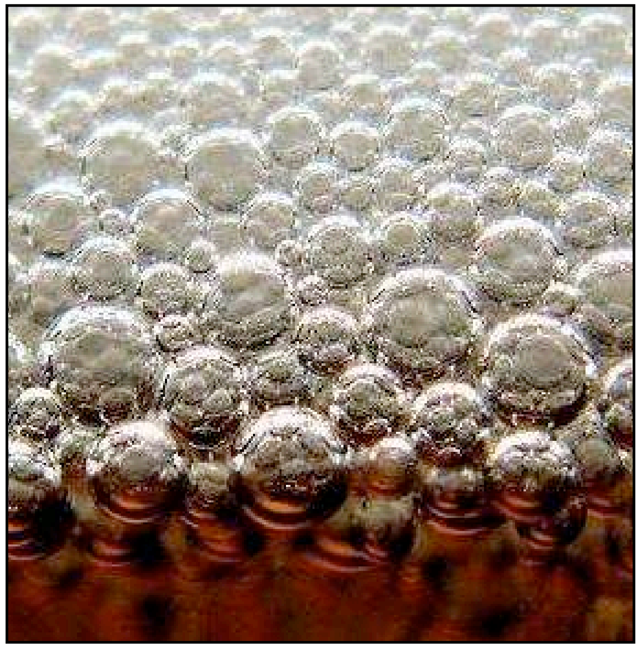
Foams and granular media

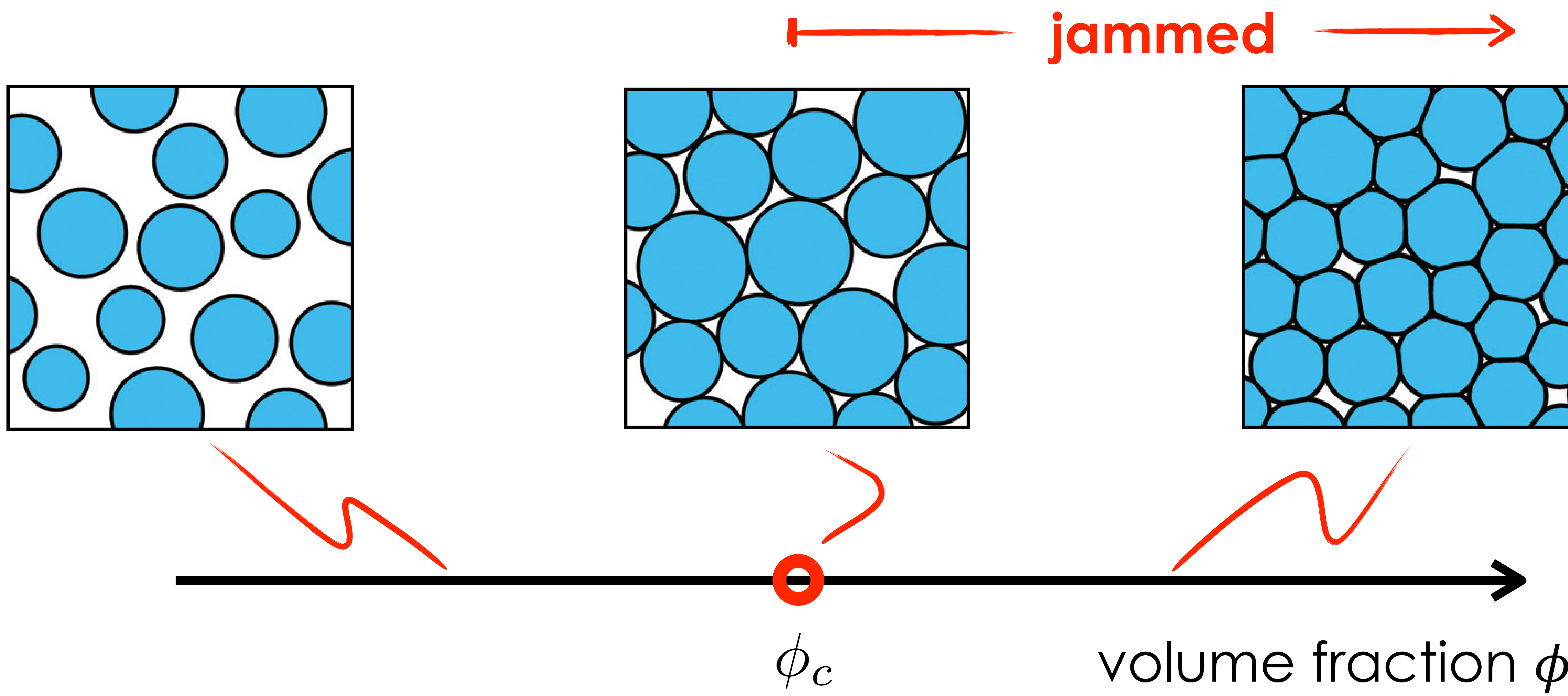
Brian Tighe



wet foams

dense grains

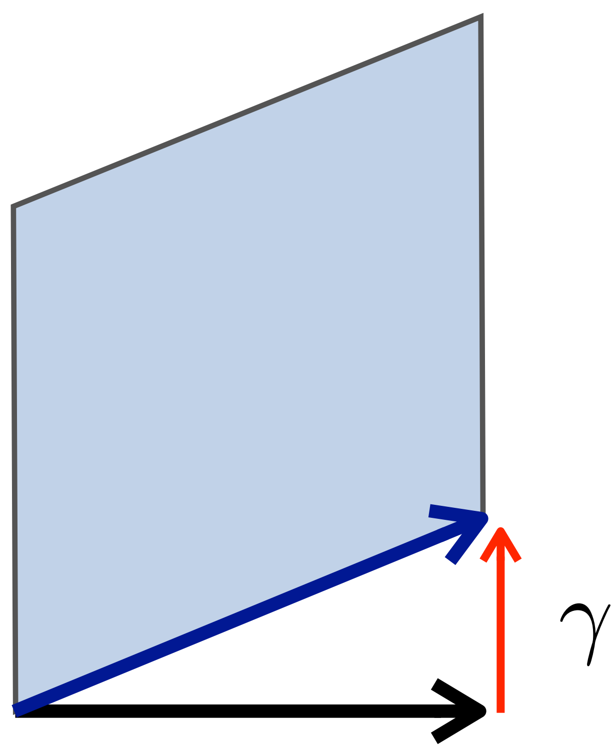
Soft repulsive spheres



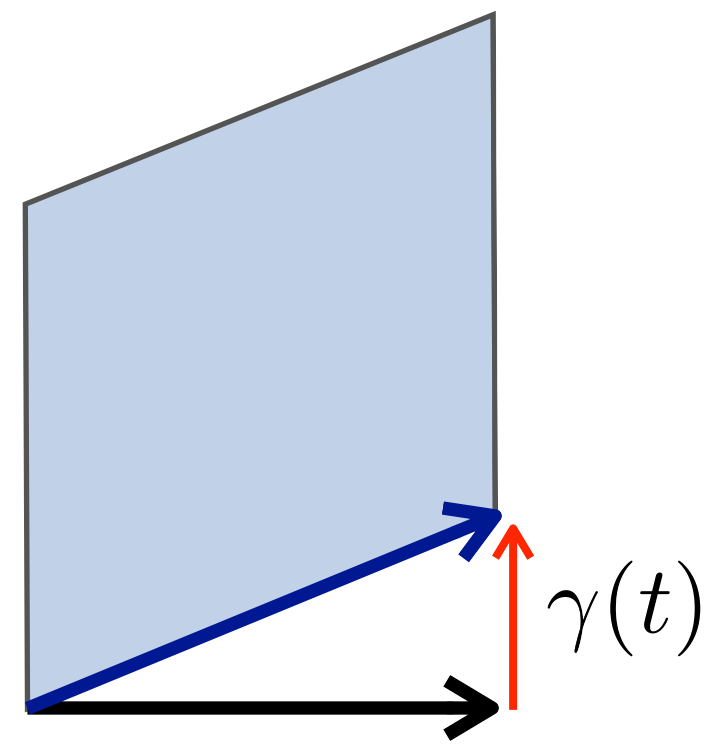
Review articles
Van Hecke, J Phys Cond Matt 2010
Liu & Nagel, Ann Rev Cond Matt Phys 2010
Liu, Nagel, Van Saarloos & Wyart 2010

Bolton & Weaire 1990
Durian PRL 1995
Liu &Nagel Nature 1998
O'Hern et al PRE 2003

How does jamming differ for (overdamped, viscous, wet) foams and (massive, frictional) grains?



quasistatic

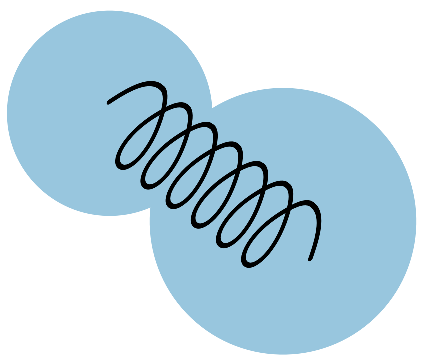


time dependent

(oscillatory)

Microscopic interactions

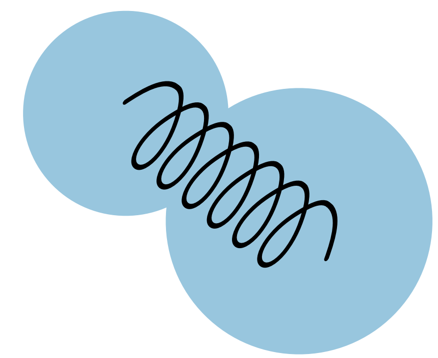
wet foams



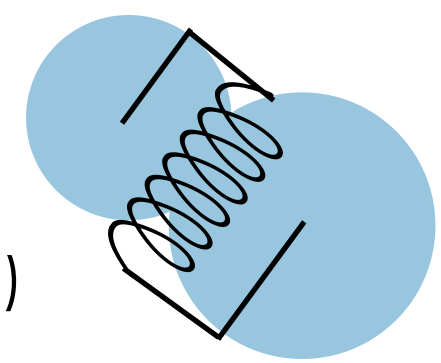
normal
elastic
force

grains

normal
elastic
force



tangential
elastic
force
(Hertz-Mindlin)



$$|f_t| \leq \mu f_n$$

Quasistatic response

Foams are isostatic at zero pressure

Grains need not be

Isostaticity: No static friction

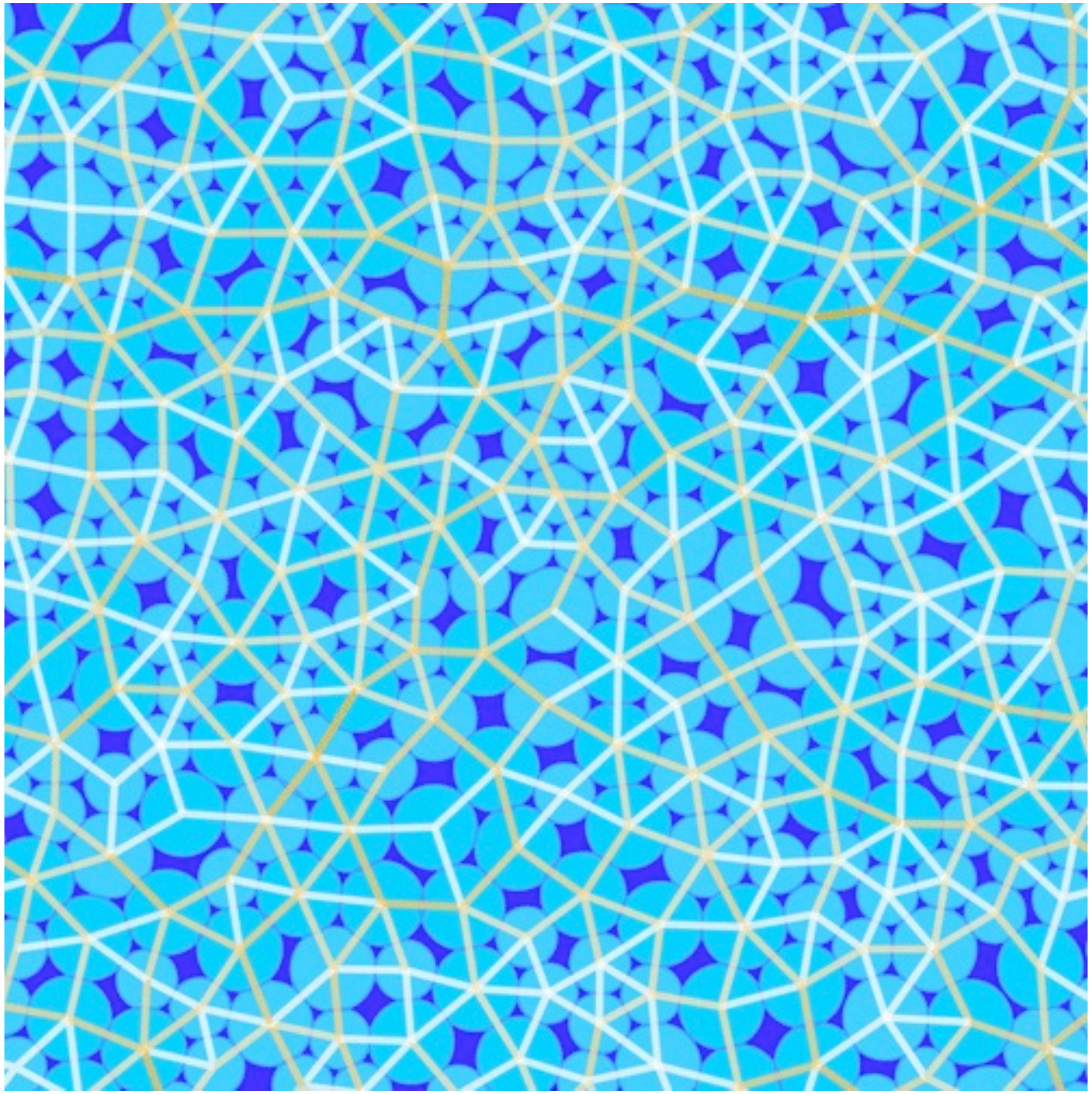


image: CS O'Hern

$$\sum_j \vec{f}_{ij} = 0$$

z ≥ 2d contacts per particle

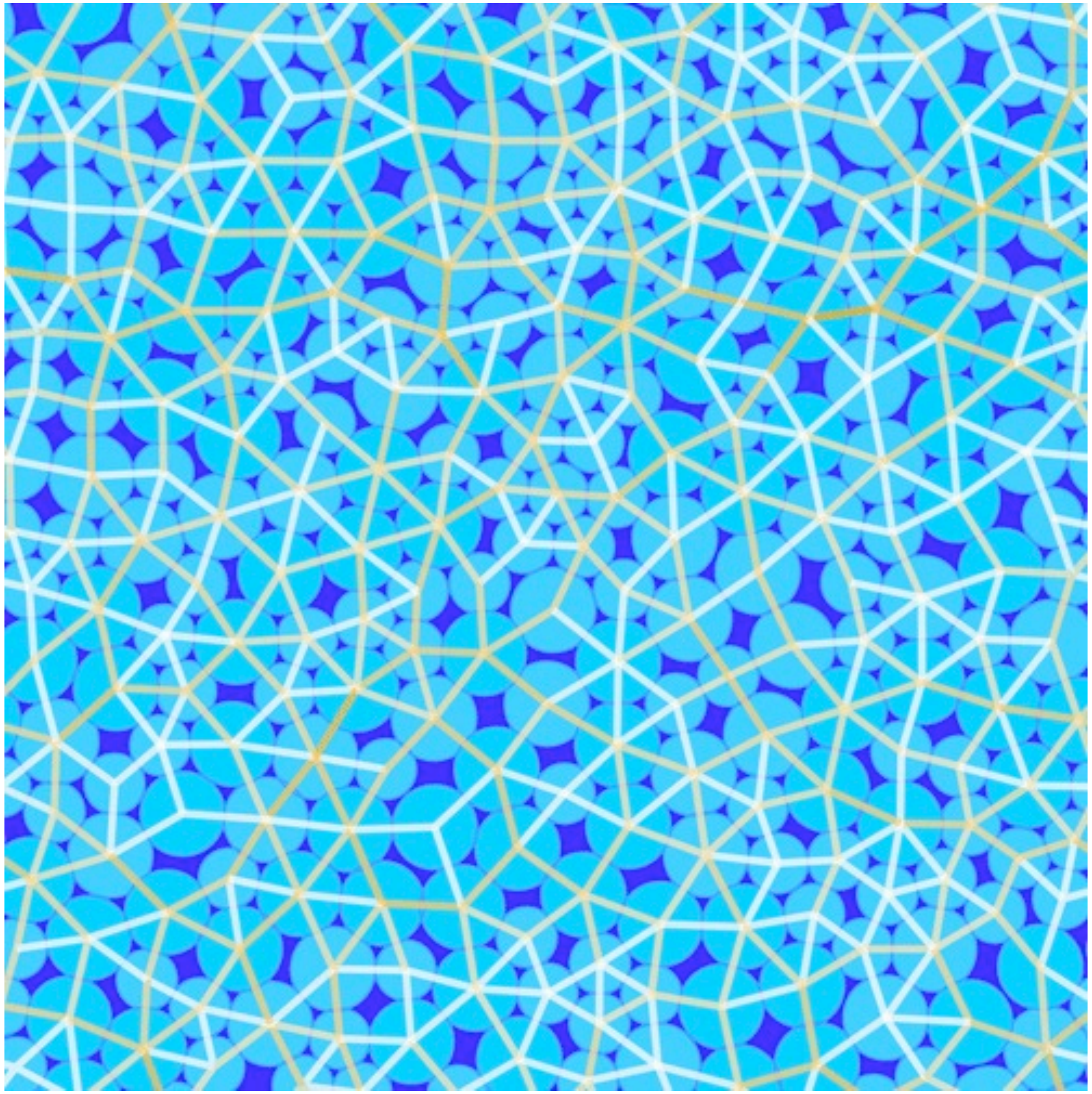
$$|\vec{r}_i - \vec{r}_j| = R_i + R_j$$

z ≤ 2d contacts per particle

thus $z_c = 2d$

Isostaticity: No static friction

$$z - z_c \sim (\phi - \phi_c)^{1/2}$$



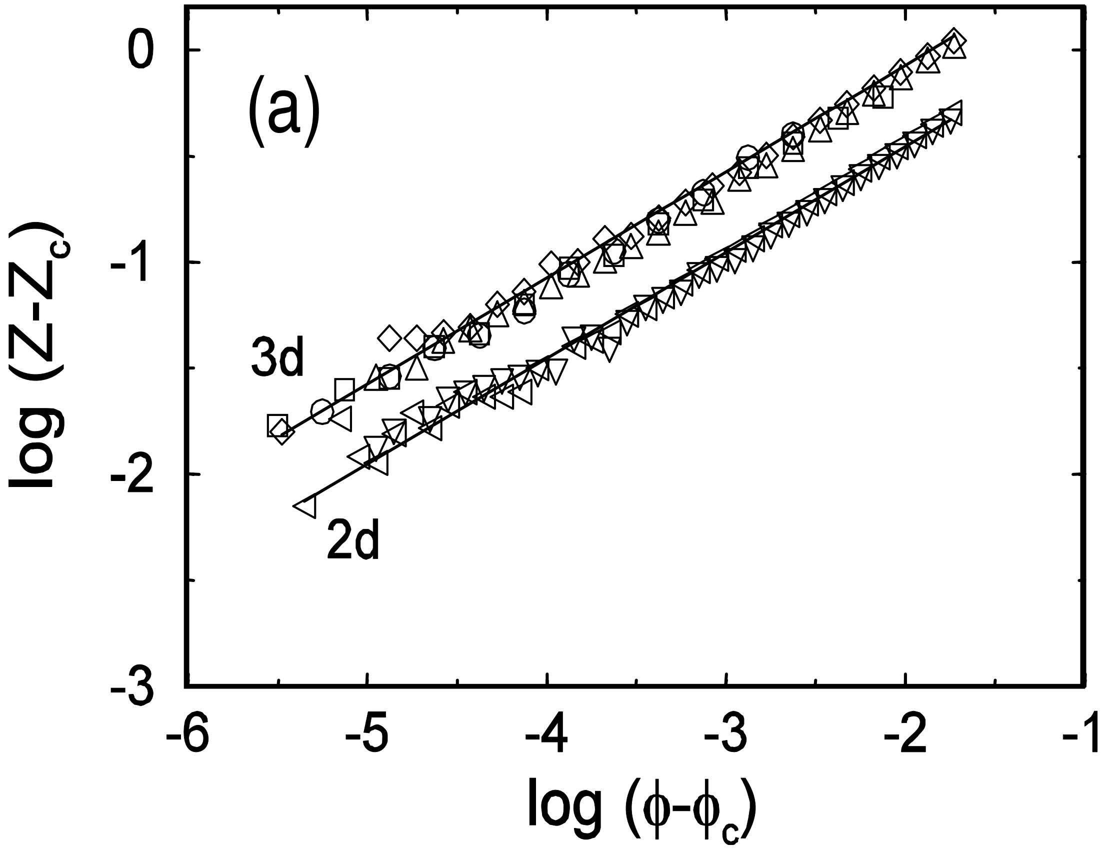


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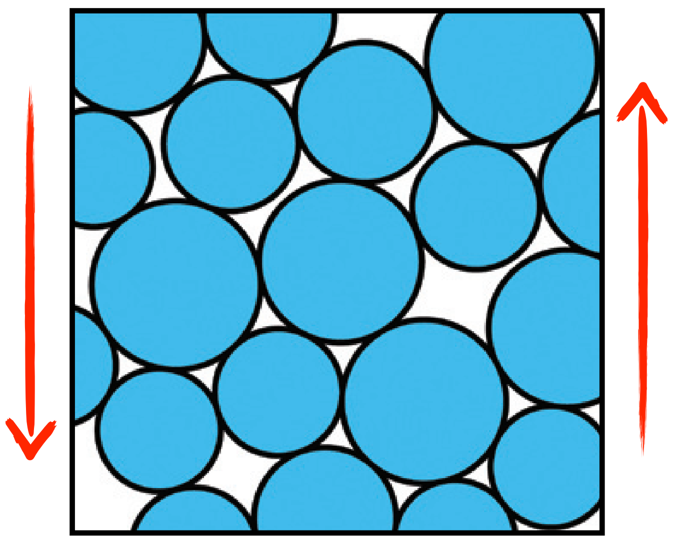
image: CS O'Hern

O'Hern et al., PRE 2003

Quasistatic response



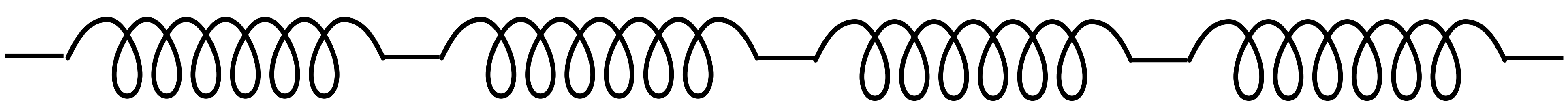
shear stress

$$\hat{K}|u\rangle = \sigma\, L^d |\hat{\gamma}\rangle$$

displacements
+ strain γ

Hessian $K_{ij} = \dfrac{\partial^2 V}{\partial u_i \partial u_j}$ elastic potential energy

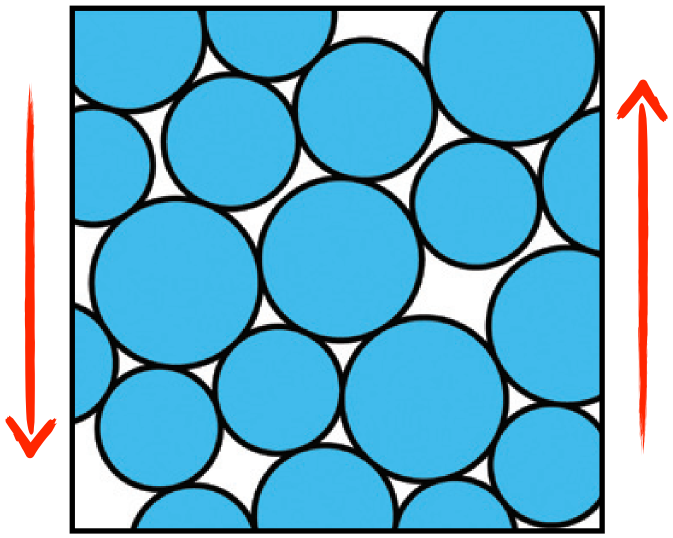
k_n k_{n+1}



$k_n \propto \omega_n^2$ = eigenvalue of the Hessian

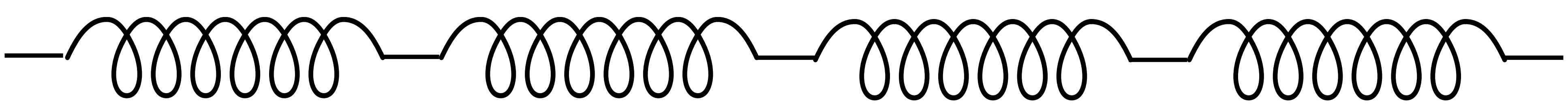
Tighe, PRL 2011

Quasistatic response



$$\frac{1}{G} \propto \sum_n \frac{1}{\omega_n^2}$$

what is the spectrum of eigenvalues?
what is the density of states?



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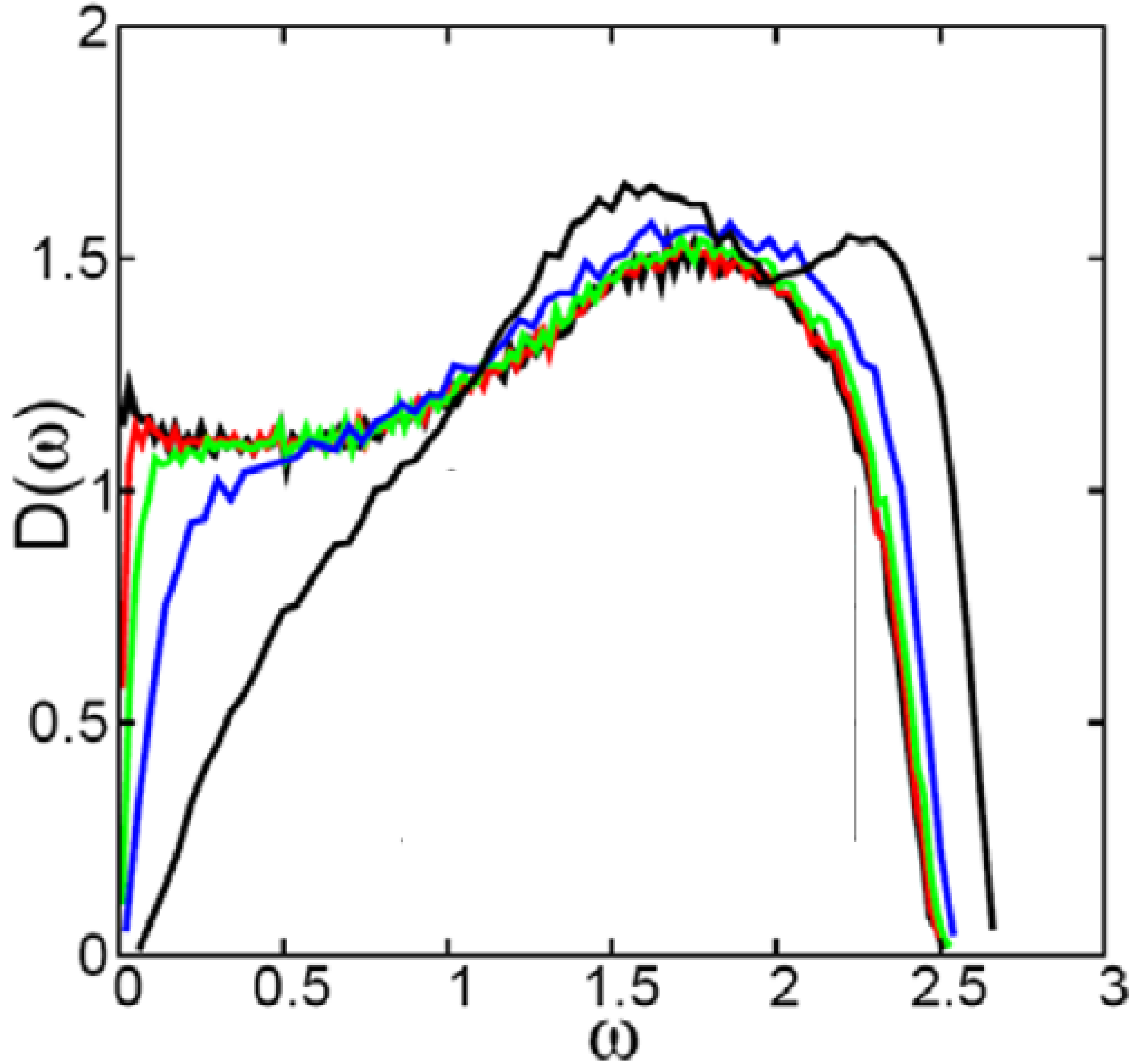
Tighe, PRL 2011

Density of states

density of states D(ω)

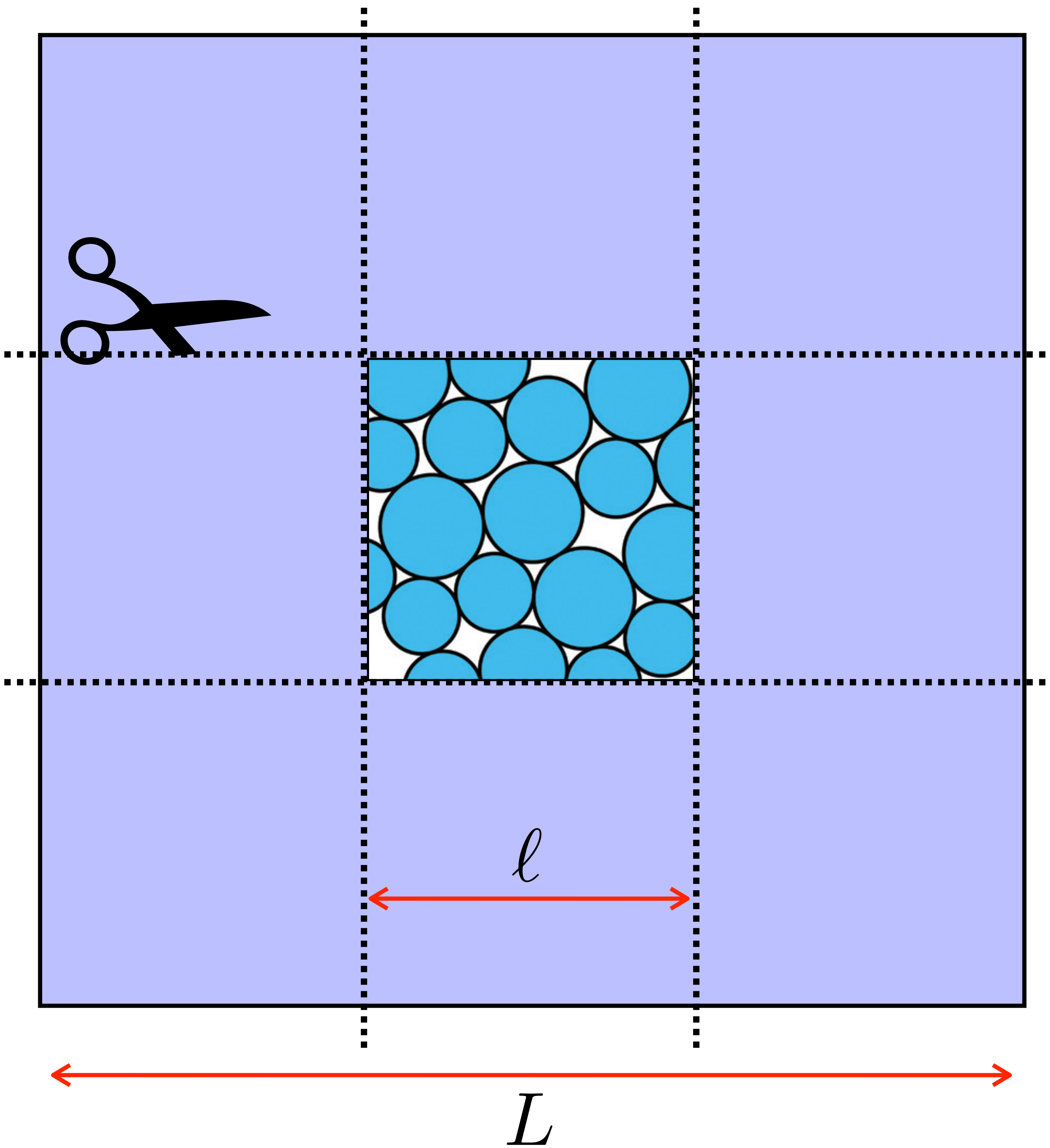
plateau at low ω

crossover frequency



Silbert et al., PRL 2005

"Cutting argument"



excess contacts ~ volume

$$N_{\mathrm{b}} \sim (z_{\mathrm{c}} + \Delta z) L^d$$

cut contacts ~ surface area

$$N_{\mathrm{c}} \sim L^d / \ell$$

create zero modes if

$$\ell < \ell^* \sim 1/\Delta z$$

Density of states

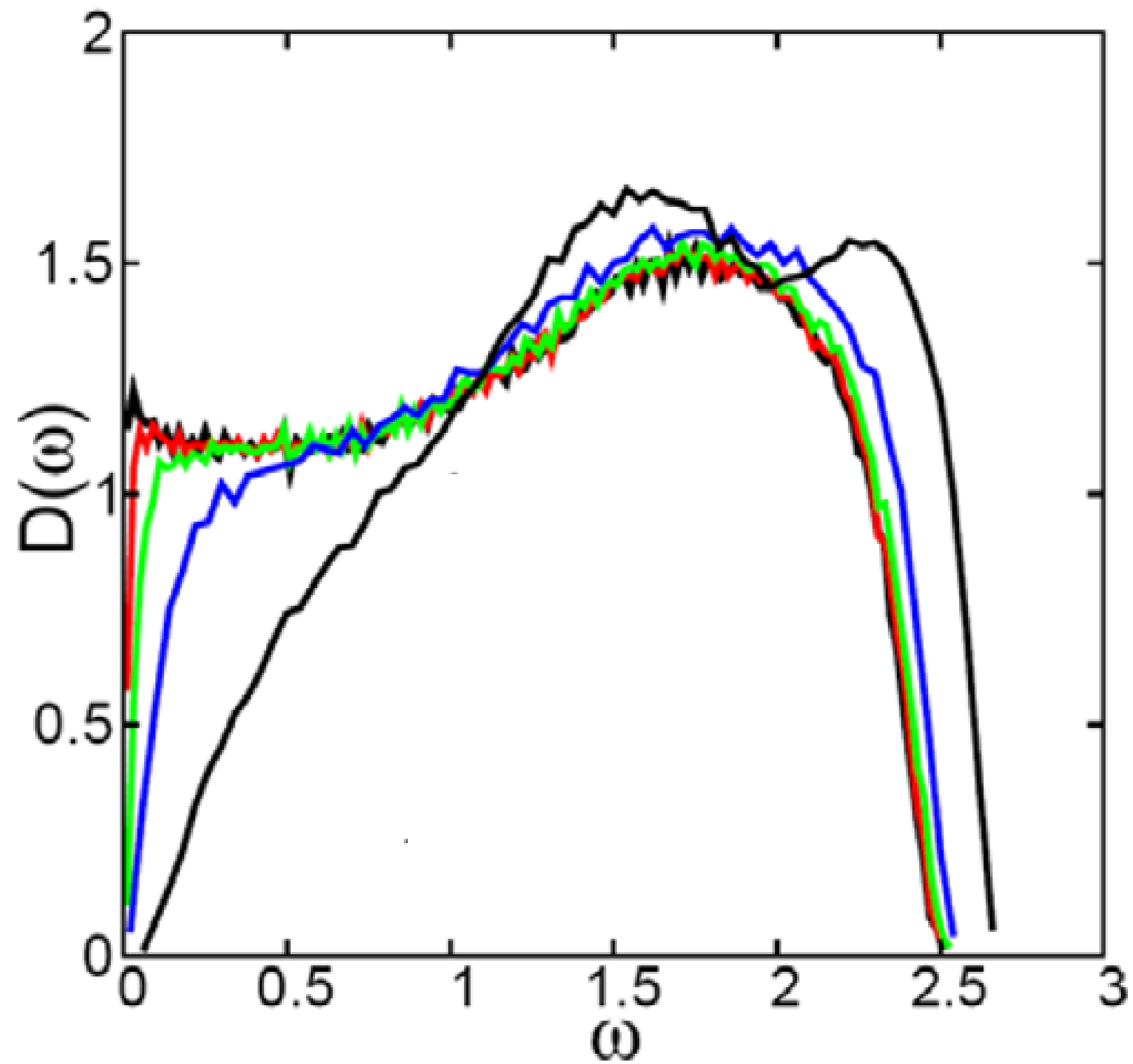
density of states D(ω)

plateau at low ω

crossover frequency

$$\omega^* \sim k^{1/2}\,\Delta z$$

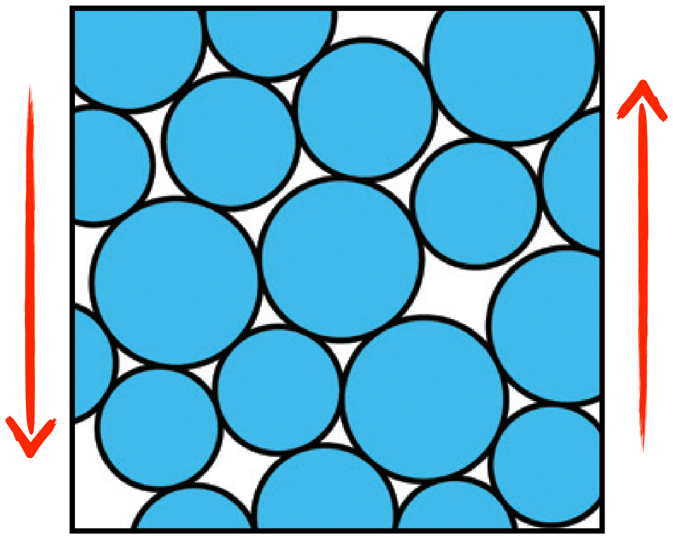
vanishes at z_c



Silbert et al., PRL 2005

Wyart et al., EPL 2006

Quasistatic response



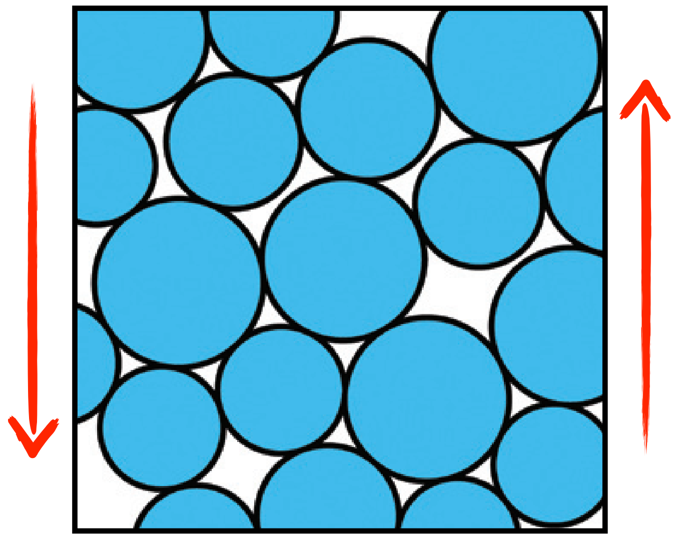
$$\frac{1}{G} \propto \sum_n \frac{1}{\omega_n^2}$$

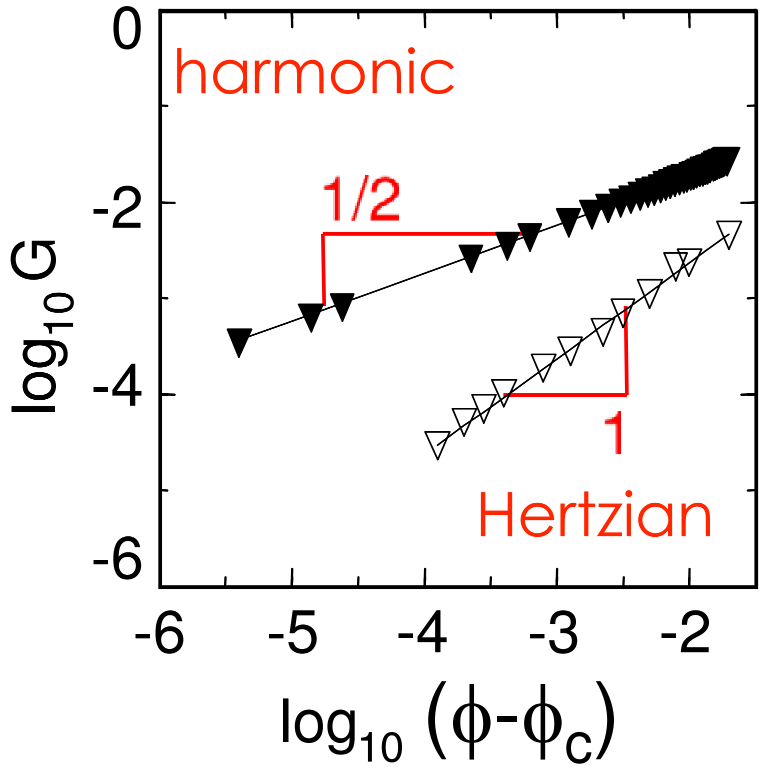
$$\frac{1}{G} \propto \int \frac{D(\omega)}{\omega^2} \mathrm{d}\omega \sim \frac{1}{k^{1/2}\,\omega^*} \sim \frac{1}{k\,\Delta z}$$

shear modulus vanishes at isostatic point!

Tighe, PRL 2011

Quasistatic response





O'Hern et al., PRE 2003

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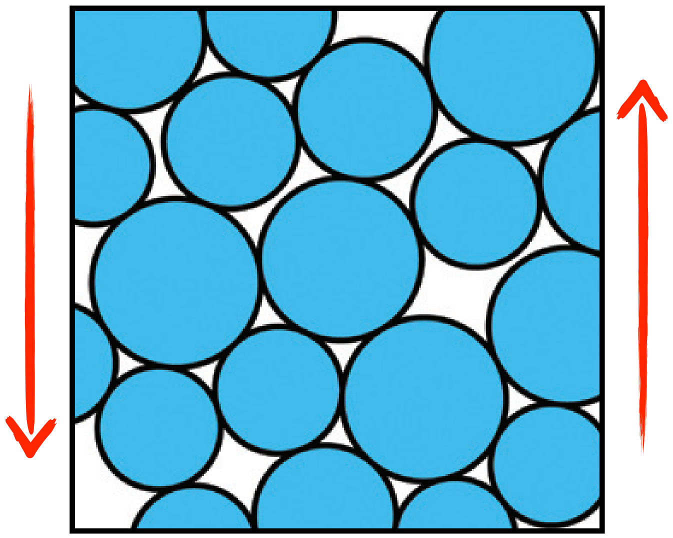
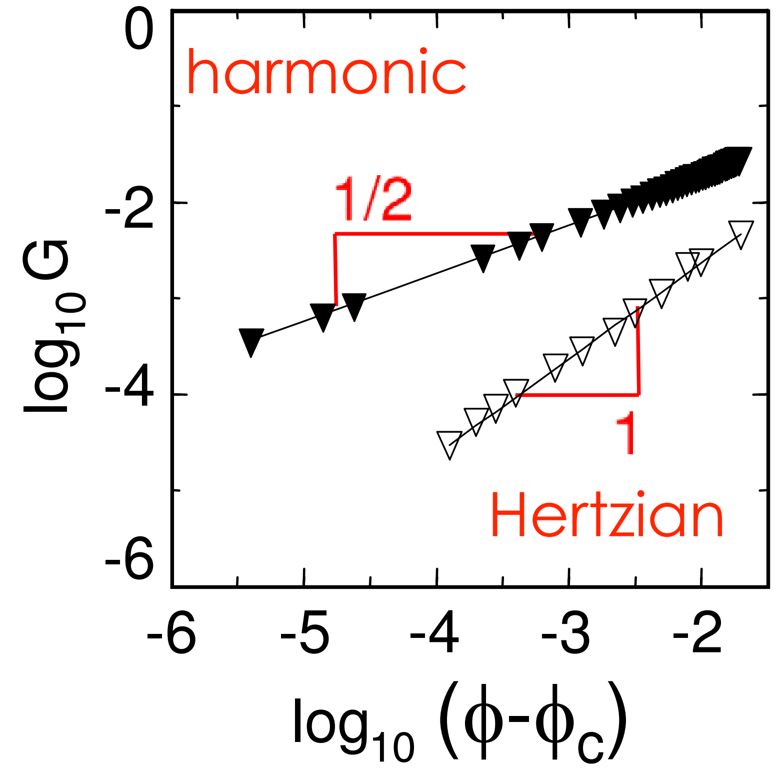
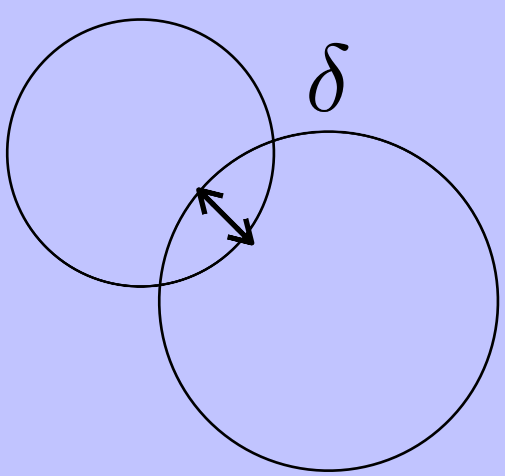




O'Hern et al.,
PRE 2003



$$V \sim \delta^{\alpha_e}$$

$$k \sim V''$$

$$\delta \sim \Delta\phi$$

$$\sim \frac{1}{k\,\Delta z}$$

tic point!

Tighe, PRL 2011

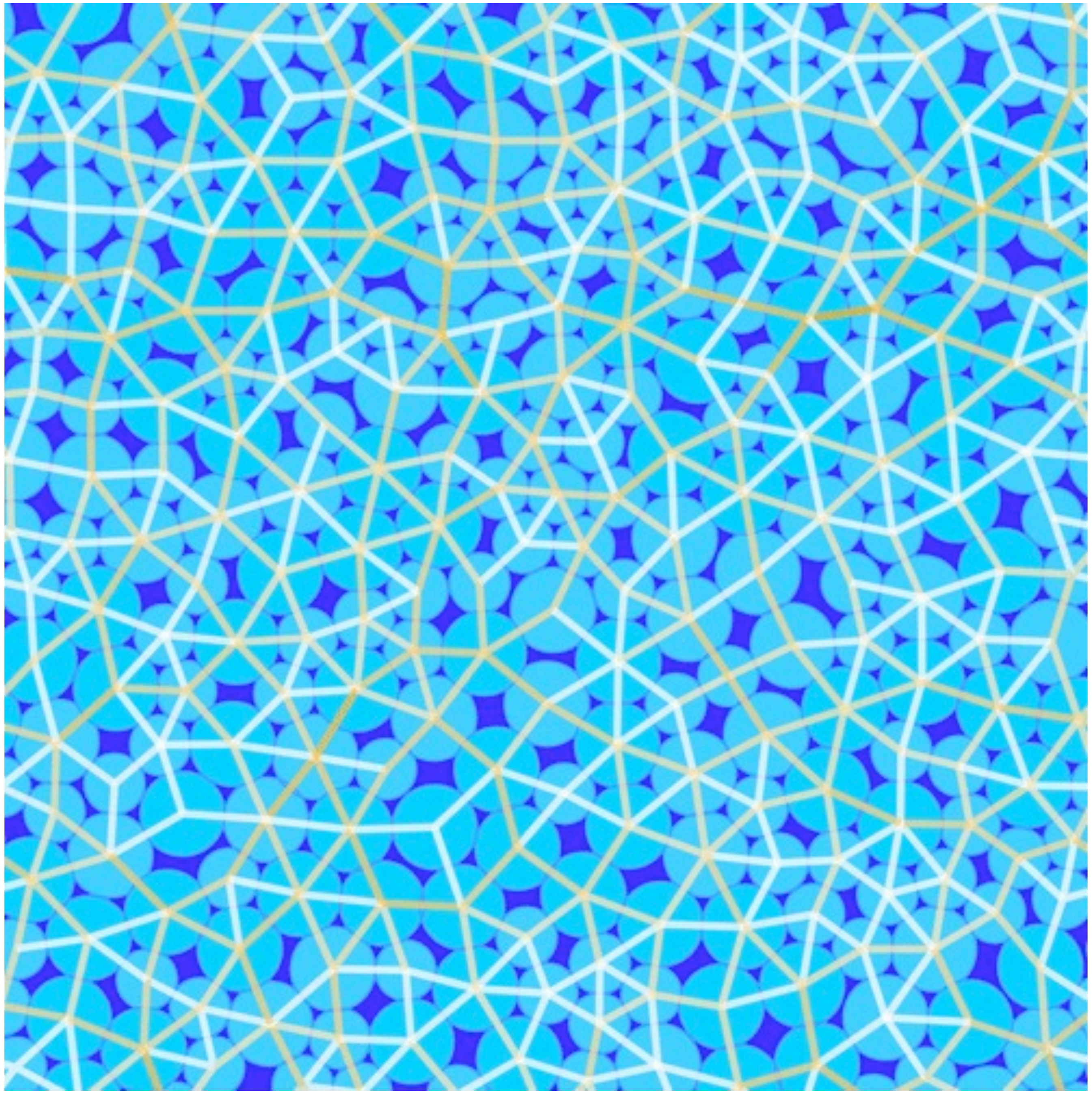
Frictionless jamming scenario

Shear modulus proportional to crossover frequency in DOS

Crossover frequency vanishes at isostatic point

Frictionless spheres (wet foams) isostatic at unjamming

Isostaticity: With static friction



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(+ torque balance)

z ≥ d+1 contacts per particle

$$|\vec{r}_i - \vec{r}_j| = R_i + R_j$$

z ≤ 2d contacts per particle

thus d+1 ≤ z ≤ 2d
at p = 0

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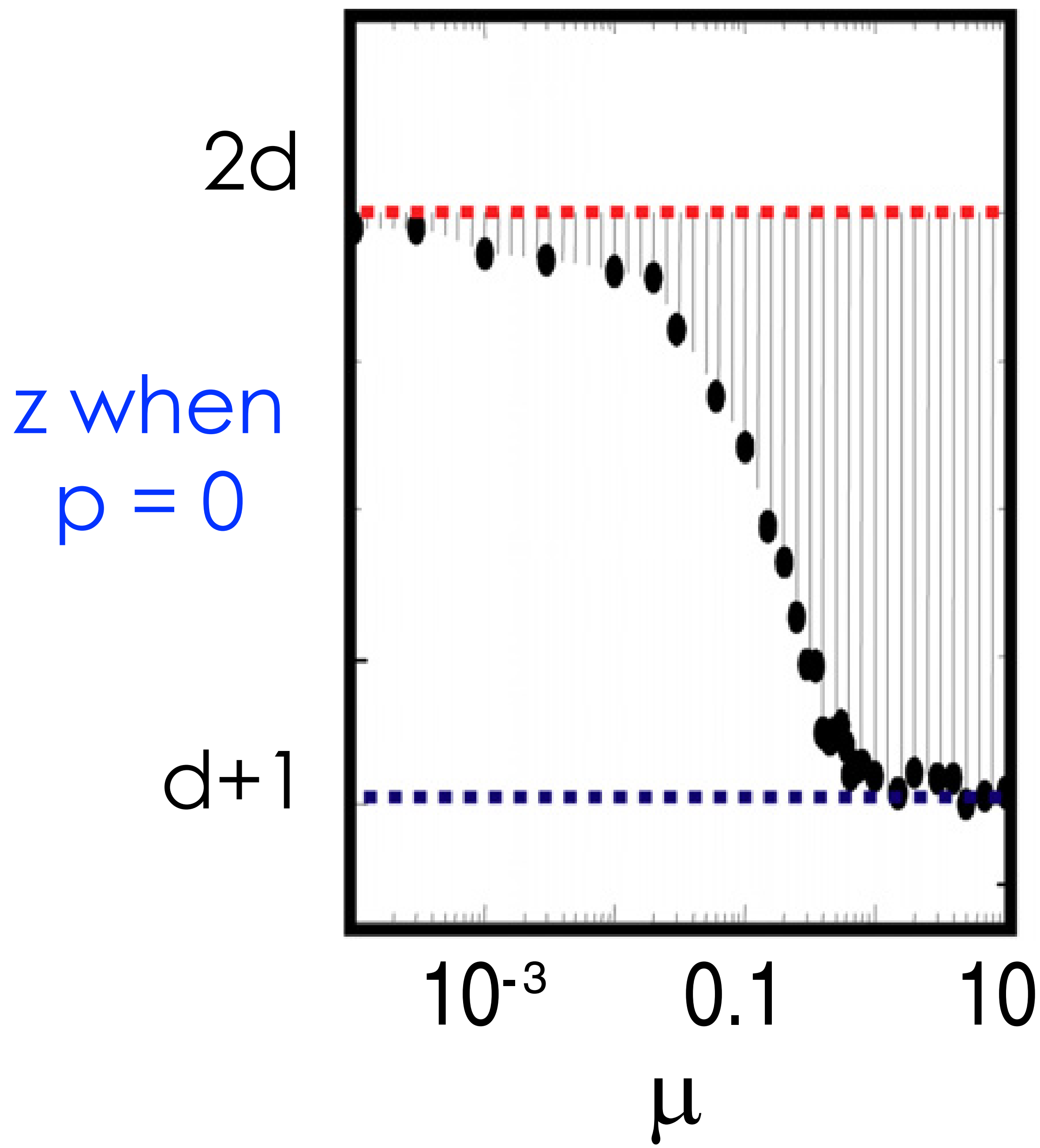

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Somfai et al., PRE 2004

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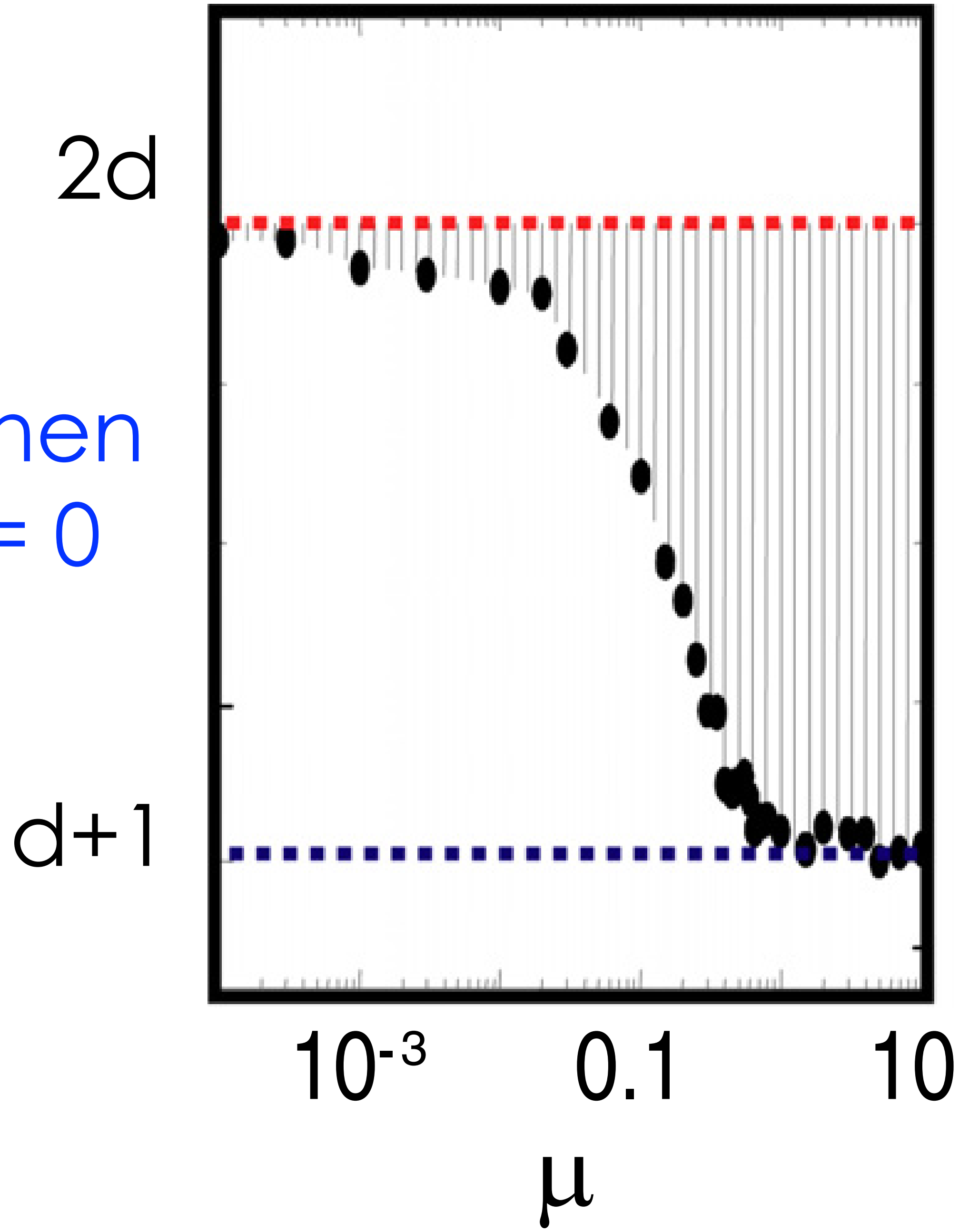

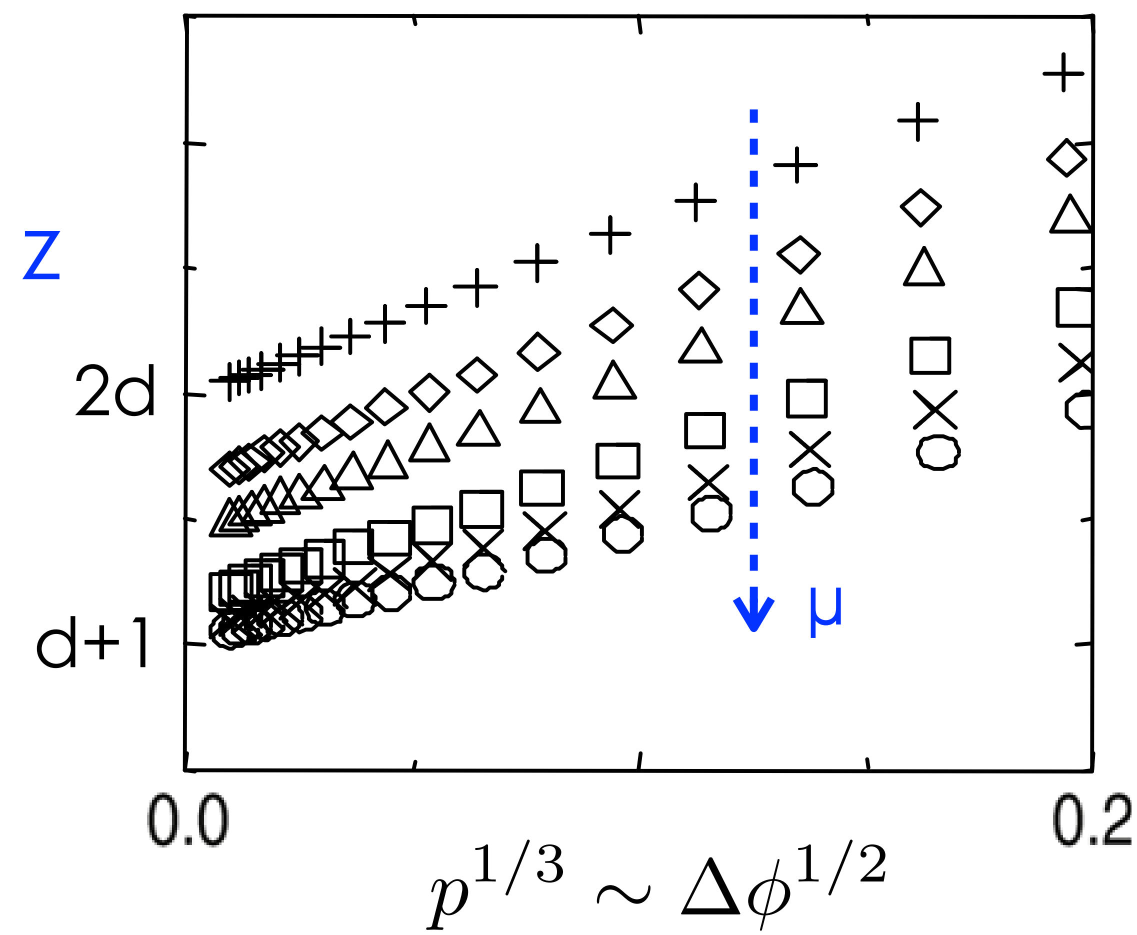

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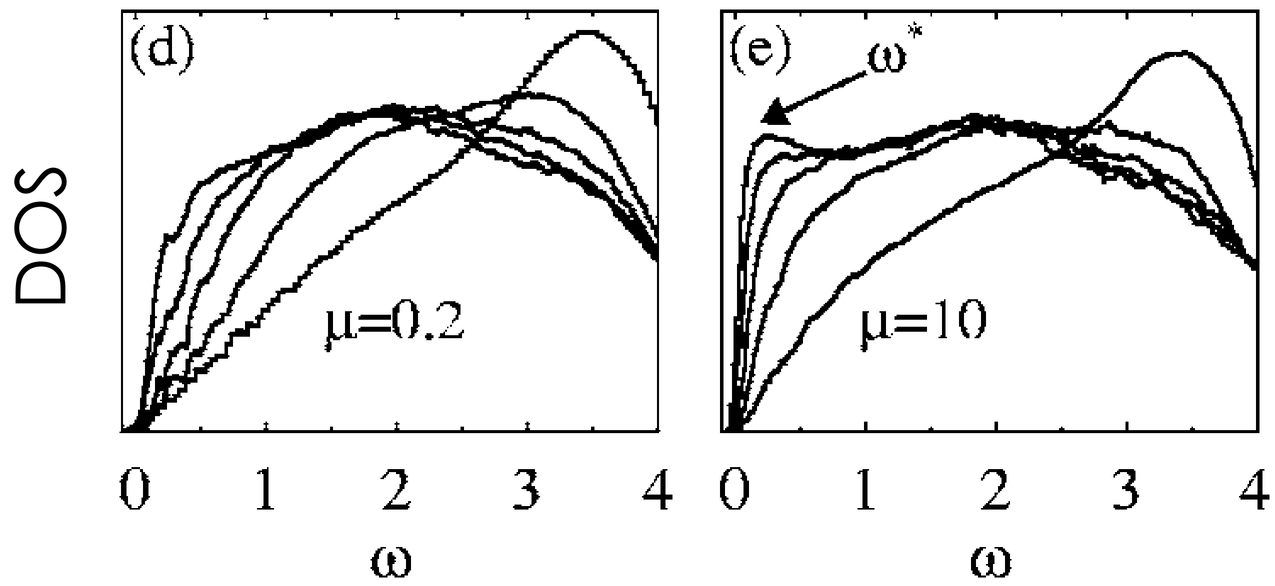
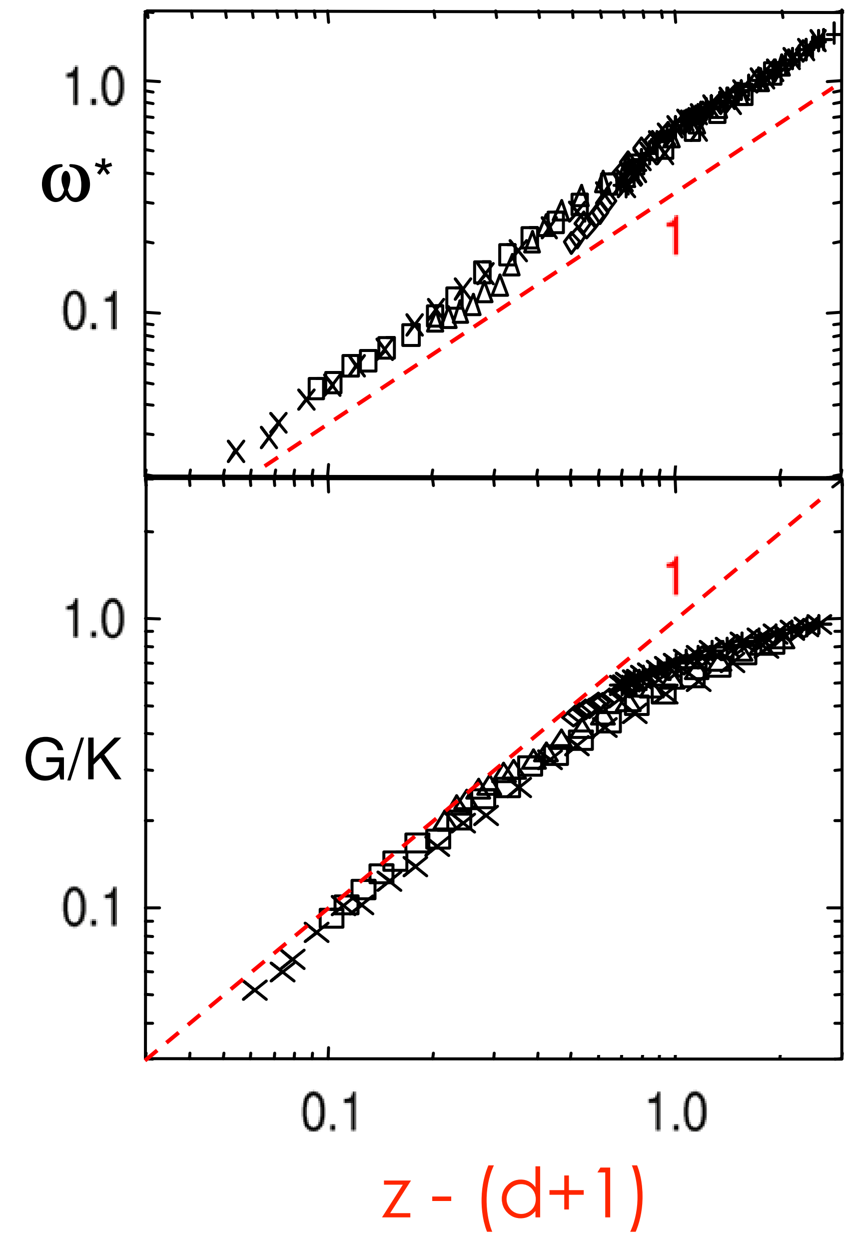
Quasistatic response with friction

$G \sim k^{1/2}\omega^*$ still true





Somfai et al., PRE 2004

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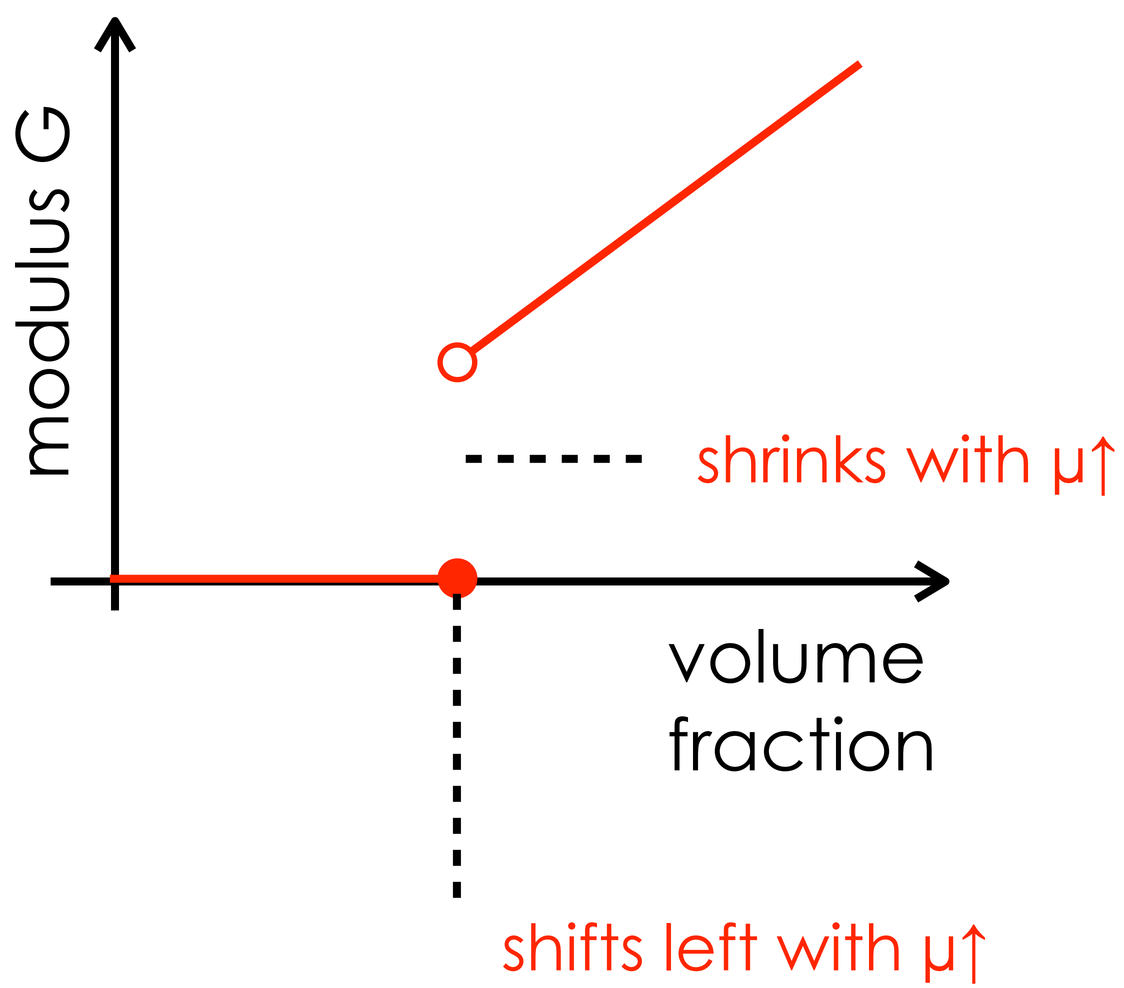

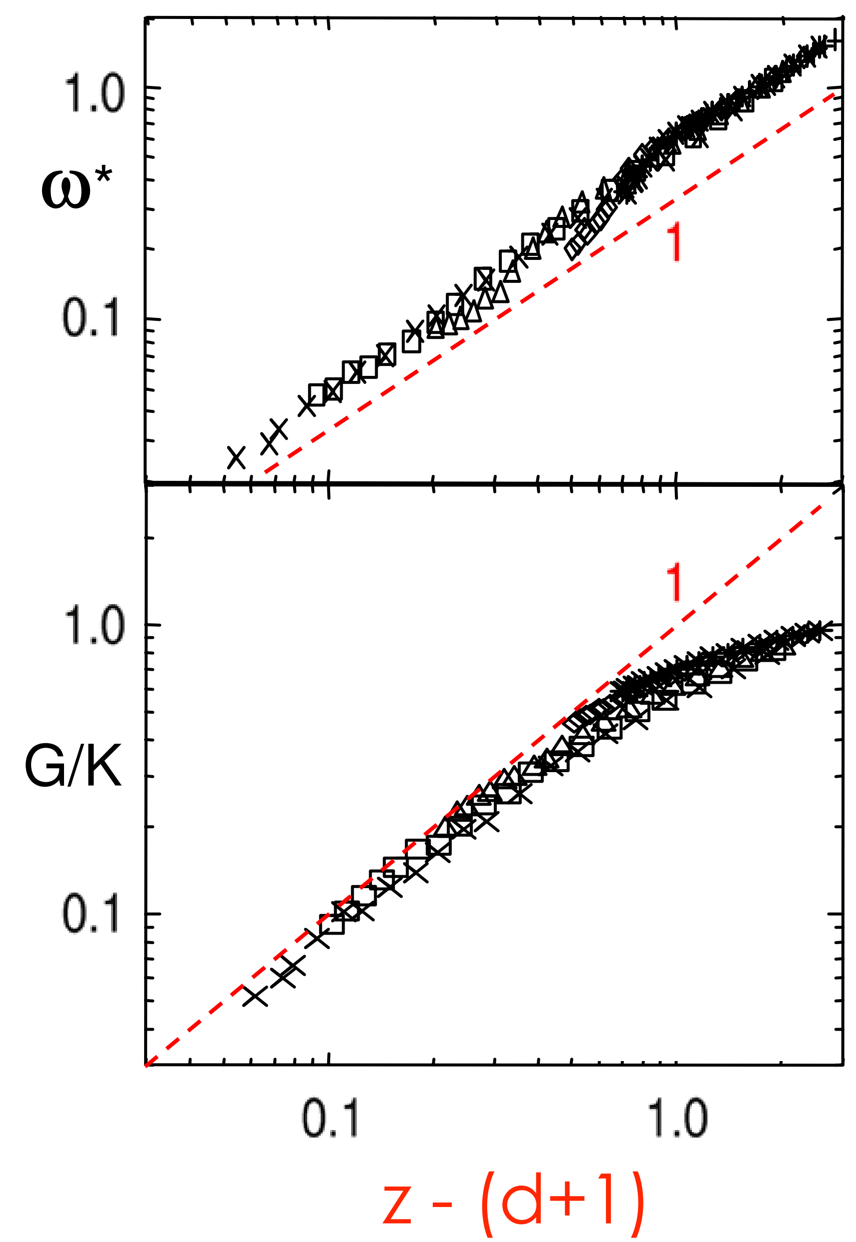

Somfai et al., PRE 2004

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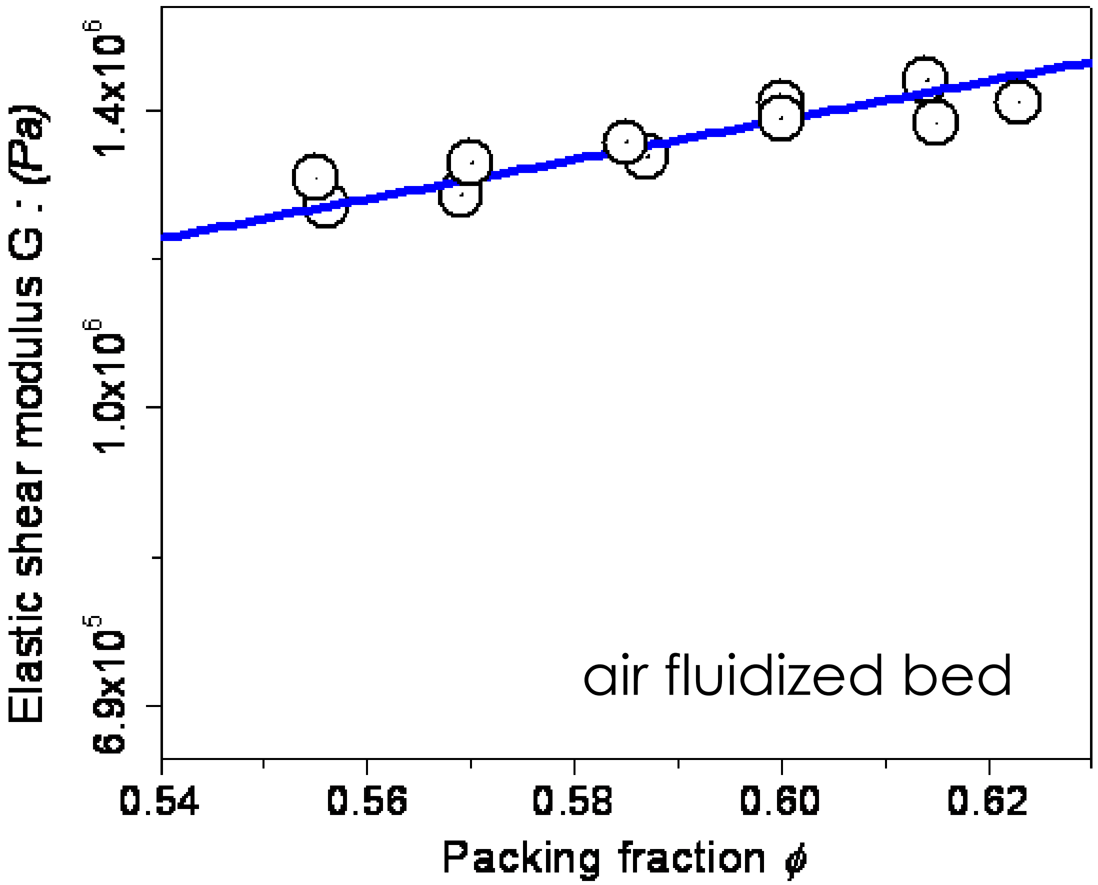
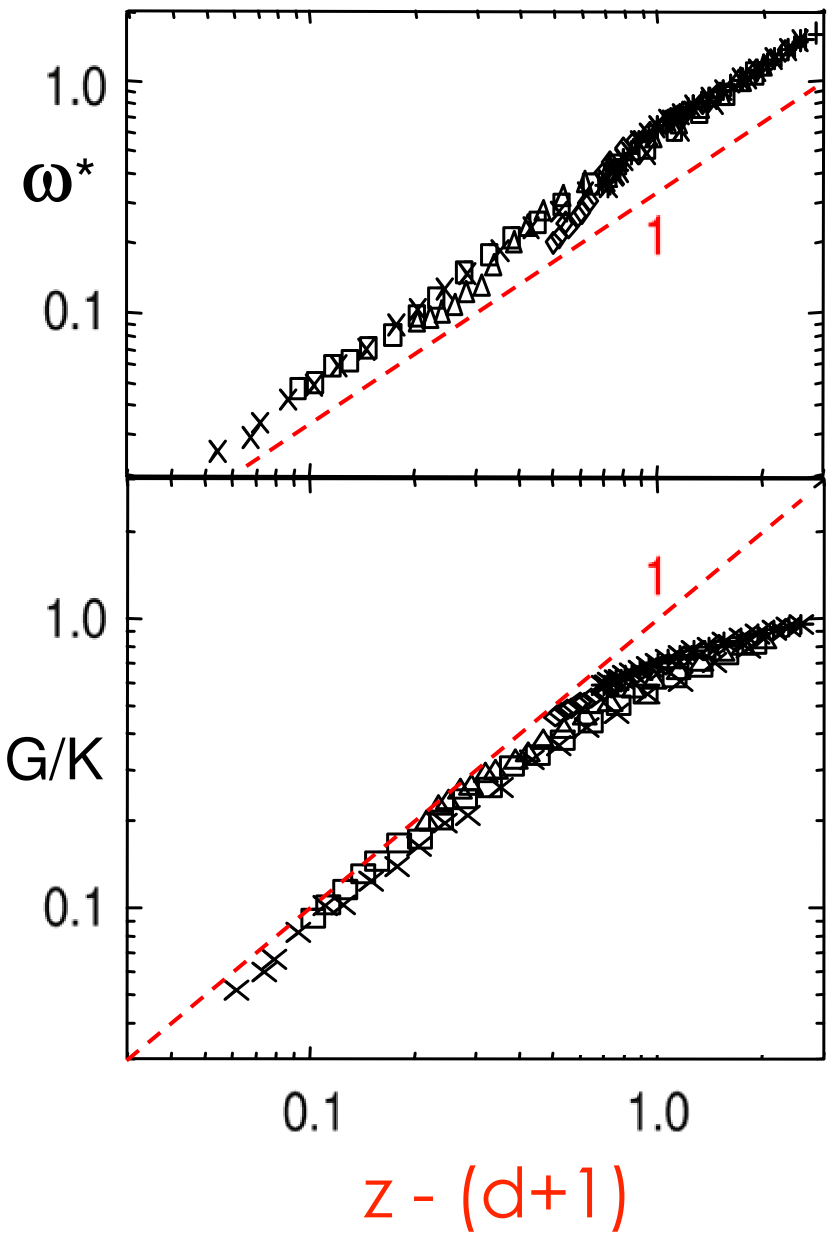




Nguyen et al., PRL 2011

Somfai et al., PRE 2004

Frictional jamming scenario

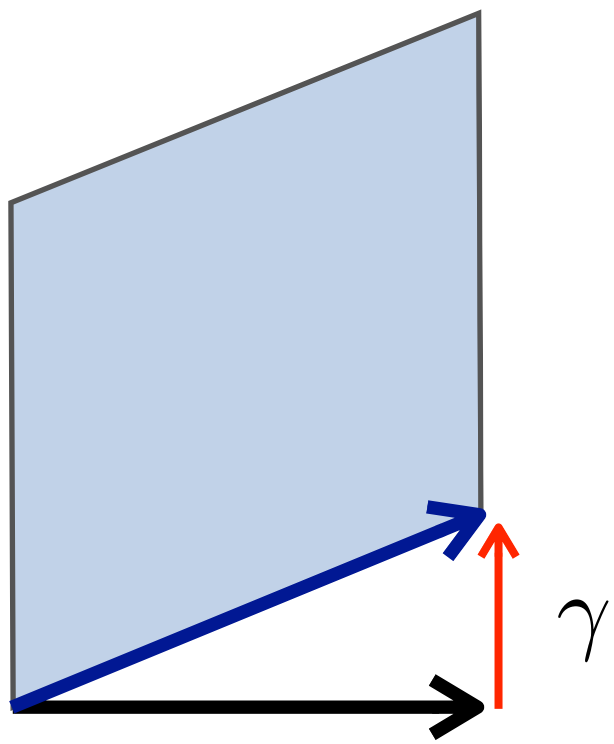
Shear modulus (still) proportional to crossover frequency in DOS

Crossover frequency (still) vanishes at isostatic point

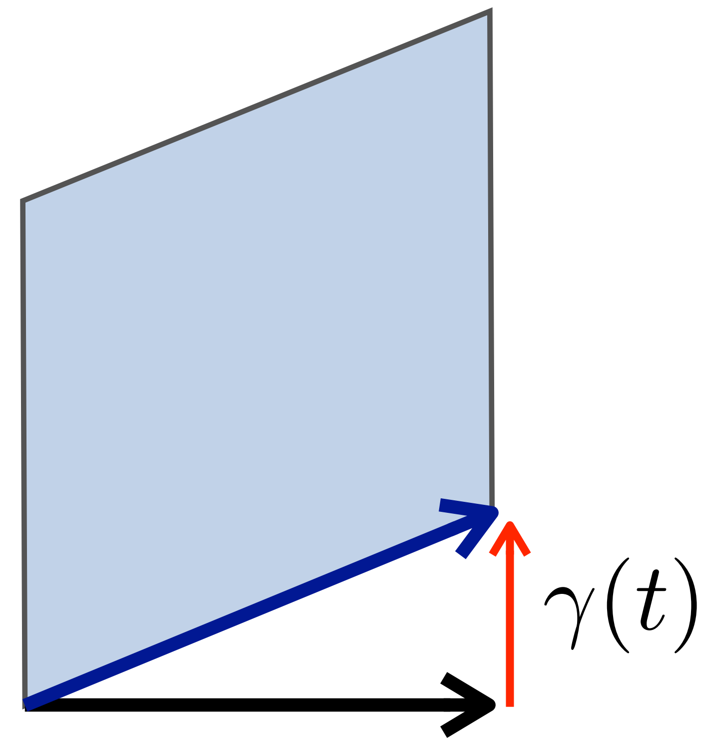
But frictional spheres (grains) hyperstatic at unjamming

Shear modulus has a jump

How does jamming differ for (overdamped, viscous, wet) foams and (massive, frictional) grains?



quasistatic

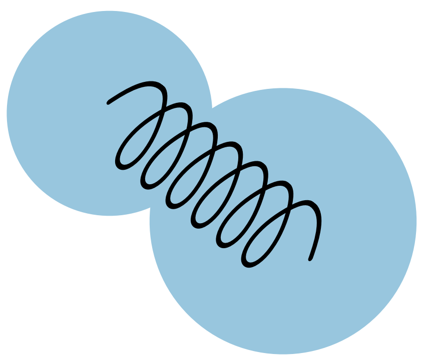


time dependent

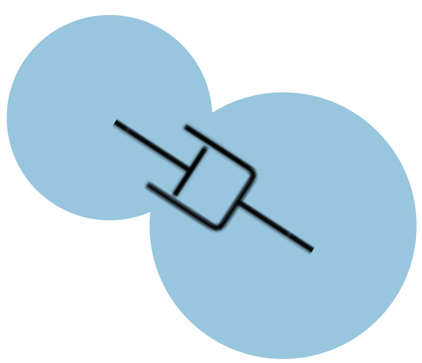
(oscillatory)

Microscopic interactions

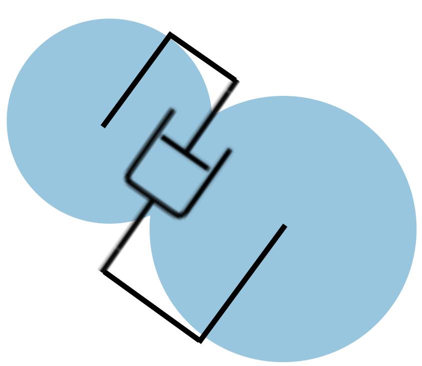
wet foams



normal
elastic
force



normal
viscous
force



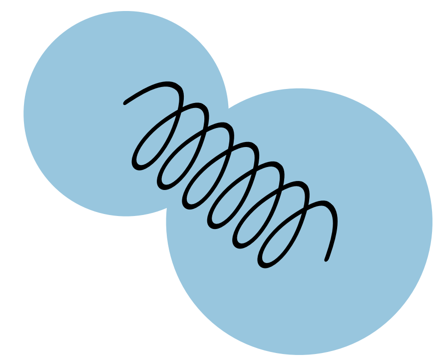
tangential
viscous
force

$$\vec{f}_{\mathrm{visc}} = -b\,\Delta\vec{v}$$

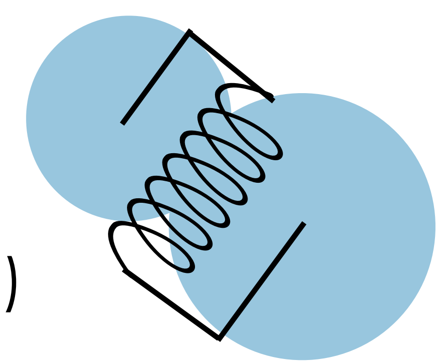
Durian, PRL 1995

grains

normal
elastic
force

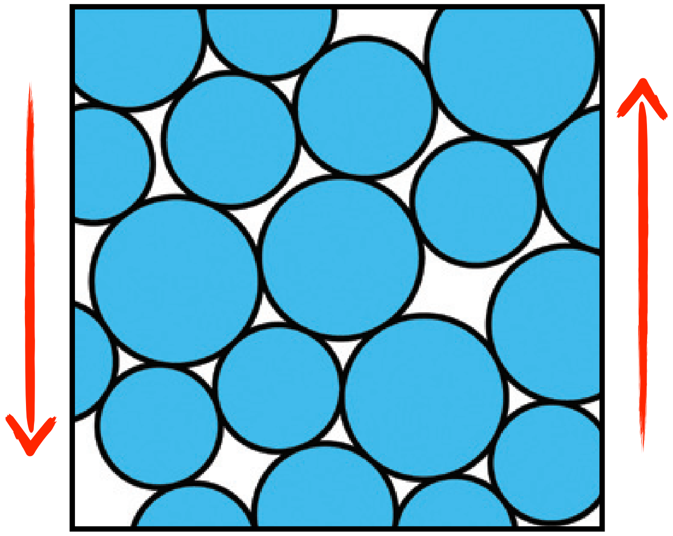


tangential
elastic
force
(Hertz-Mindlin)



$$|f_t| \leq \mu f_n$$

Viscoelastic linear response



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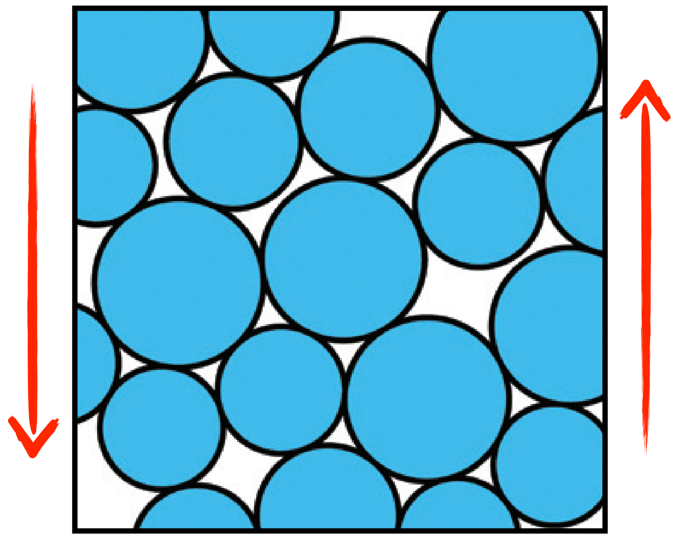

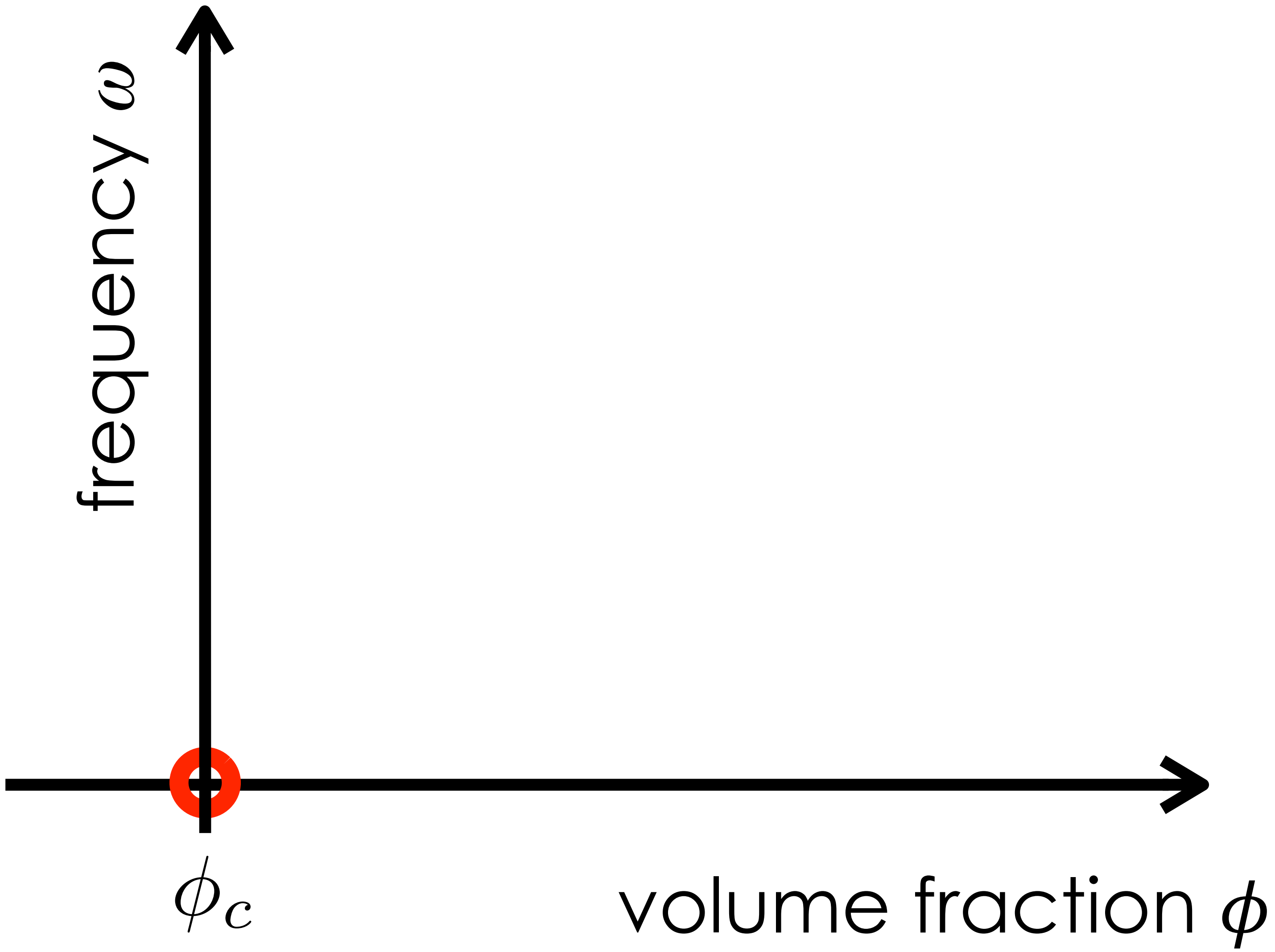

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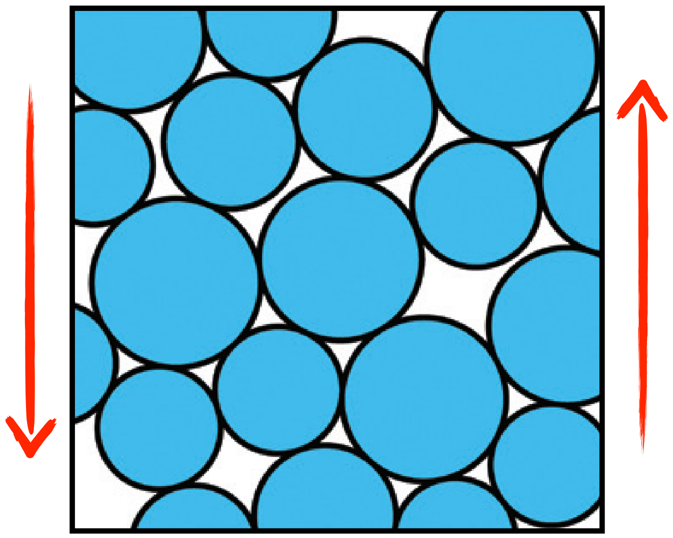

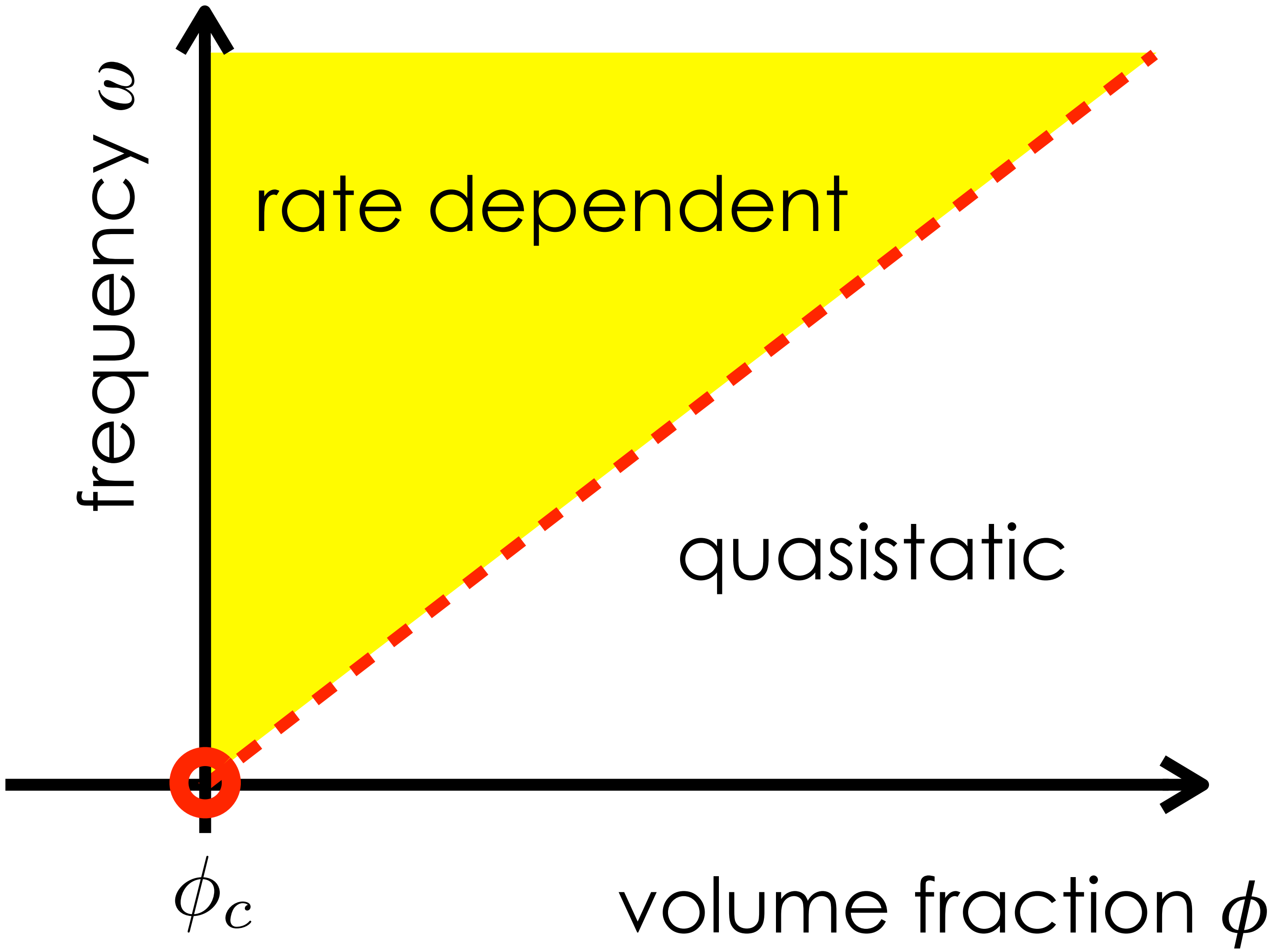

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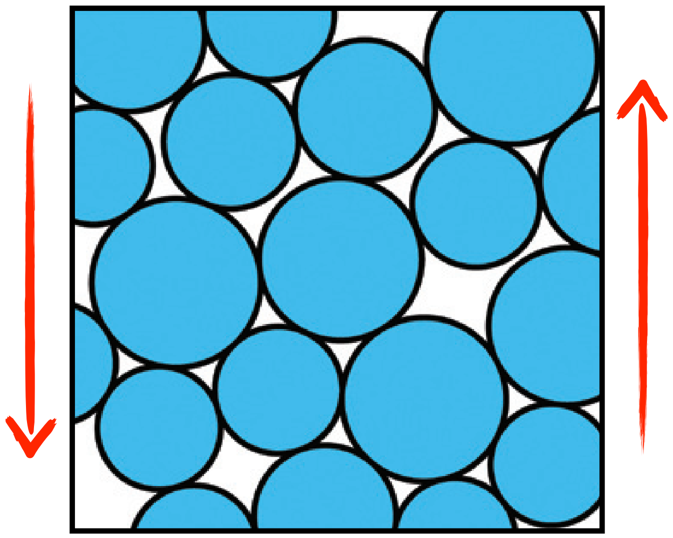


driving frequency

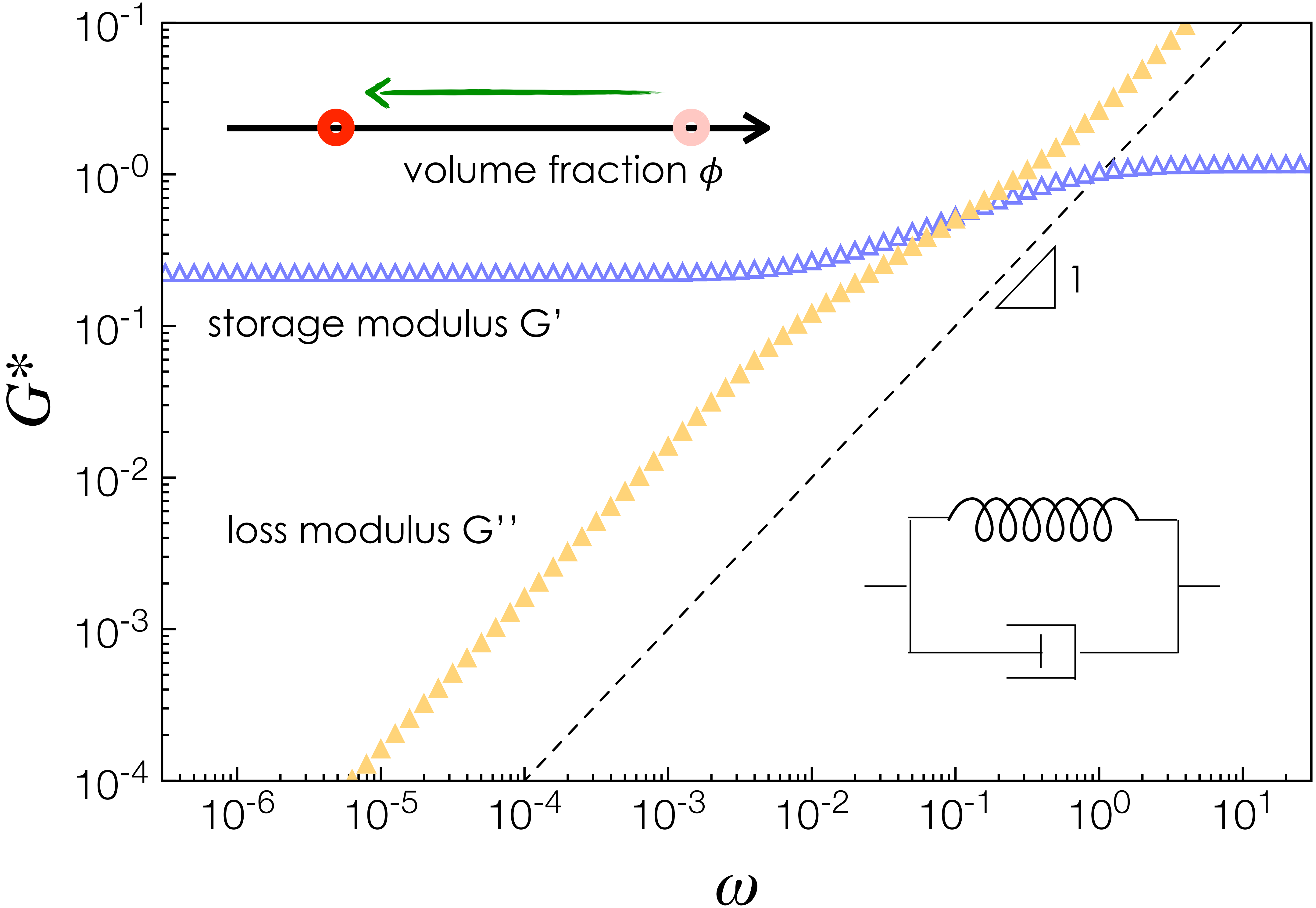
$$(\hat{K} + \imath\omega\,\hat{B})|u\rangle = \sigma(\omega)\,L^d|\hat{\gamma}\rangle$$

viscous friction

$$B_{ij} = \frac{\partial^2 R}{\partial \dot{u}_i \partial \dot{u}_j}$$

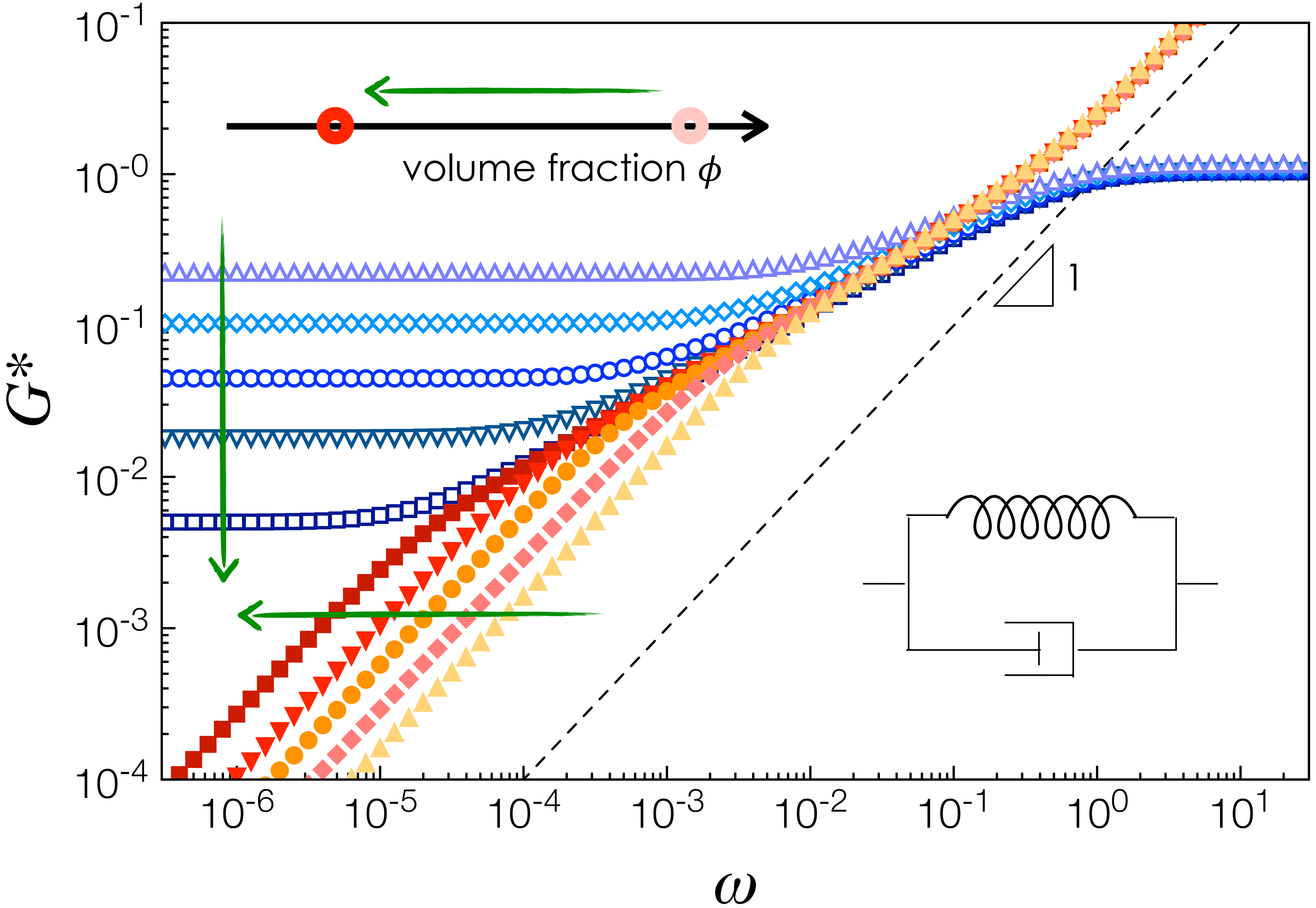
Rayleigh dissipation funcion

Storage and loss moduli



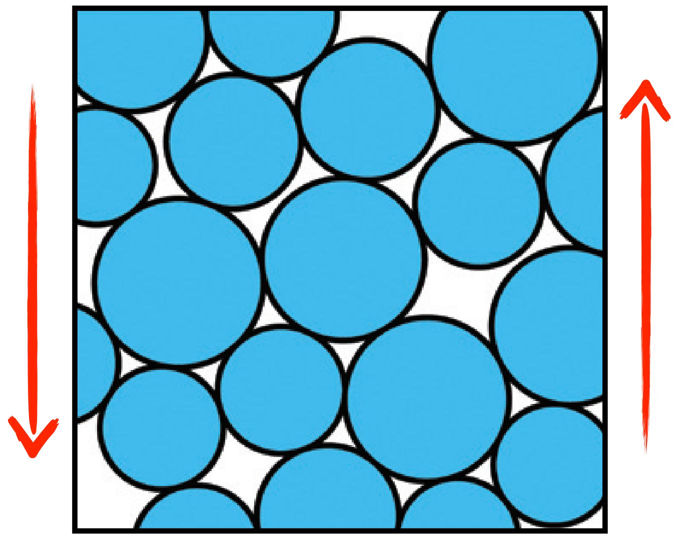
Tighe, PRL 2011

Storage and loss moduli



Tighe, PRL 2011

Viscoelastic linear response



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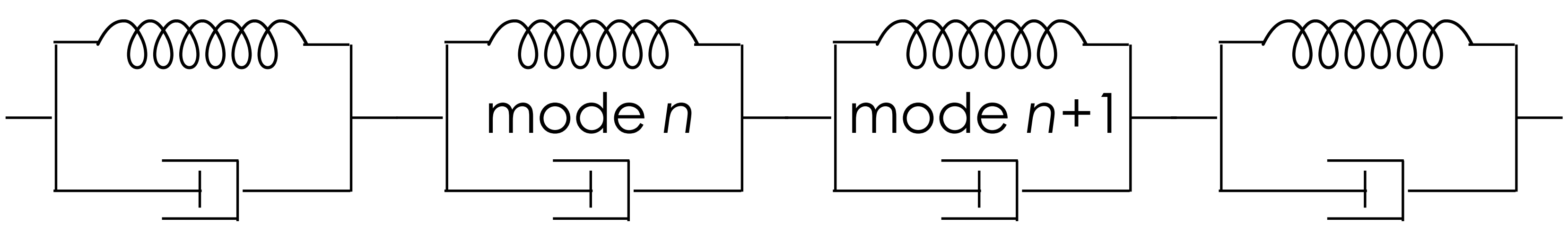
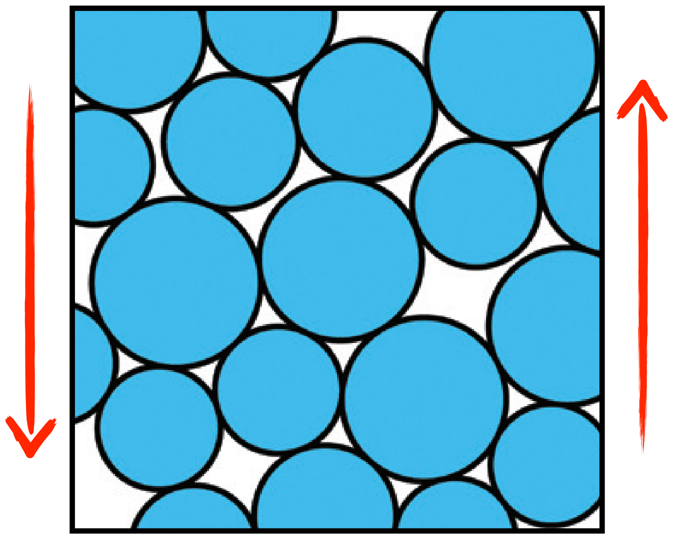


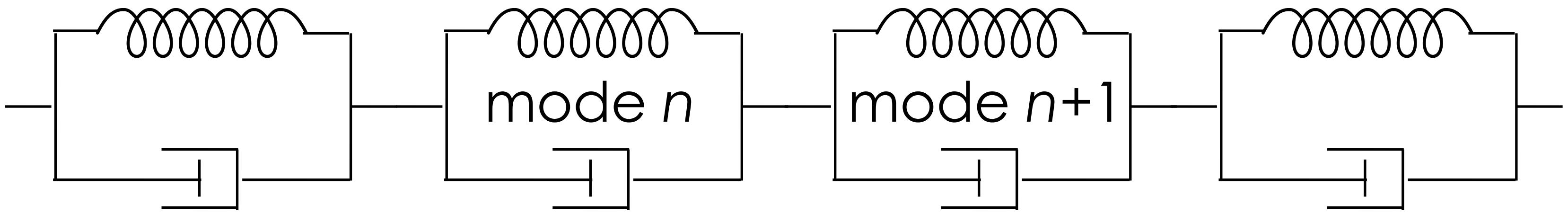
$$g_n^* \propto s_n + \imath\omega$$

s_n = generalized eigenvalue of $\{\hat{K}, \hat{B}\}$

Viscoelastic linear response



$$\frac{1}{G^*} \propto \sum_n \frac{1}{s_n + \imath\omega}$$



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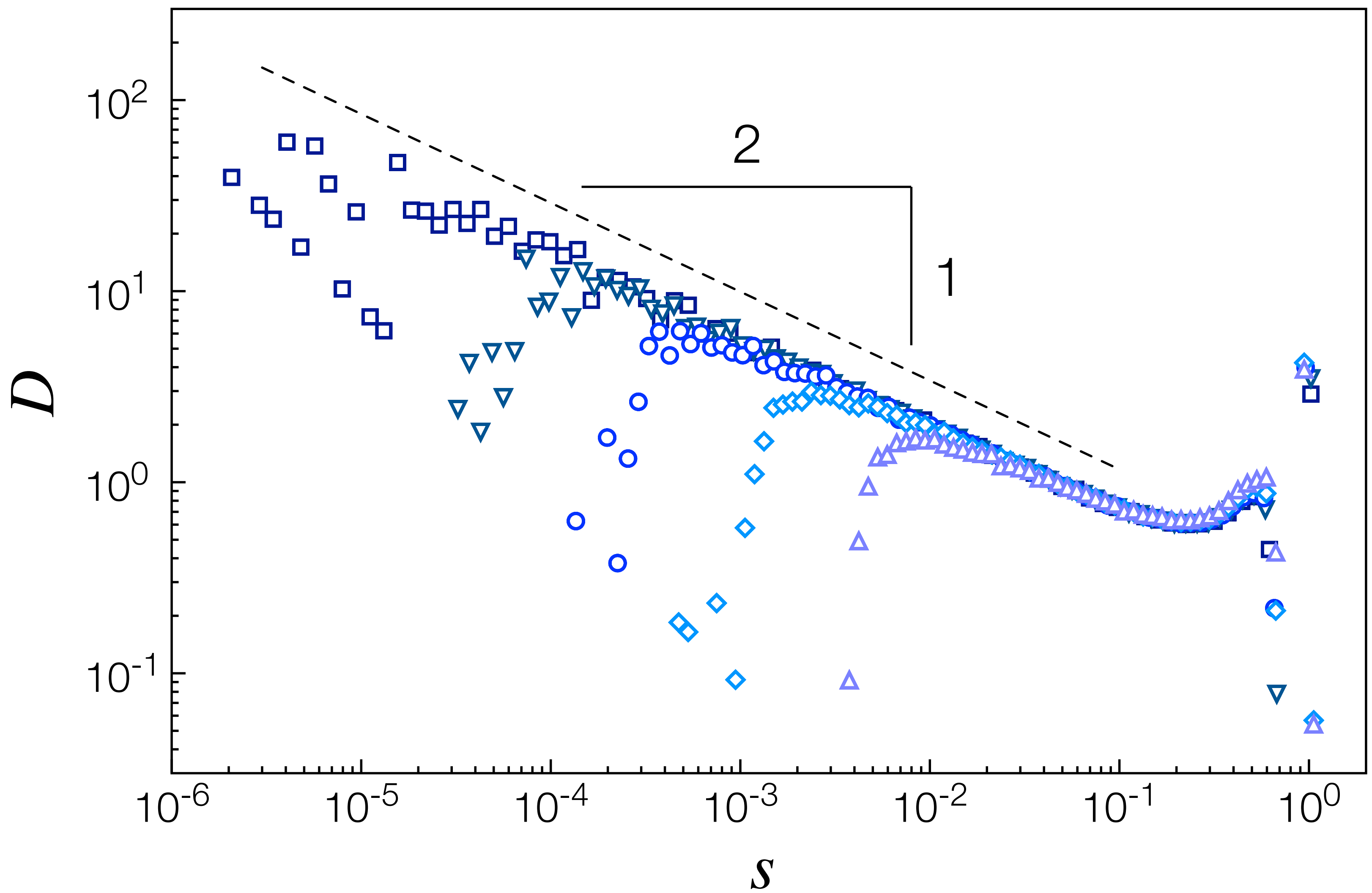
Density of states (*DOS*)



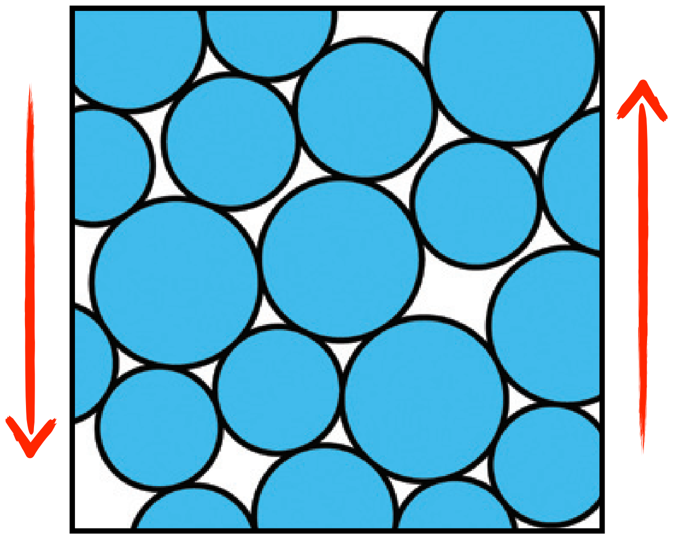
$$DOS \sim \frac{1}{s^{1/2}}$$

$$s^* := \frac{1}{\tau^*} \sim \Delta\phi$$

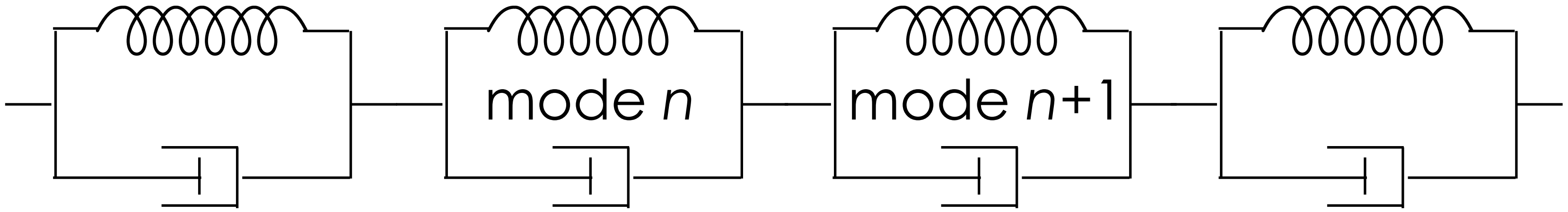
generalize "cutting argument" of Wyart Nagel & Witten (EPL 2005) to damped dynamics

Tighe, PRL 2011

Viscoelastic linear response



$$\frac{1}{G^*(\omega)} \propto \int \frac{D(s)\,\mathrm{d}s}{s + \imath\omega}$$



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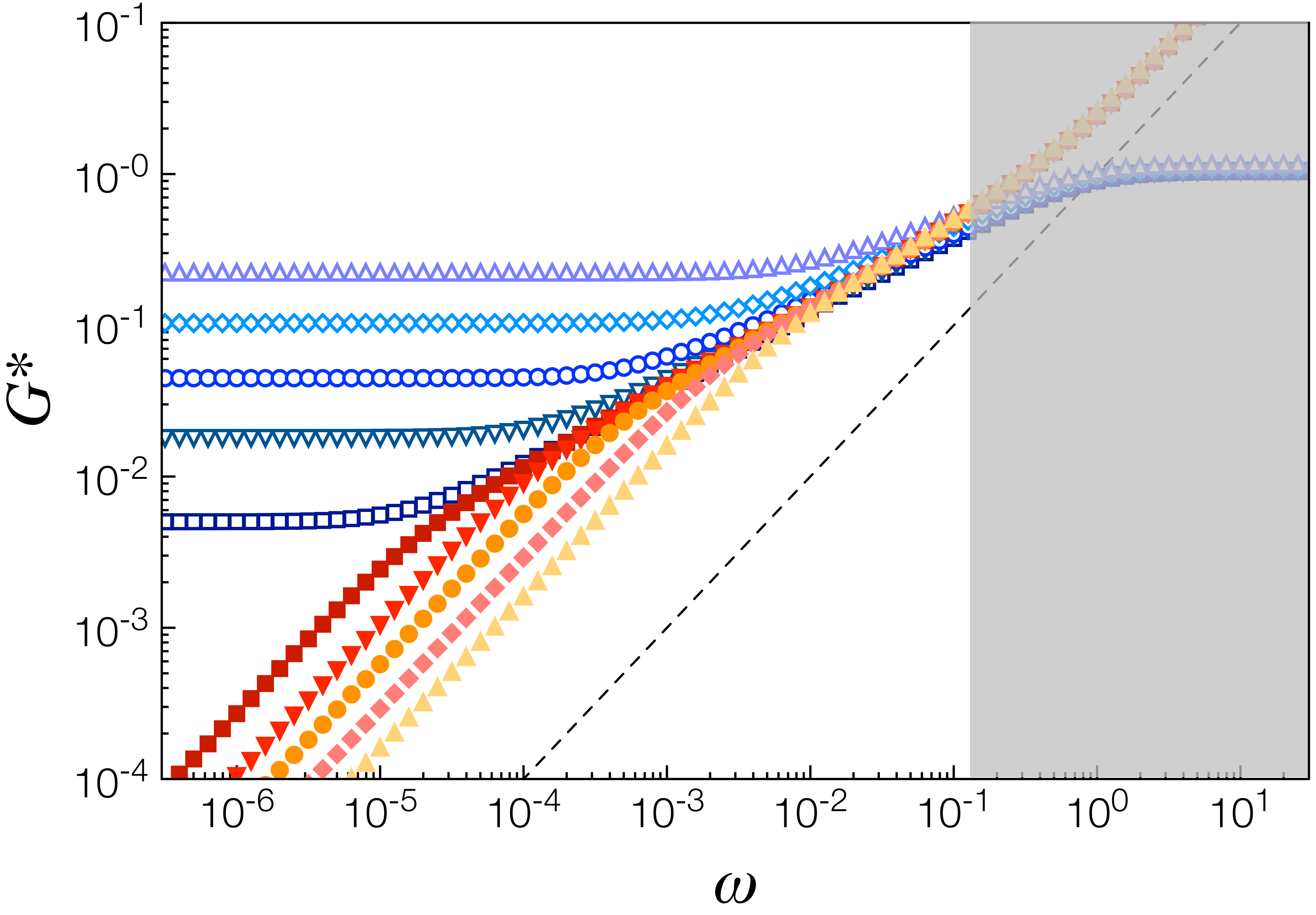
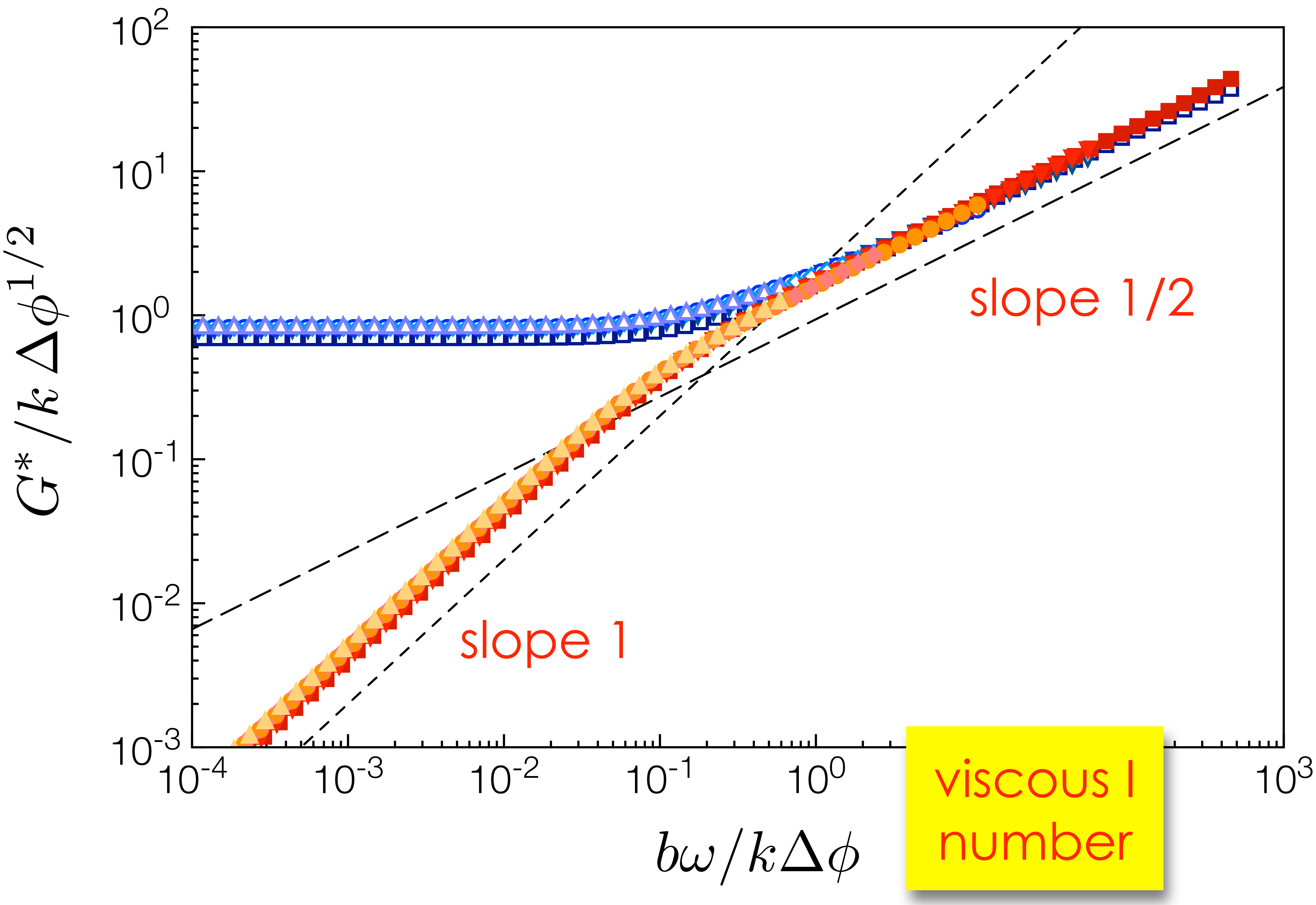
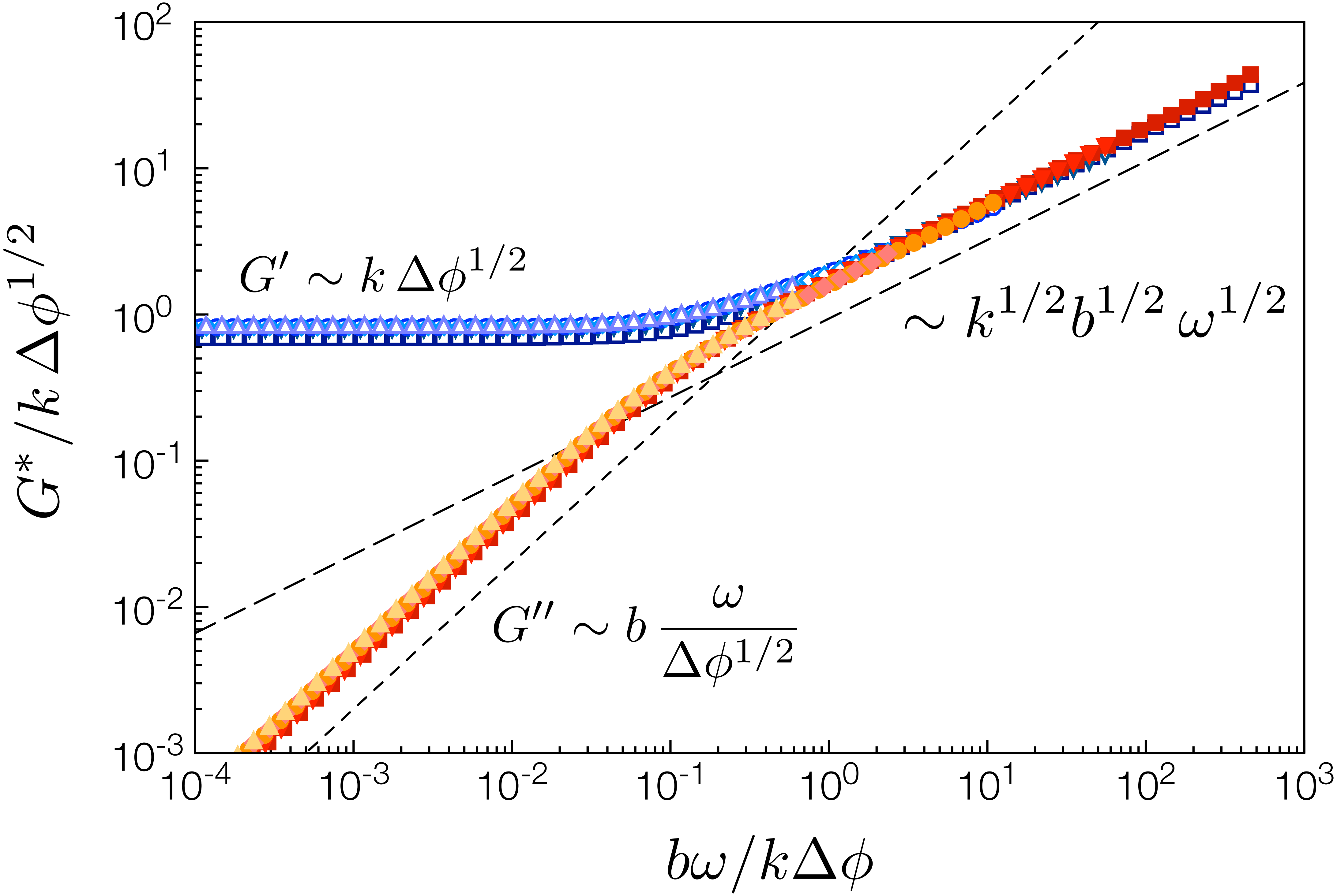
Storage and loss moduli



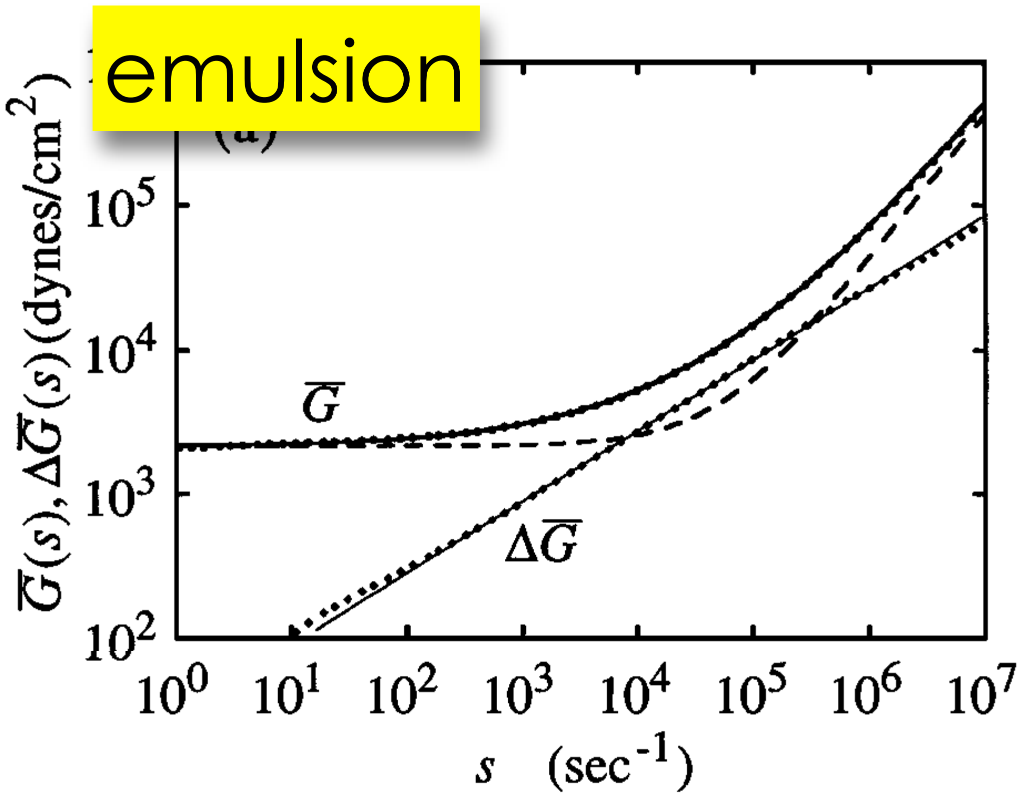
Dynamical critical scaling



Dynamical critical scaling



Shear thinning in experiment



Liu et al., PRL 1996

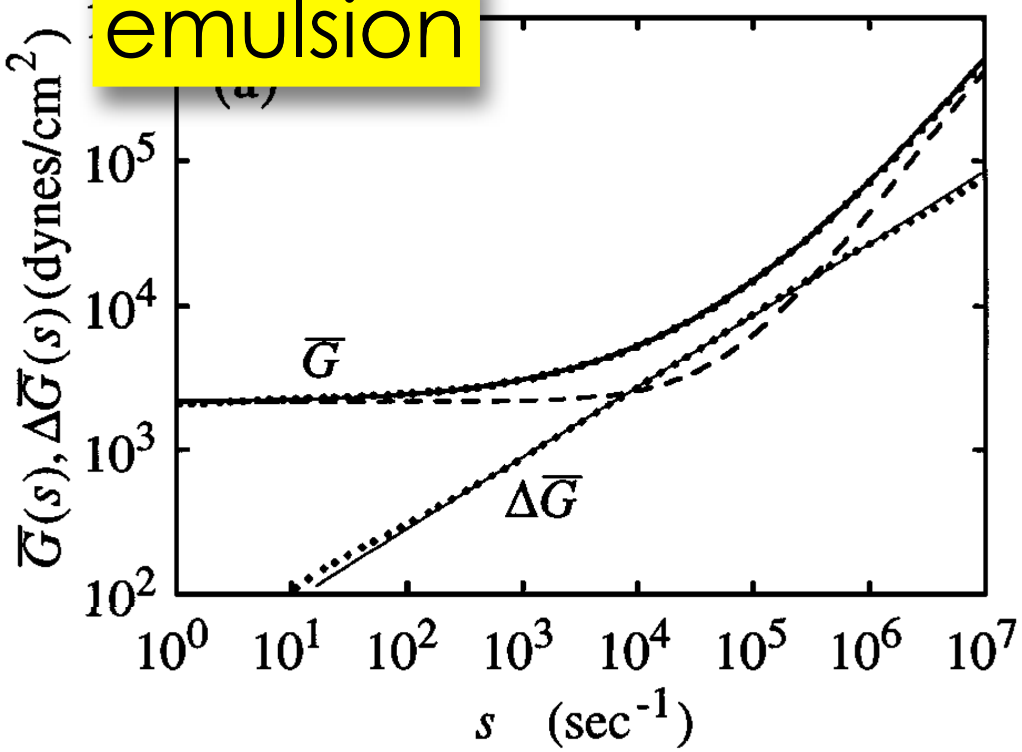
$$G^* \sim (\imath\omega)^{1/2}$$

what's new?

- clear connection to jamming
- multiple regimes with crossovers
- no fluctuation-dissipation

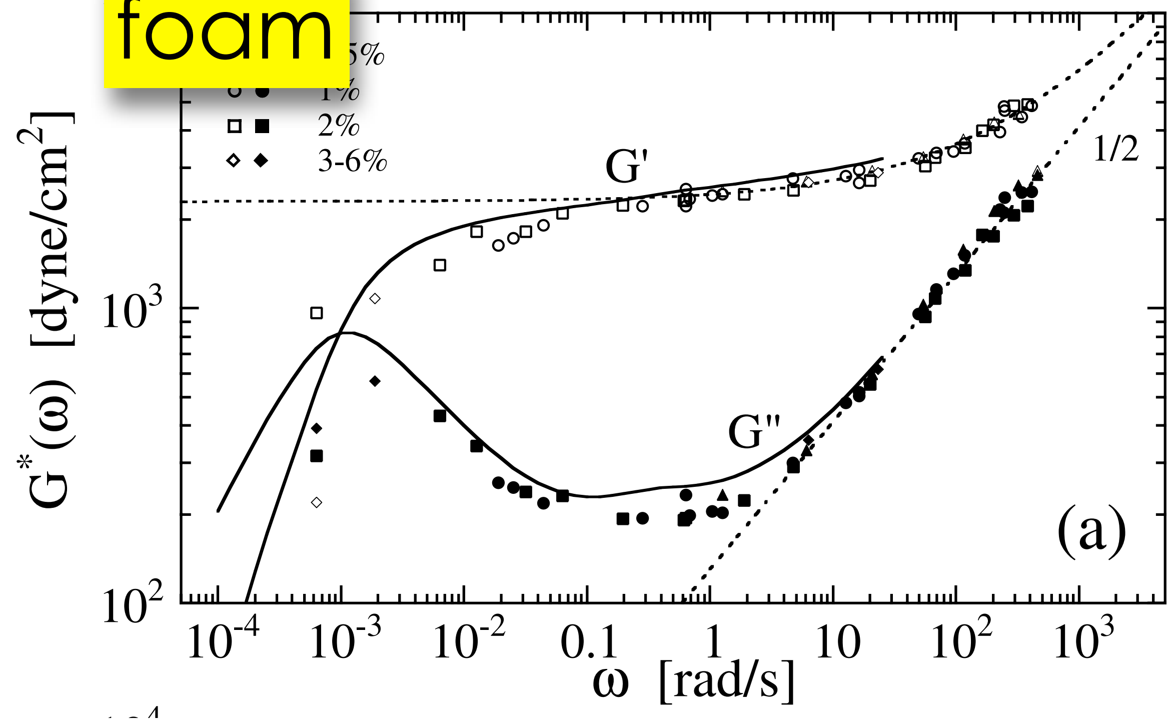
Shear thinning in experiment

emulsion



Liu et al., PRL 1996

foam



Gopal & Durian, PRL 2003

$$G^* \sim (\imath\omega)^{1/2}$$

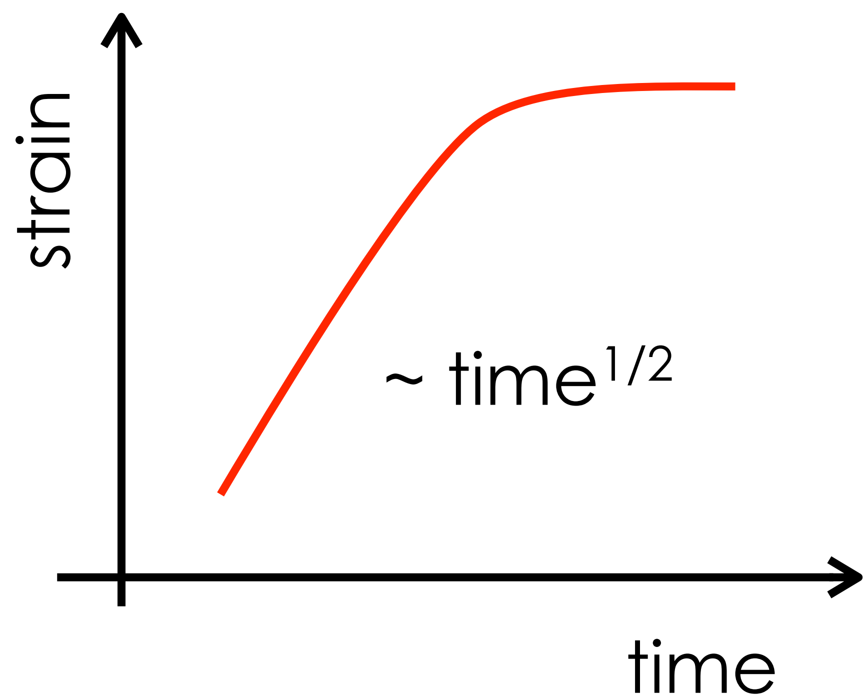
emulsions, liquid foams, organic foams, microgel suspensions…

Höhler & Cohen-Addad, PRE 1998

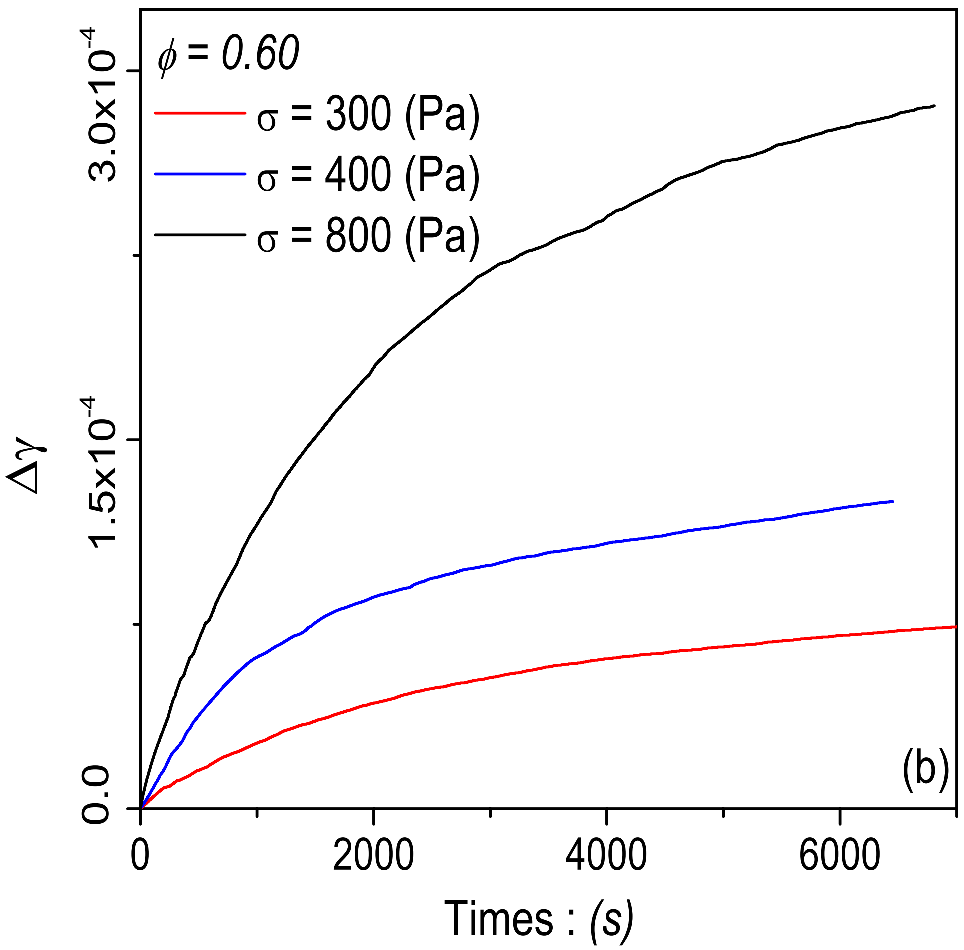
Hébraud et al., Langmuir 2000

Kropka & Celina, JCP 2010

Creep compliance



$$\gamma \sim \log\left(1 + t/\tau\right)$$



viscous soft spheres

grains

Nguyen et al., PRL 2011

see also McDowell & Khan, Gran. Matt. 2003

Conclusions

anomalous slow modes near isostaticity
control response

frictionless/foams

necessarily isostatic
at $p = 0$

vanishing G

diverging viscosity

shear thinning

frictional/grains

not necessarily isostatic
at $p = 0$

jump in G