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Modeling Random Wet 2D Foams with Controlled Polydispersity

"Back to the Future?"

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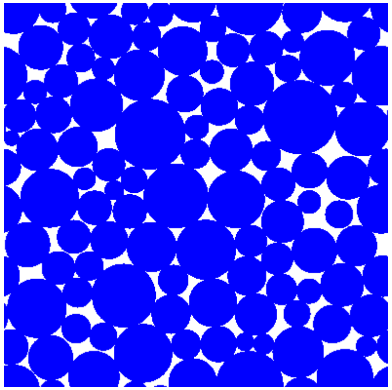
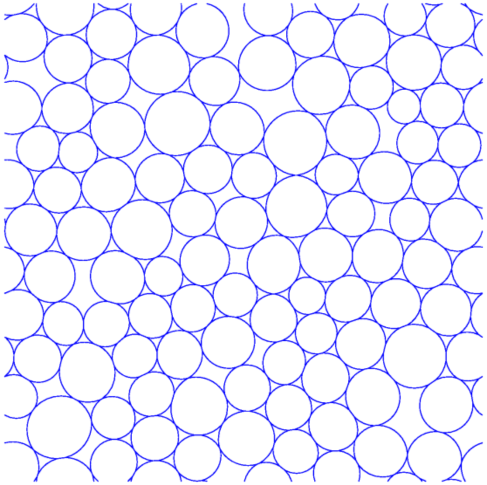
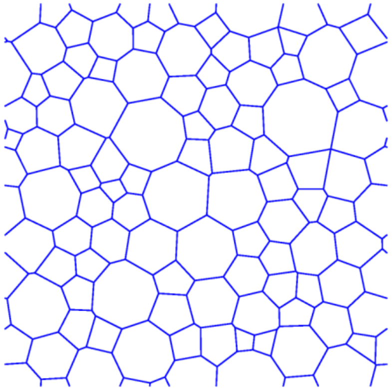
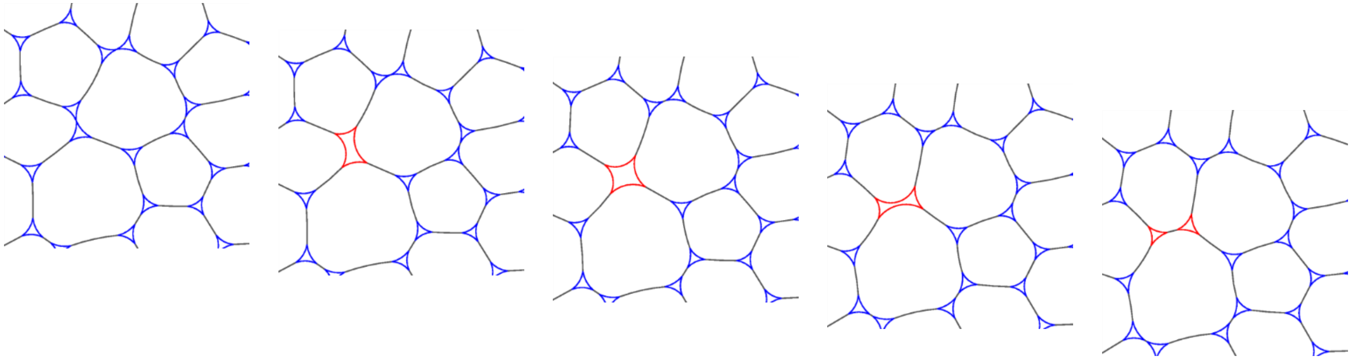
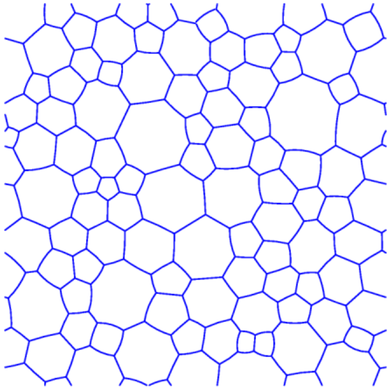
Imperial College London







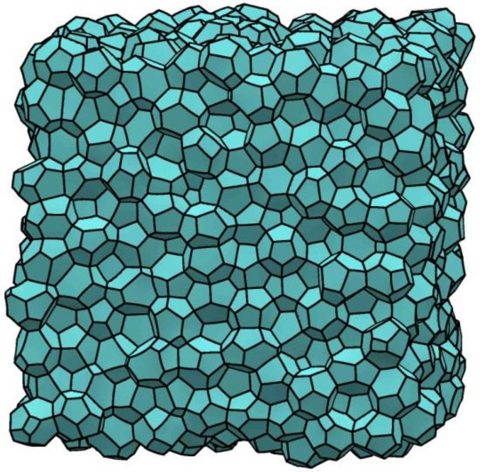




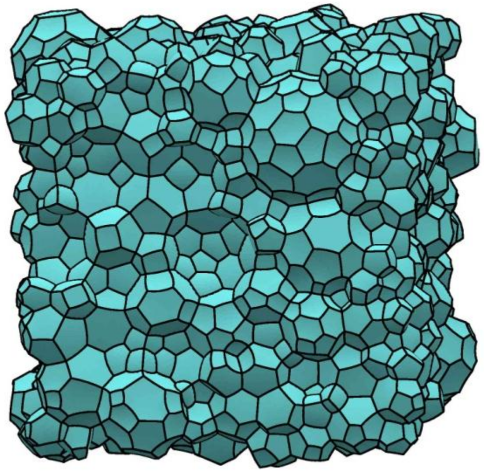
Random Foams

Kraynik, Reinelt & van Swol (2003) *Phys Rev E* **67**, 031403;

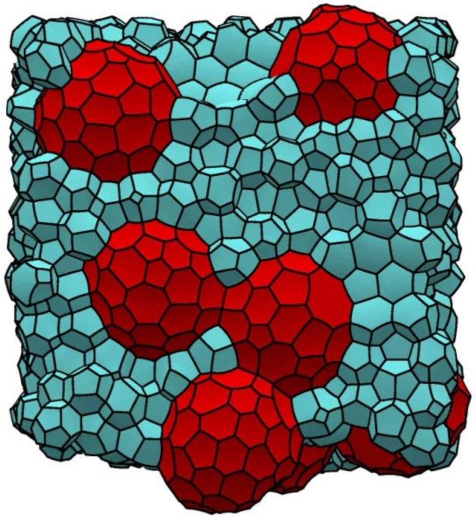
(2004) *Phys Rev Lett* **93**, 208301; (2005) *Colloids Surfaces A* **263** 11-17.



Monodisperse



Polydisperse



Bidisperse

Spatially periodic structure

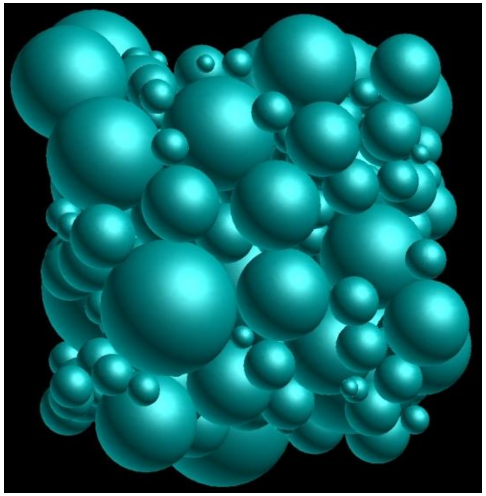
1728 cells

Cell volumes vary by three orders of magnitude

Outline

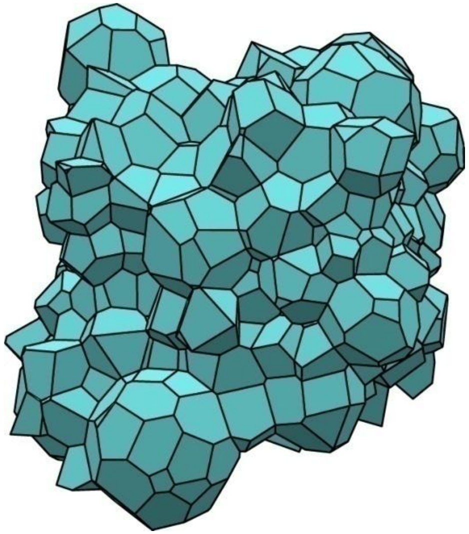
- Motivation
- Simulating random 2D foams with controlled polydispersity
- Dry
  - Energy
  - Shear modulus
- Wet
  - Increasing liquid fraction toward the jamming transition
  - Shear modulus

Simulating Random Polydisperse Foam

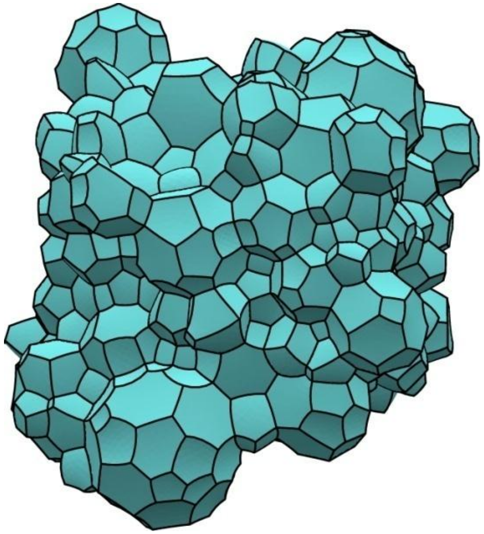


Molecular Dynamics

Random Close Packed (RCP) Spheres

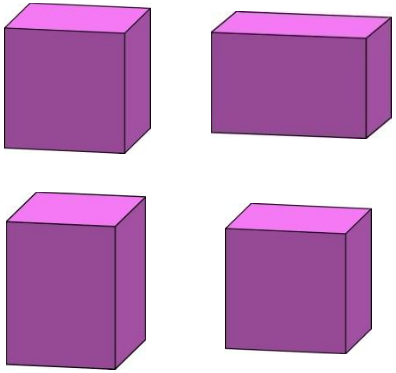


Laguerre (Weighted Voronoi) Tessellation



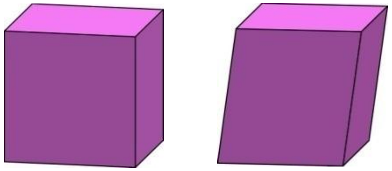
Surface Evolver

Relaxed Foam



Tension-Compression Cycles

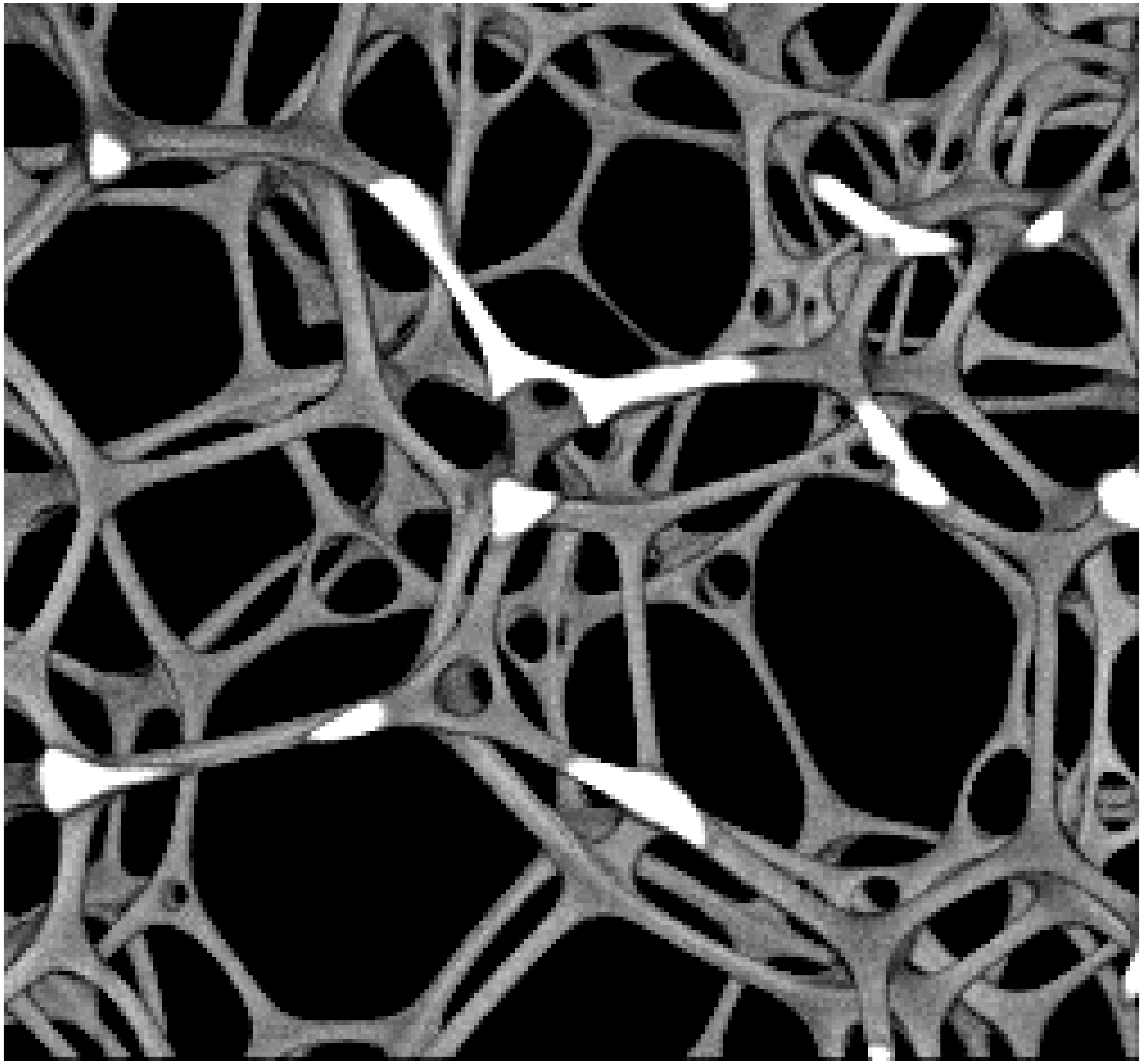
Annealing



Relax the lattice to achieve isotropic stress

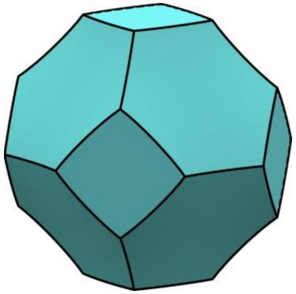
Elastic Recoil

Confocal microscopy of Plateau borders in emulsions

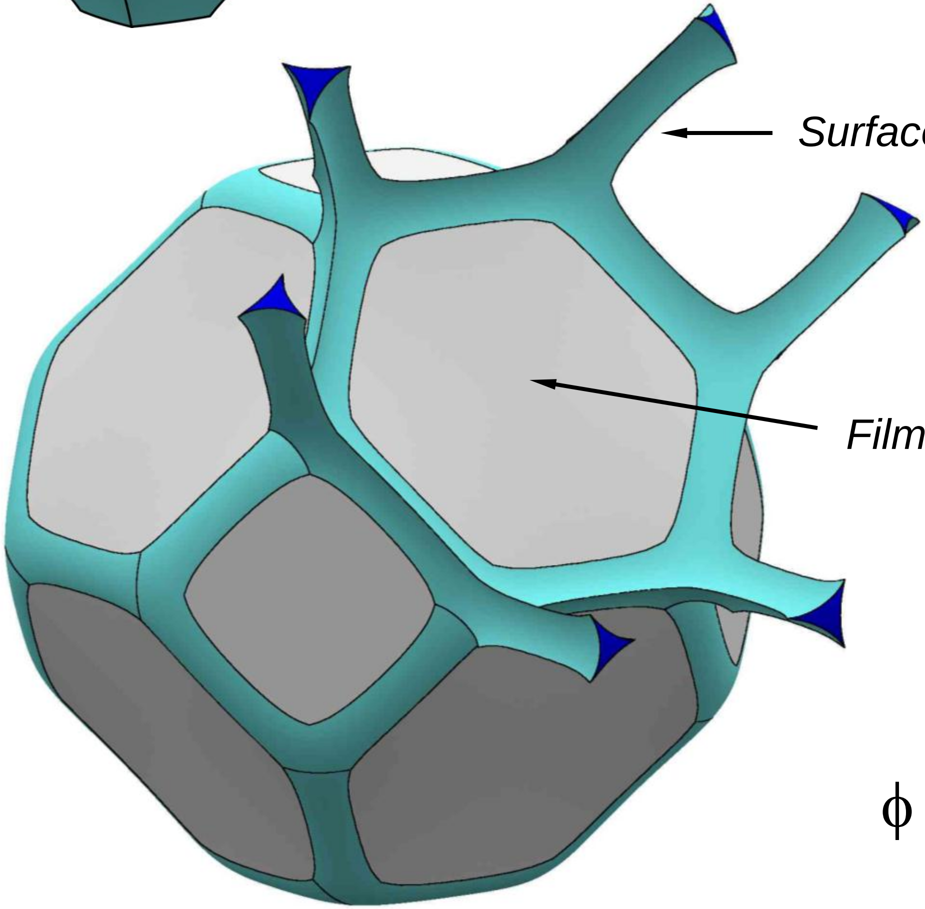


Eric Weeks, Physics, Emory University and Doug Wise, Physics, Harvard University

Plateau Borders in a Wet Kelvin Foam

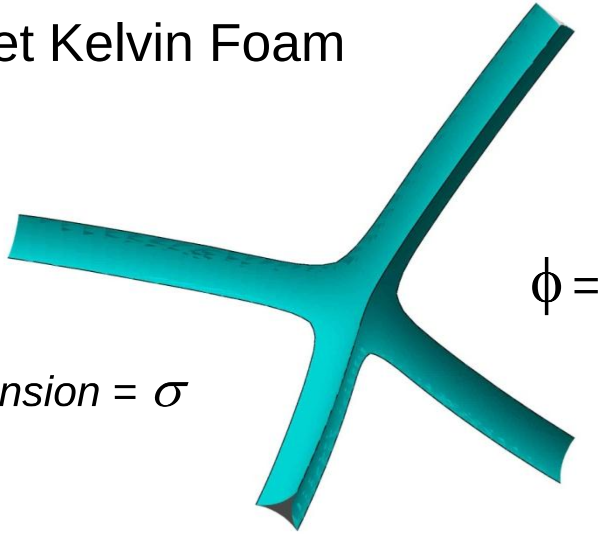


Kelvin Cell

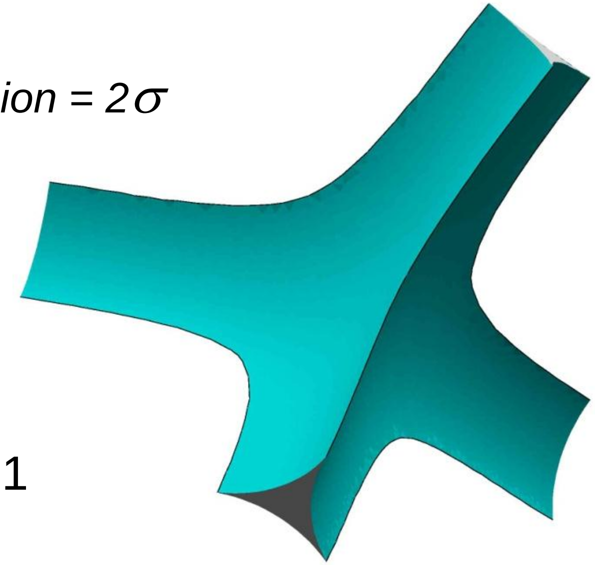


Surface Tension = σ

Film Tension = 2σ



$\phi = 0.001$



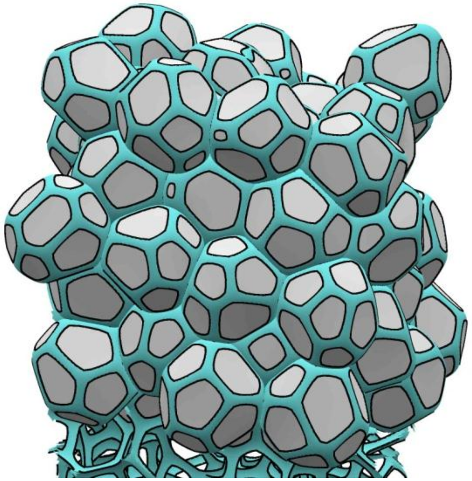
$\phi = 0.01$

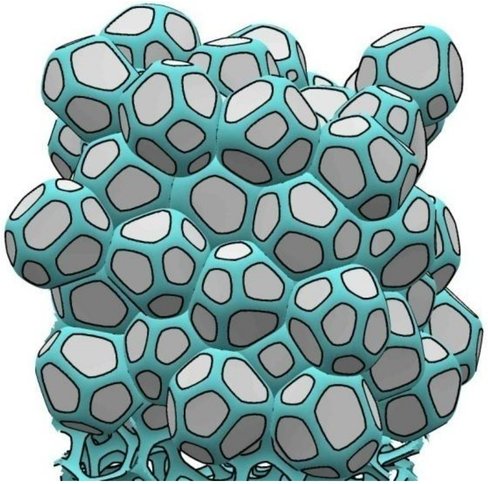
Plateau borders in liquid foams correspond to struts in solid foams

Wet monodisperse foams with 64 cells and isotropic stress

ϕ = 0.02

5 lost contacts



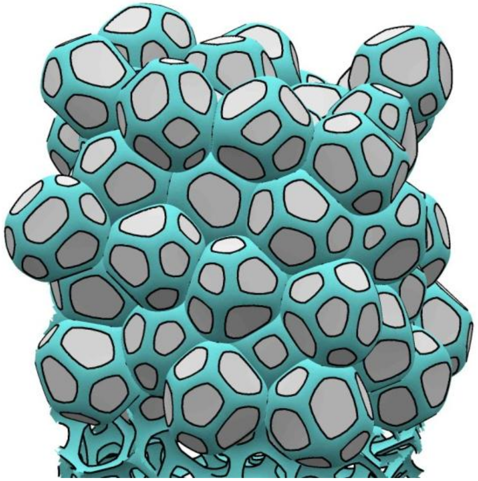


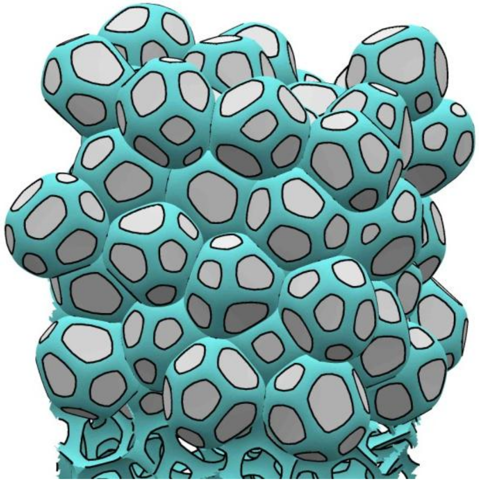
ϕ = 0.03

10

ϕ = 0.04

13

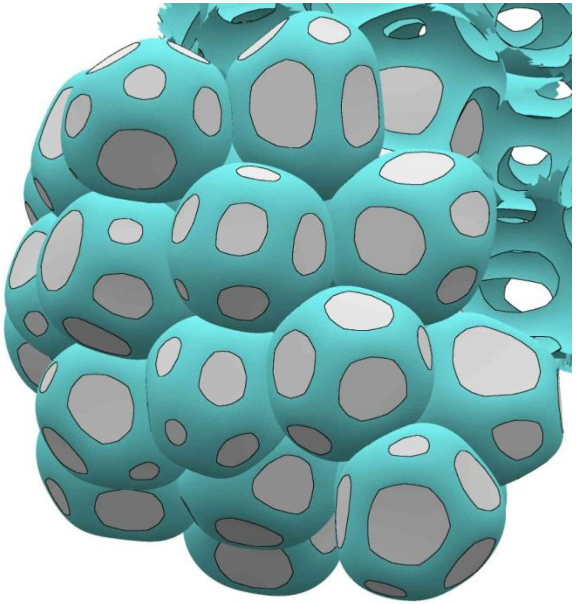




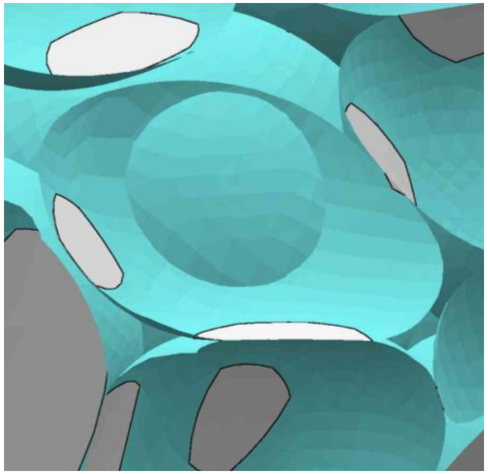
ϕ = 0.05

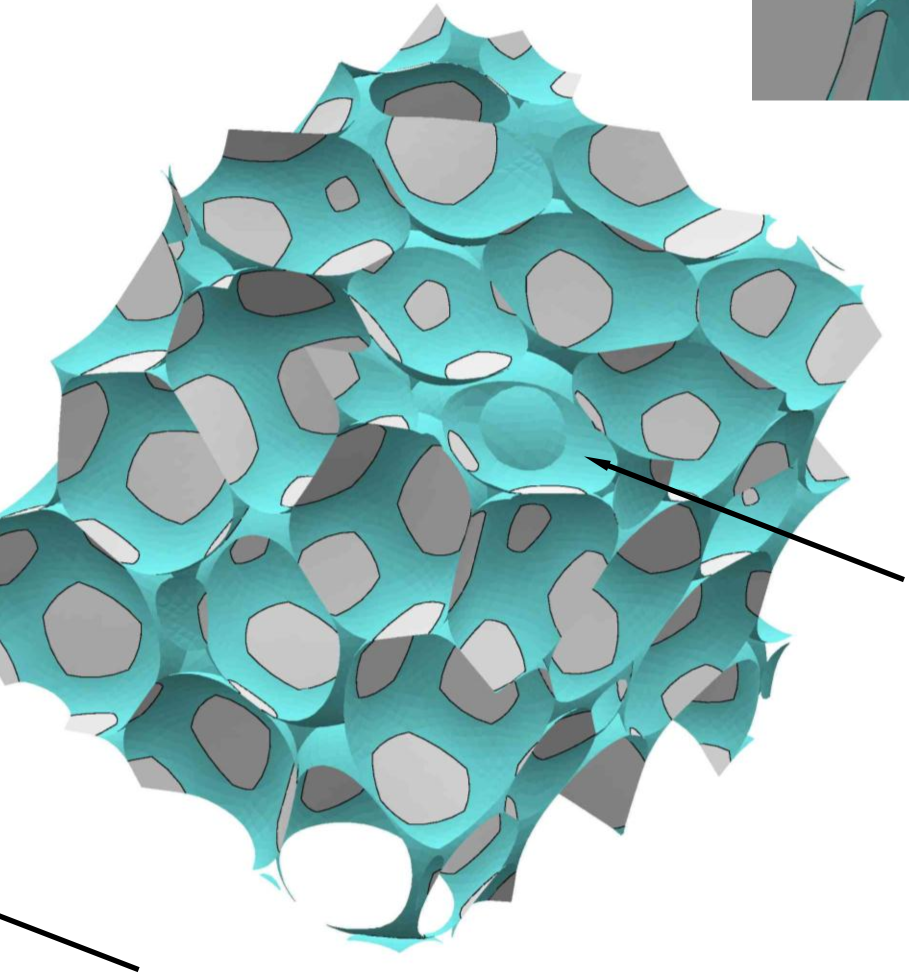
16

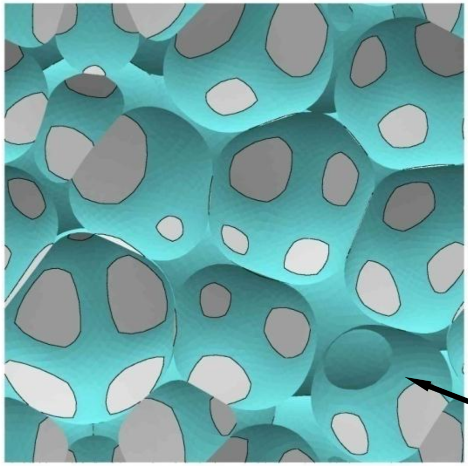
Bubble Overlap



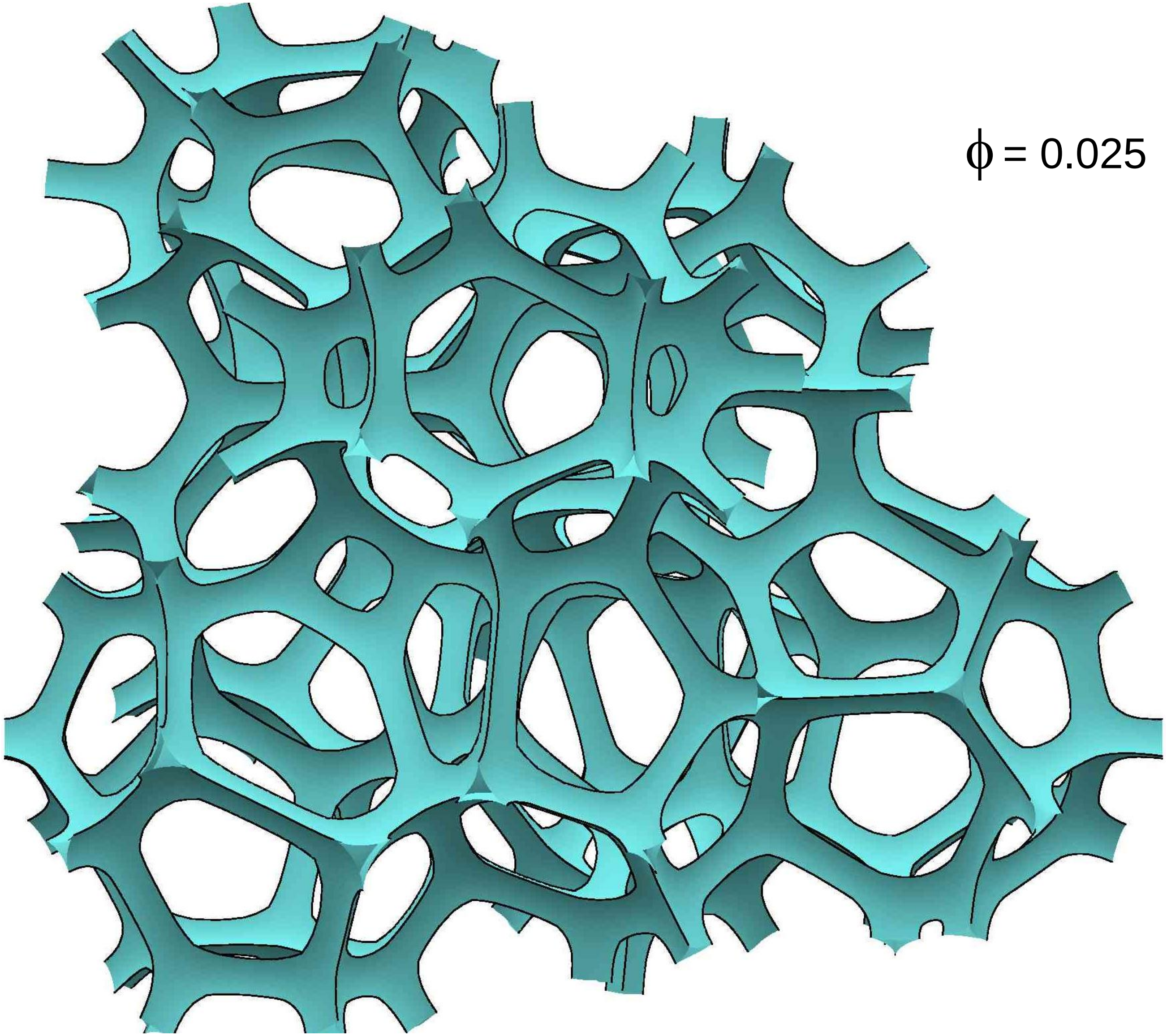
$\phi = 0.08$ 19 lost contacts





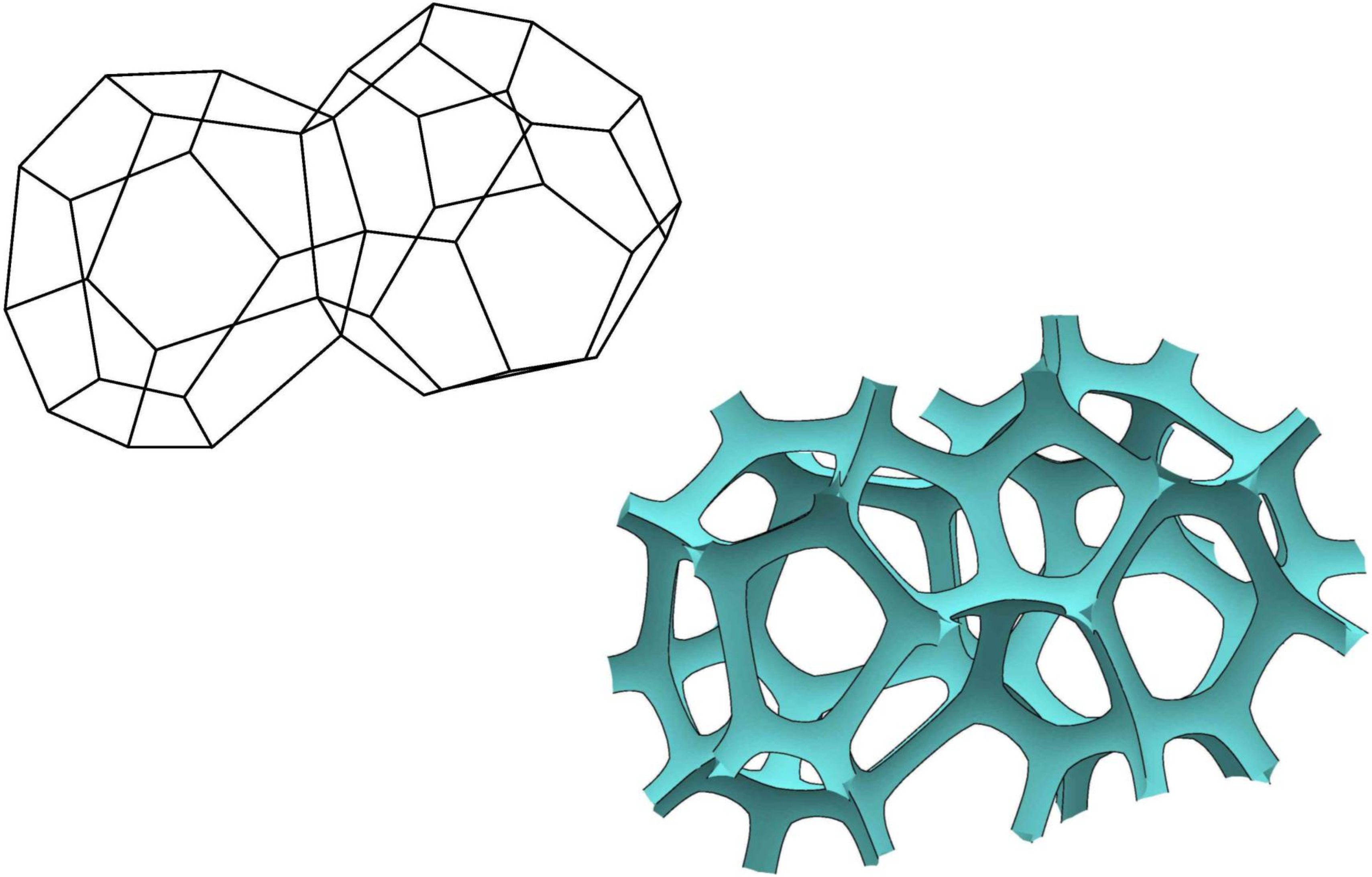


Random Monodisperse Foam



$\phi = 0.025$

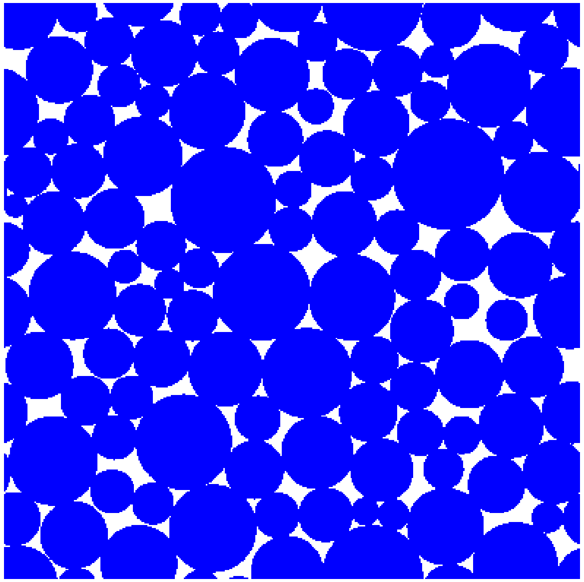
Modeling the strut-level geometry of open-cell foams



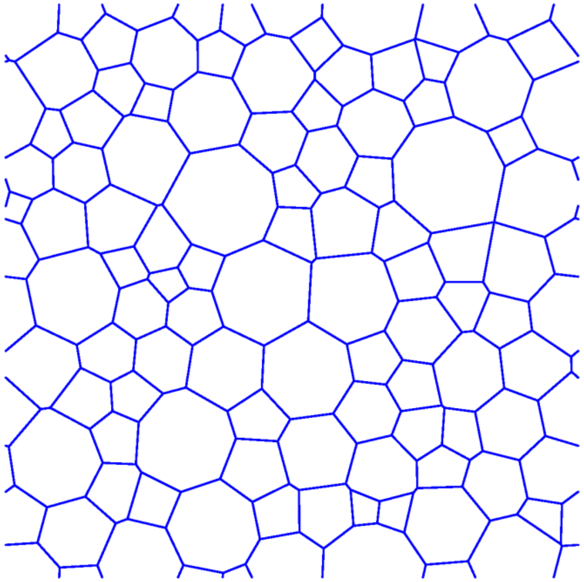
Multiple solutions when the thin film tension is 2σ

The junction between the Plateau borders and the thin films is not unique computationally and can lead to overlapping interfaces near the cusp when the film tension is 2σ.

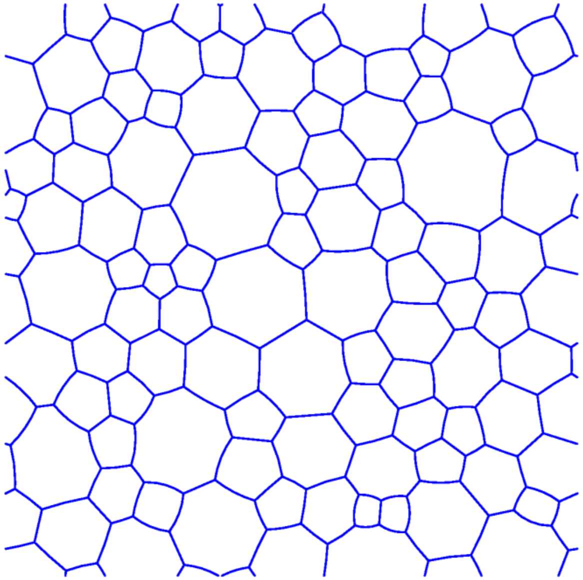
Random Polydisperse 2D Foams



Packed Disks



Voronoi Polygons



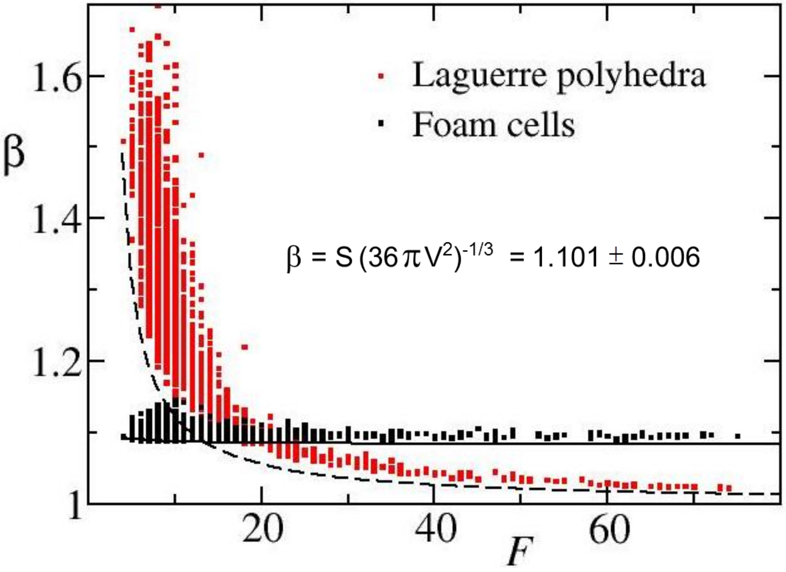
Equilibrium

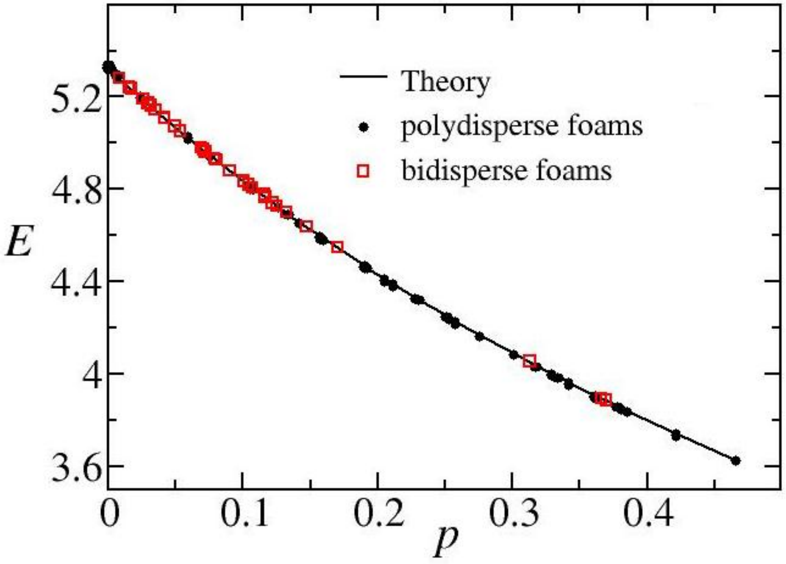
Surface Free Energy Density in 3D

$$E = \sigma \frac{S_F}{V_F} = \sigma \frac{\sum S}{\sum V} = 3\beta\sigma \frac{\langle R^2 \rangle}{\langle R^3 \rangle} = 3\beta \frac{\langle R^3 \rangle^{1/3}}{R_{32}} \frac{\sigma}{\langle R^3 \rangle^{1/3}} = \frac{\beta\,(36\pi)^{1/3}}{1 + \left[\frac{R_{32}}{\langle R^3 \rangle^{1/3}} - 1\right]} \frac{\sigma}{\langle V \rangle^{1/3}}$$

$$E = \frac{3\beta\sigma}{R_{32}} = \frac{\beta\,(36\pi)^{1/3}}{1+p} \frac{\sigma}{\langle V \rangle^{1/3}} = \frac{5.32}{1+p} \frac{\sigma}{\langle V \rangle^{1/3}}$$

Sauter mean radius $R_{32} = \frac{\langle R^3 \rangle}{\langle R^2 \rangle}$ polydispersity $p = \frac{R_{32}}{\langle R^3 \rangle^{1/3}} - 1 \geq 0$



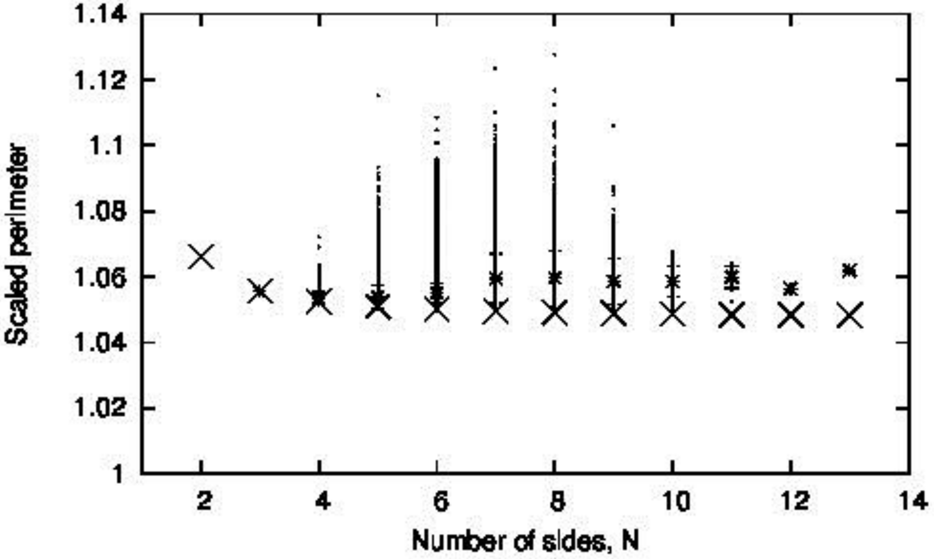


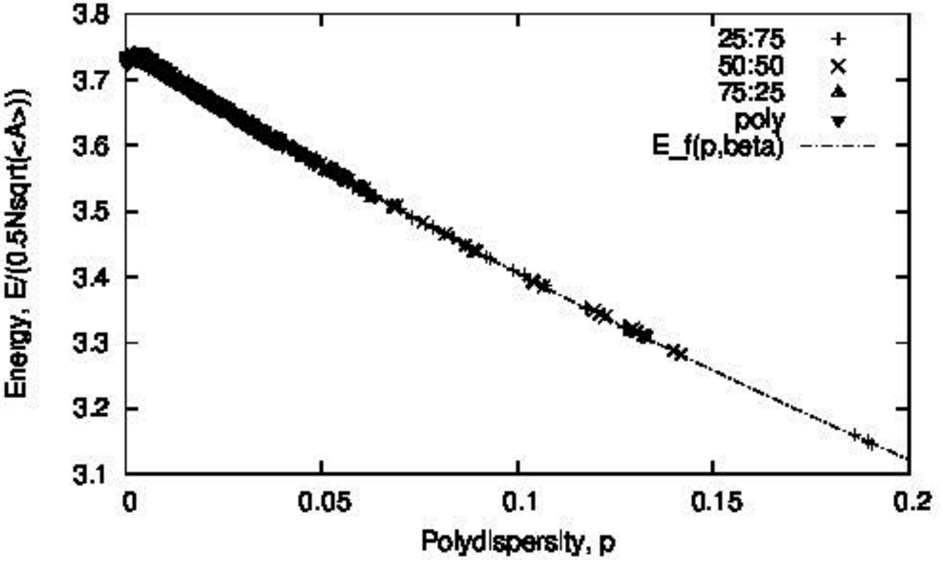
Surface Free Energy Density in 2D

$$E = \sigma \frac{L_F}{A_F} = \sigma \frac{\sum L}{\sum A} = 2\beta\sigma \frac{\langle R\rangle}{\langle R^2\rangle} = 2\beta \frac{\langle R^2\rangle^{1/2}}{R_{21}} \frac{\sigma}{\langle R^2\rangle^{1/2}} = \frac{2\,\beta\,\pi^{1/2}}{1+\left[\frac{R_{21}}{\langle R^2\rangle^{1/2}} - 1\right]} \frac{\sigma}{\langle A\rangle^{1/2}}$$

$$E = \frac{2\beta\sigma}{R_{21}} = \frac{2\beta\pi^{1/2}}{1+p} \frac{\sigma}{\langle A\rangle^{1/2}} = \frac{3.75}{1+p} \frac{\sigma}{\langle A\rangle^{1/2}}$$

Sauter mean radius $R_{21} = \frac{\langle R^2\rangle}{\langle R\rangle}$ polydispersity $p = \frac{R_{21}}{\langle R^2\rangle^{1/2}} - 1 \geq 0$

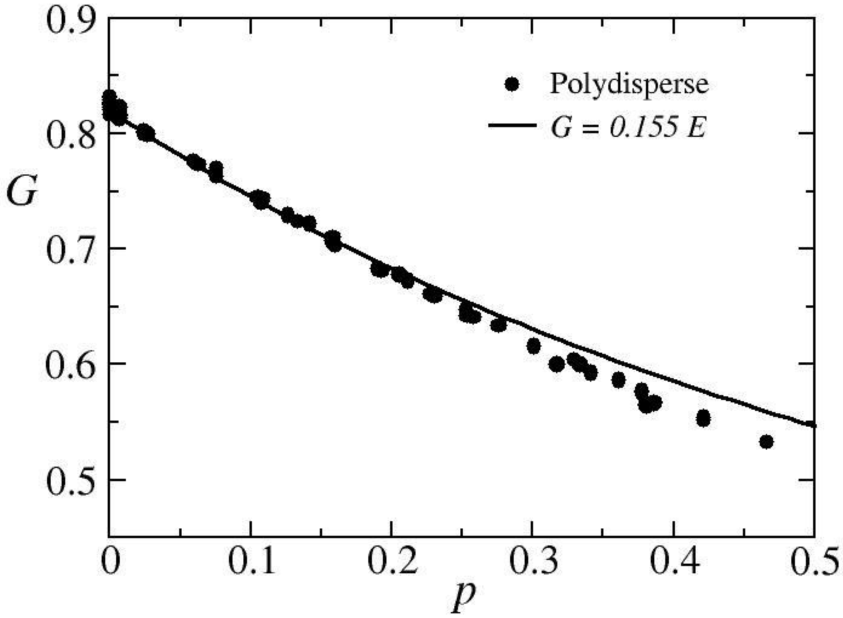


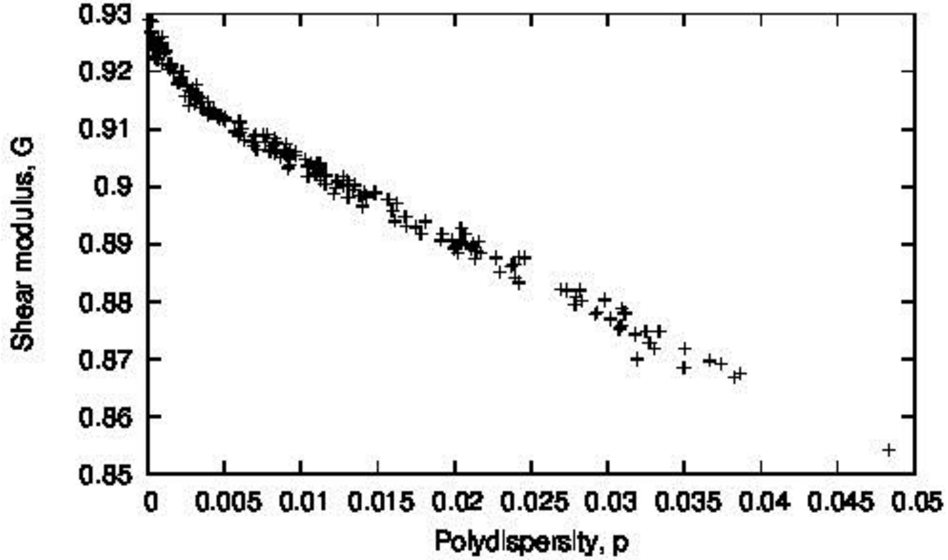


Shear Modulus

3D

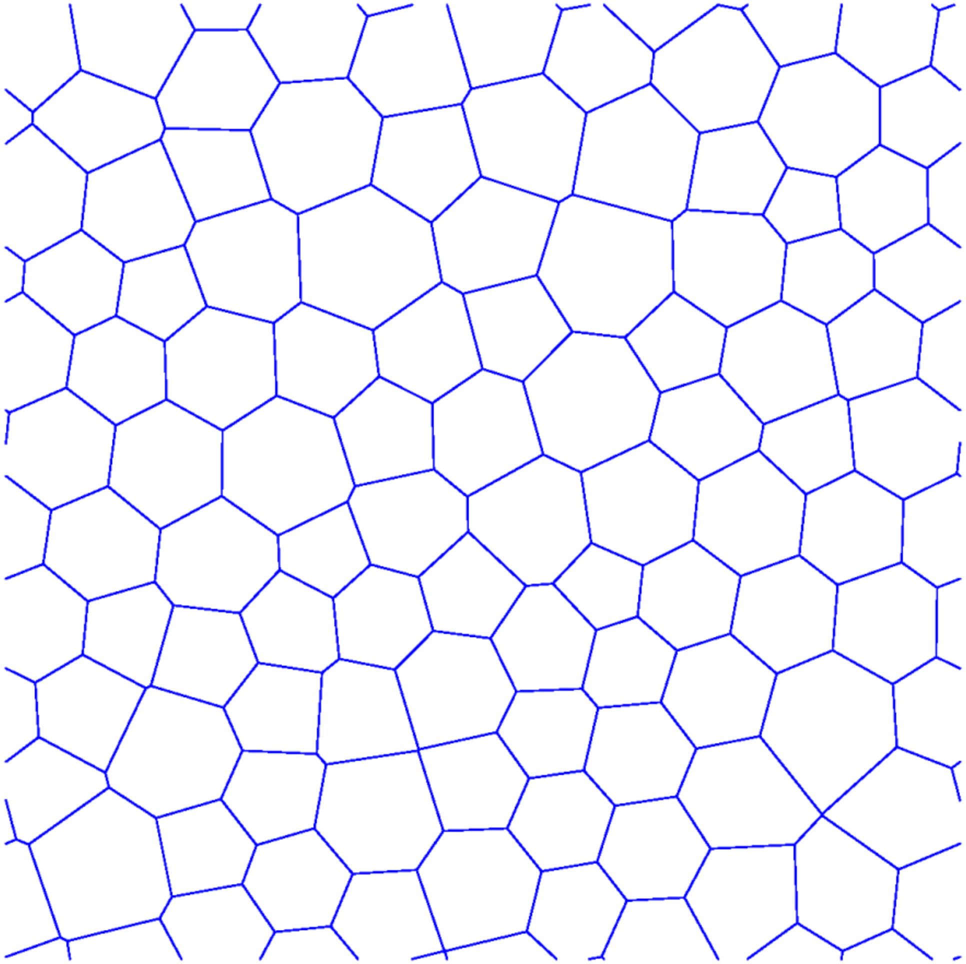
2D





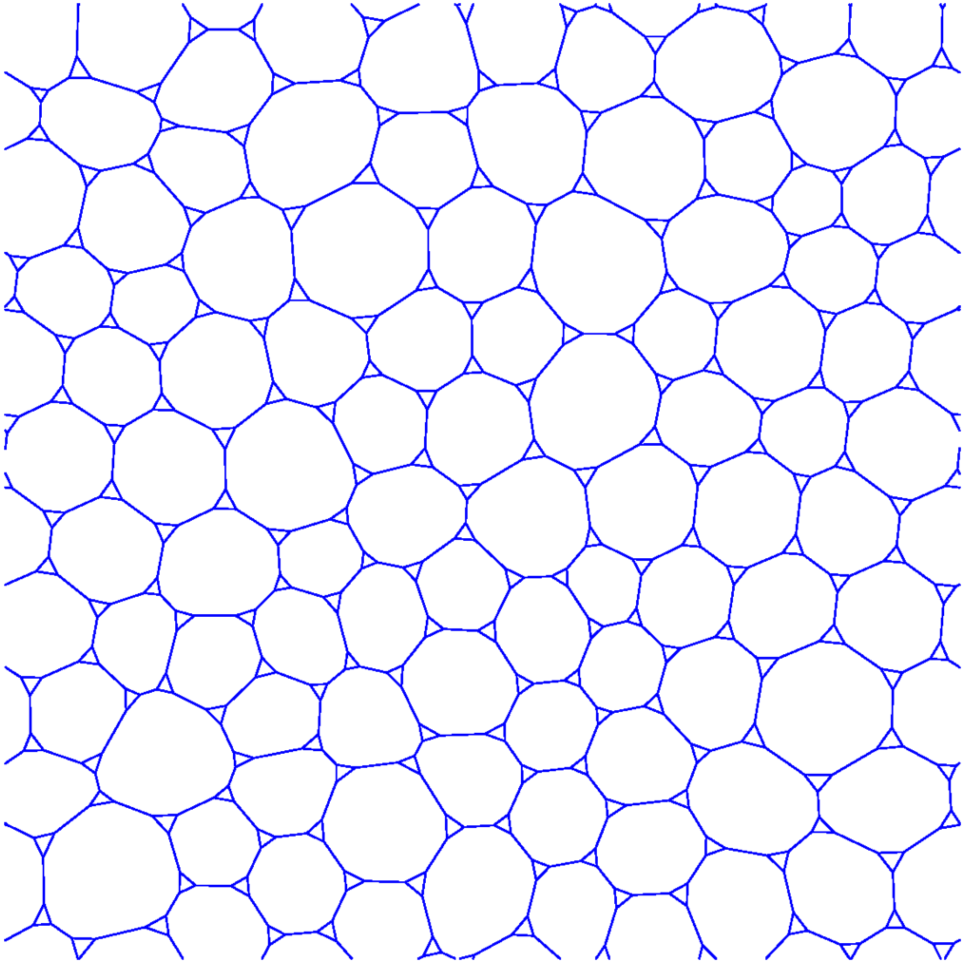
$$G \approx 0.155\ E = 0.512\ \frac{\sigma}{R_{32}}$$

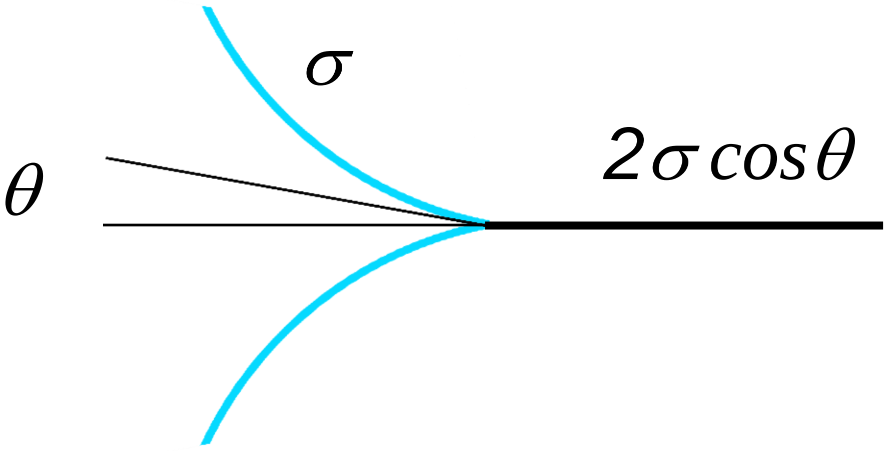
Creating Wet Random Polydisperse 2D Foams



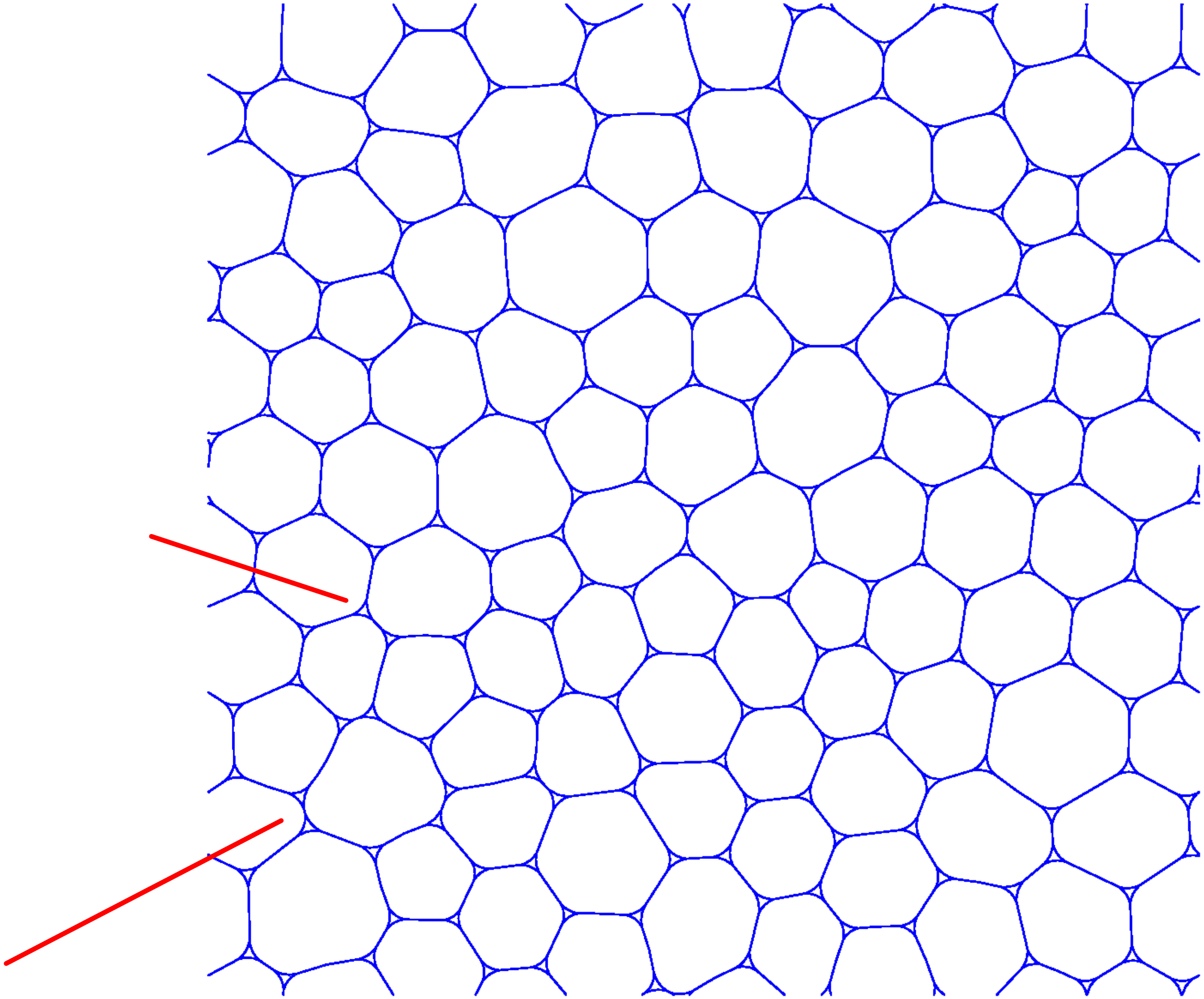
Weighted-Voronoi (Laguerre) Polygons

Weighted-Voronoi Polygons with Primitive Plateau Borders




σ
$2\sigma\cos\theta$
θ

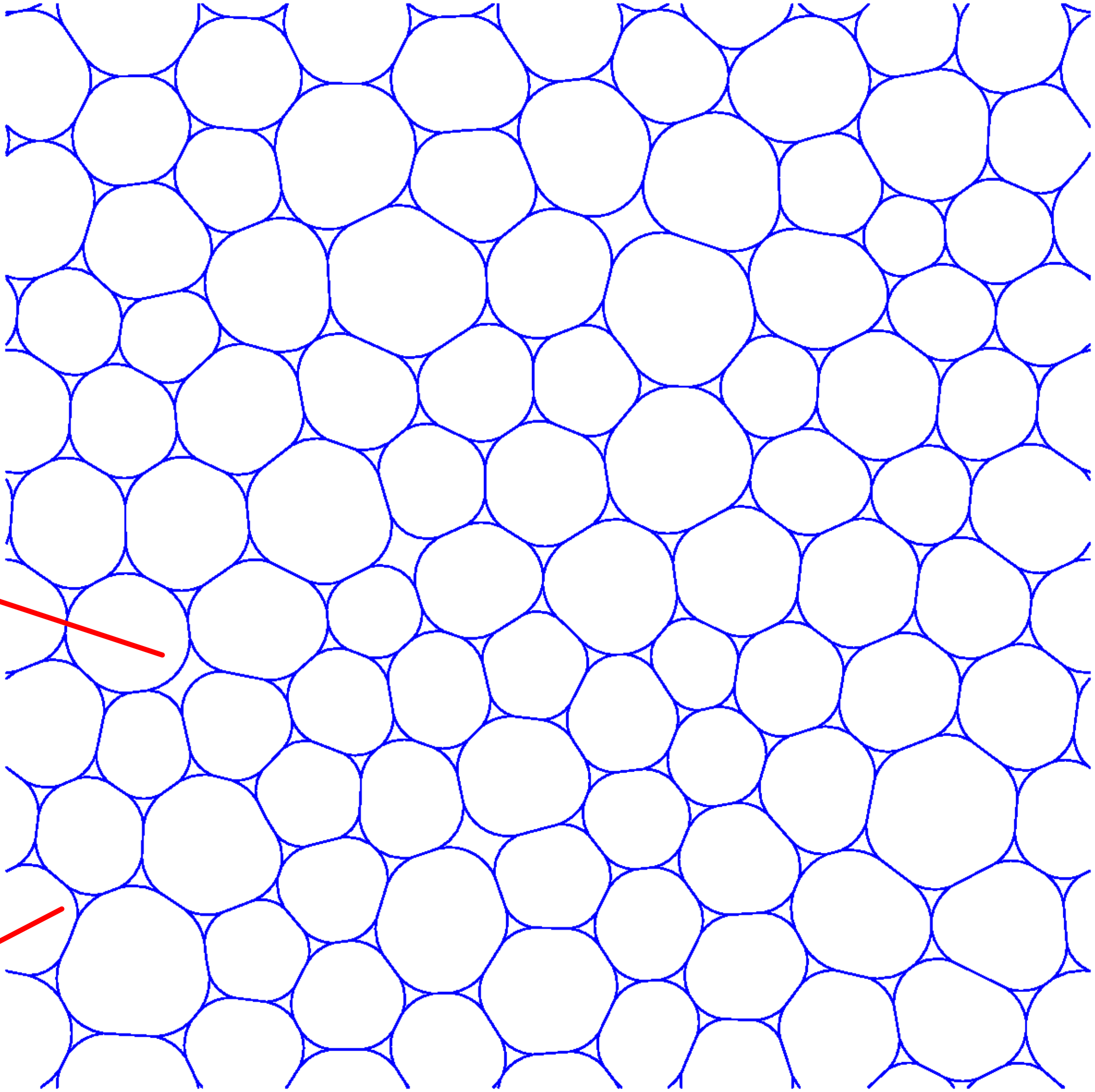
Wet Topological Transitions



$\theta = 8^o$

$\rho = 0.02$

Wet Topological Transitions



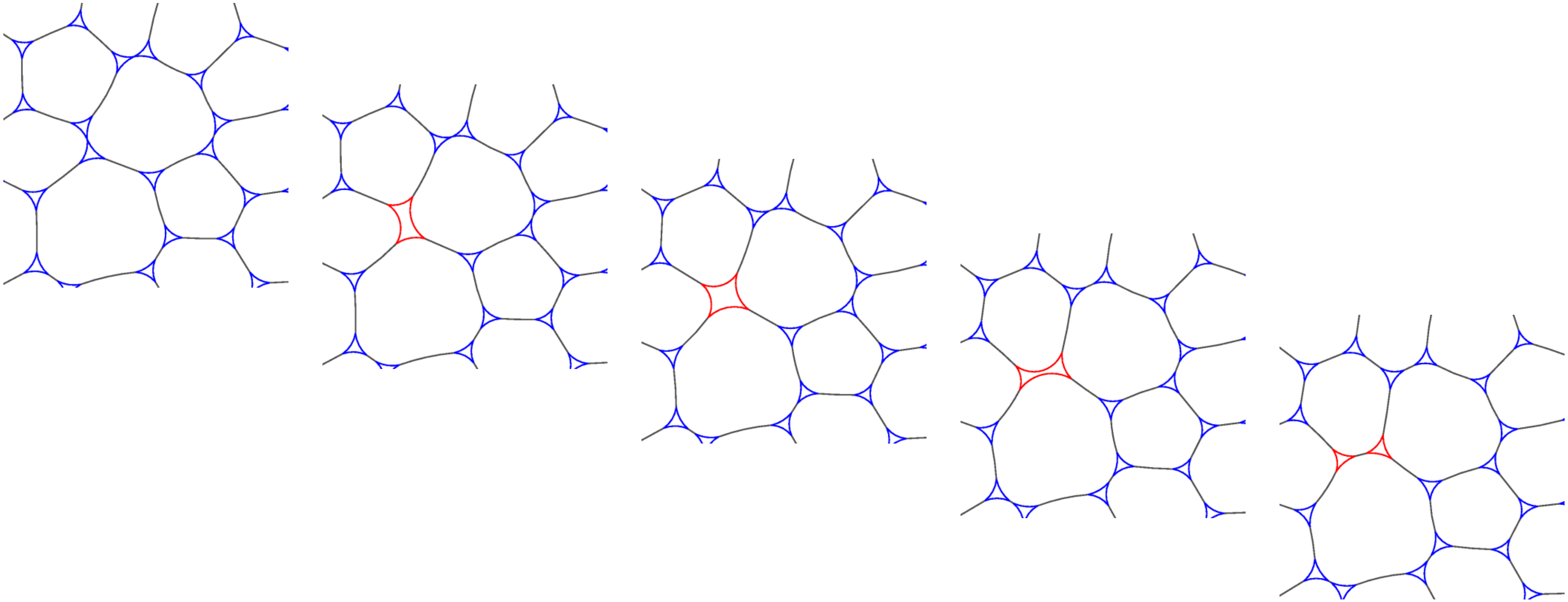
Neighbor
Loss

Neighbor
Switch

$\theta = 8^o$

$\rho = 0.05$

Wet Topological Transition
Neighbor Switching



Outlook for Random Wet 2D Foams

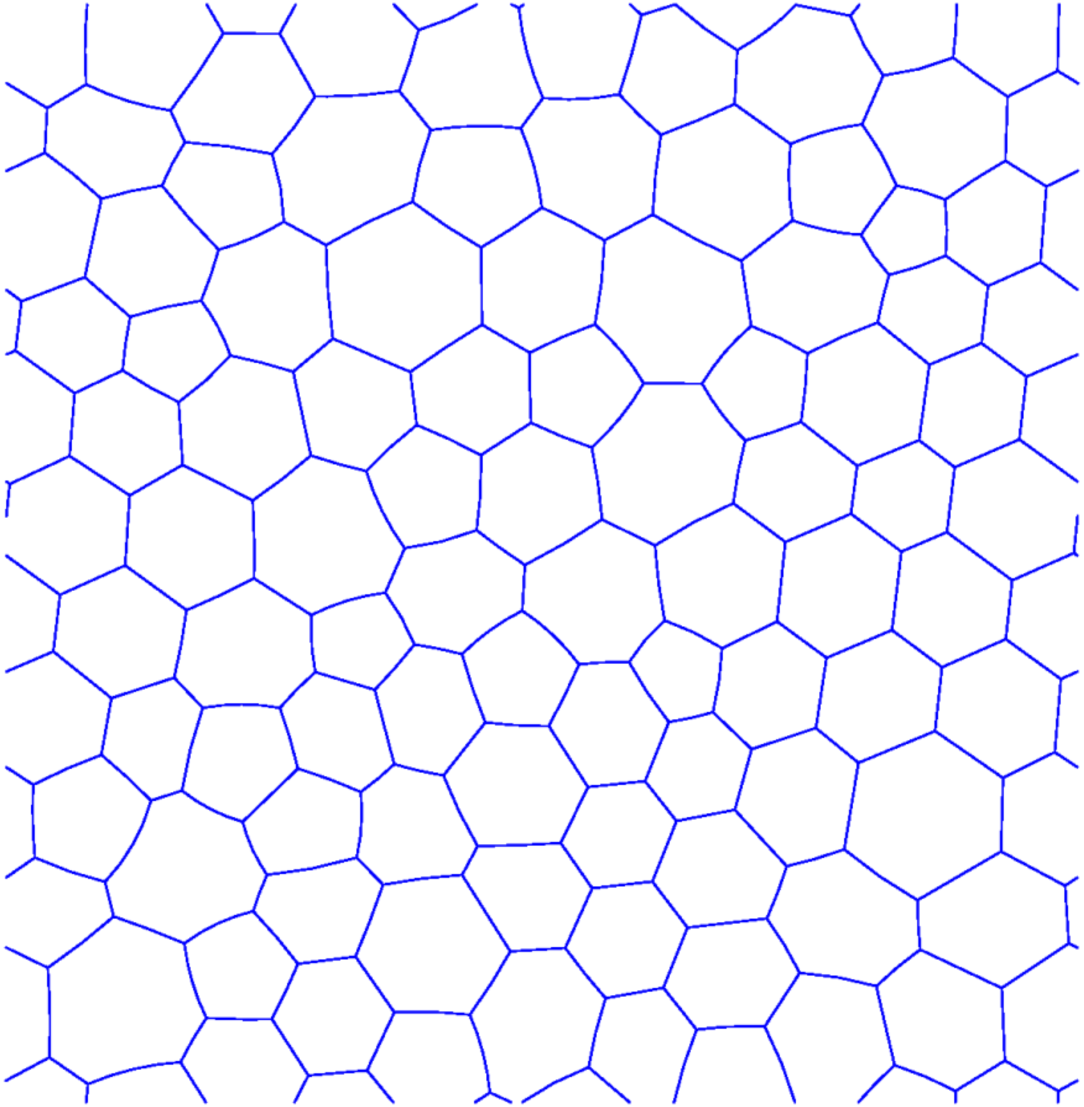
Topology

Geometry

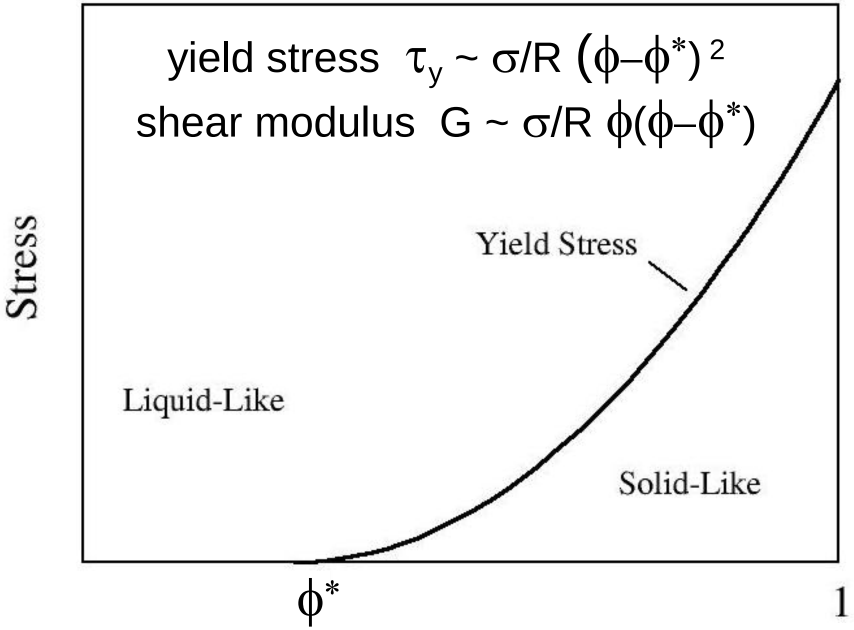
Rheology

Diffusive Coarsening

Swelling a foam by adding liquid



Transition Between Liquid-Like and Solid-Like Behavior



yield stress $\tau_y \sim \sigma/R\,(\phi-\phi^*)^2$

shear modulus $G \sim \sigma/R\,\phi(\phi-\phi^*)$

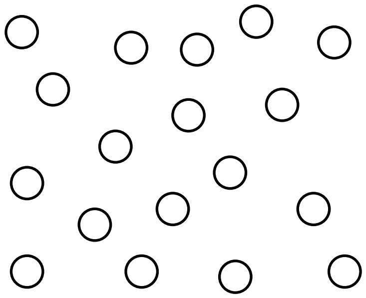
ϕ = volume fraction of dispersed phase

ϕ^* = onset of jamming

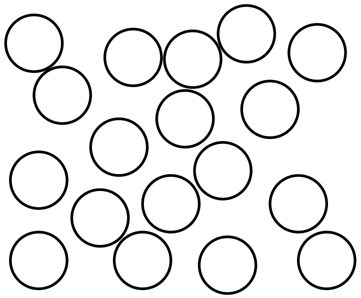
σ = surface tension

R = bubble/drop size

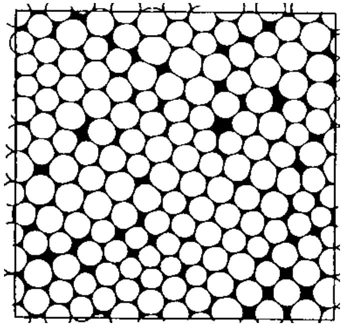
Hohler & Cohen-Addad (2005)



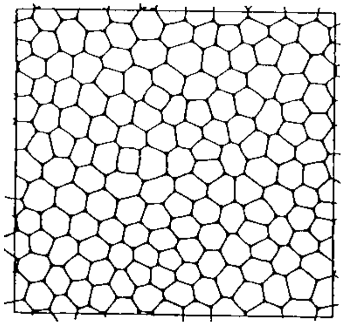
Semi-dilute gas-bubble suspension
or liquid-liquid emulsion



Concentrated suspension
or emulsion

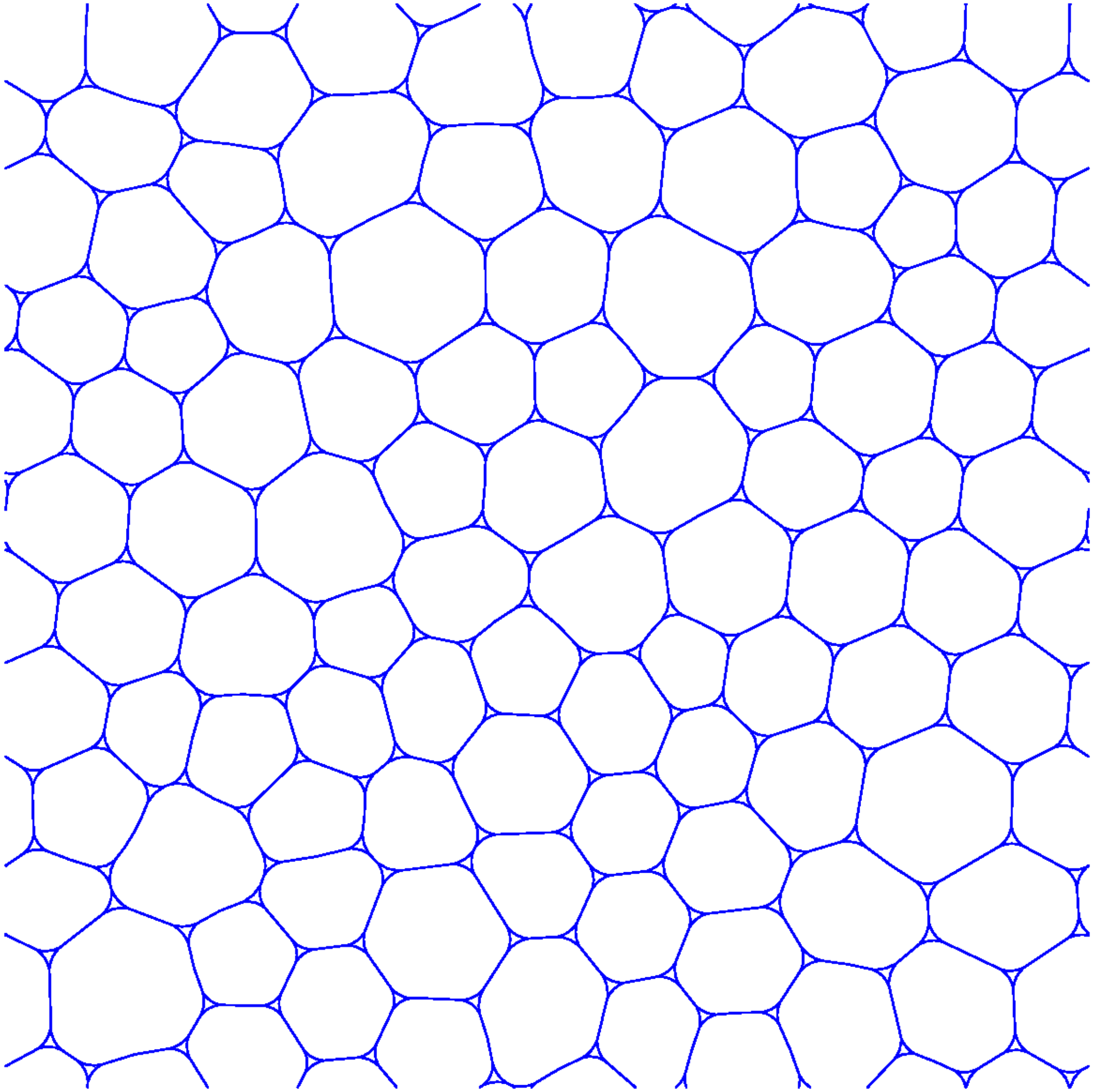


Wet Foam



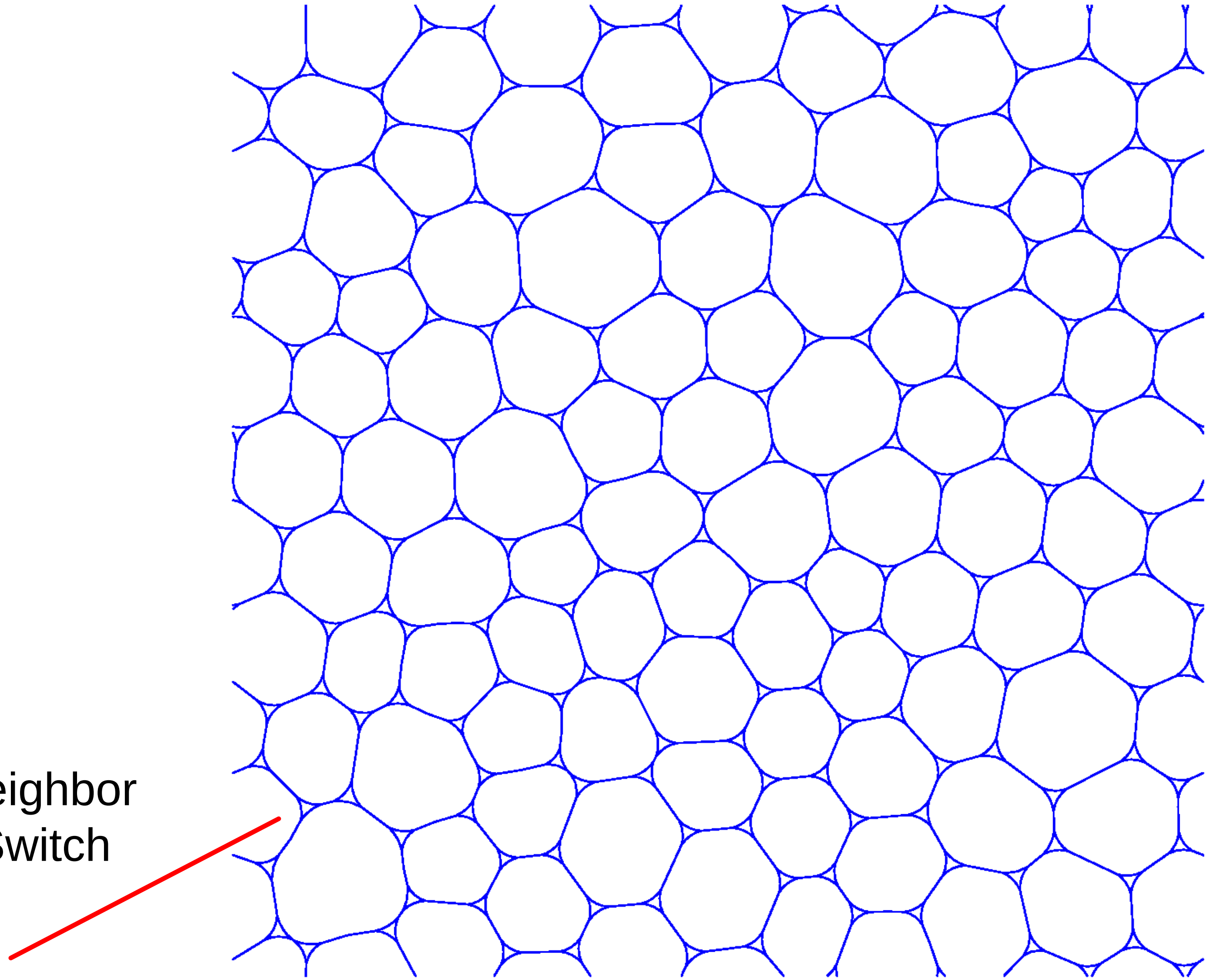
Dry Foam

Highly Concentrated Emulsion
High Internal Phase Emulsion (HIPE)

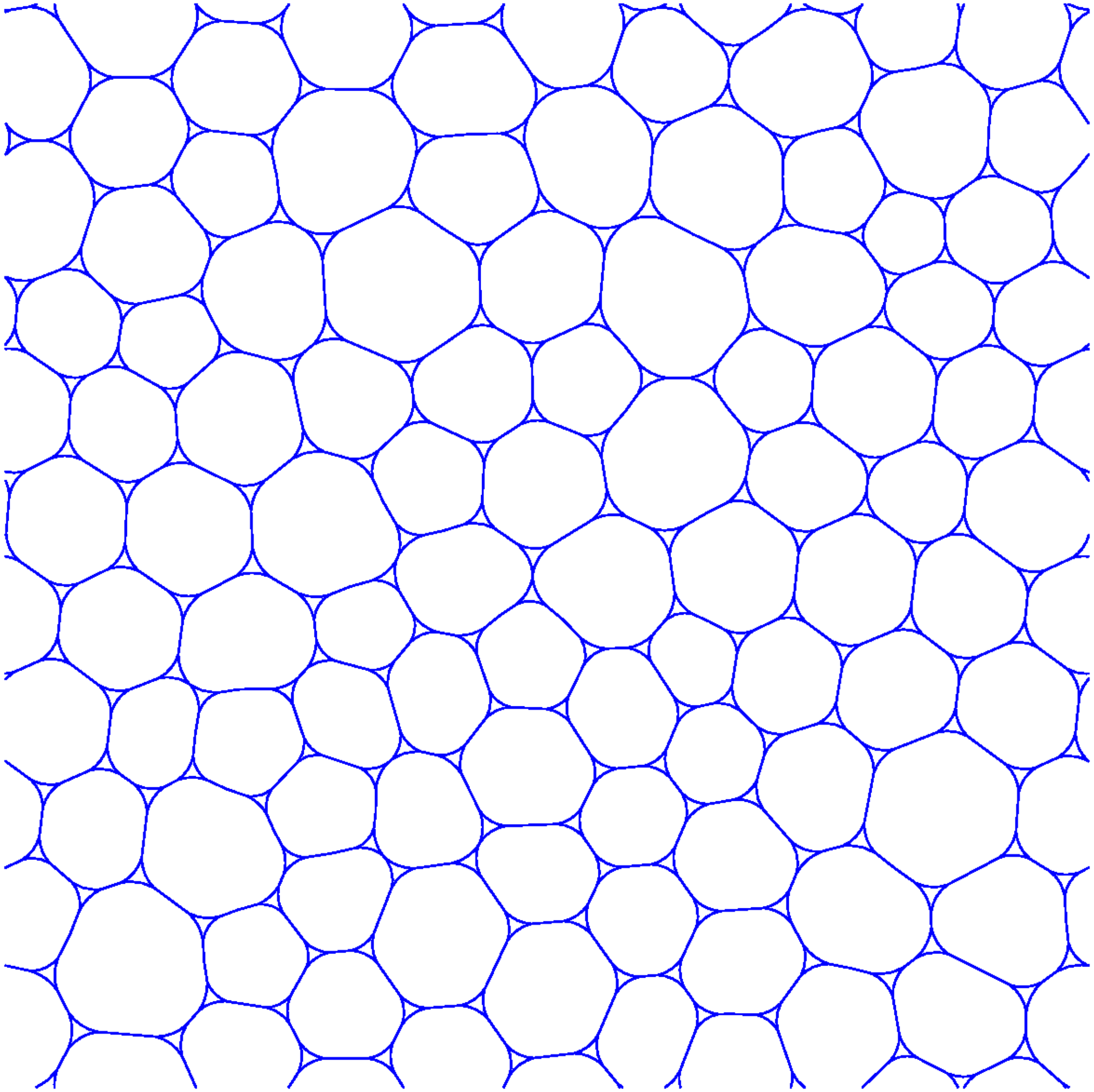


$\theta = 8^{o}$

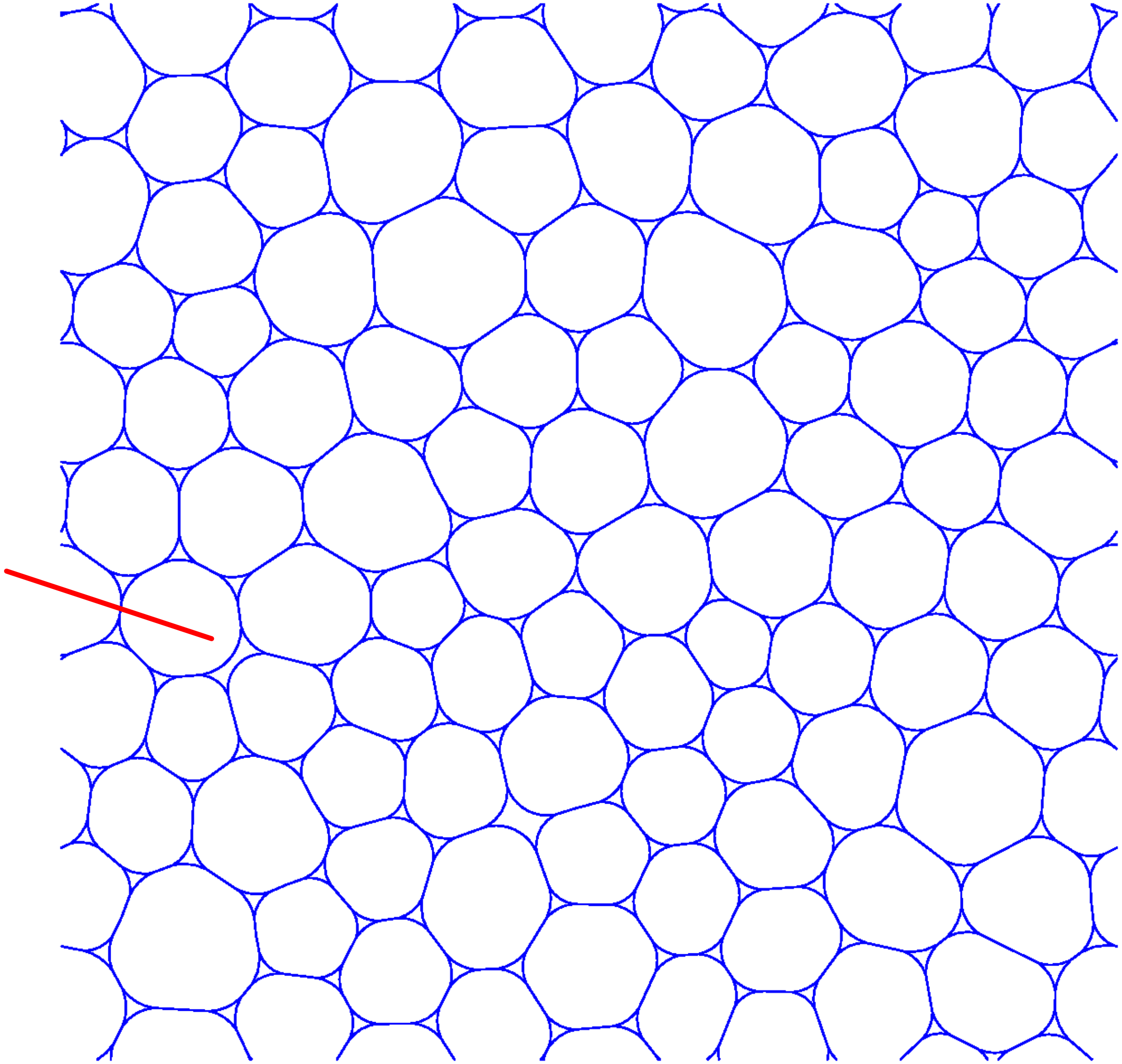
$\rho = 0.02$


Neighbor
Switch

ρ = 0.03

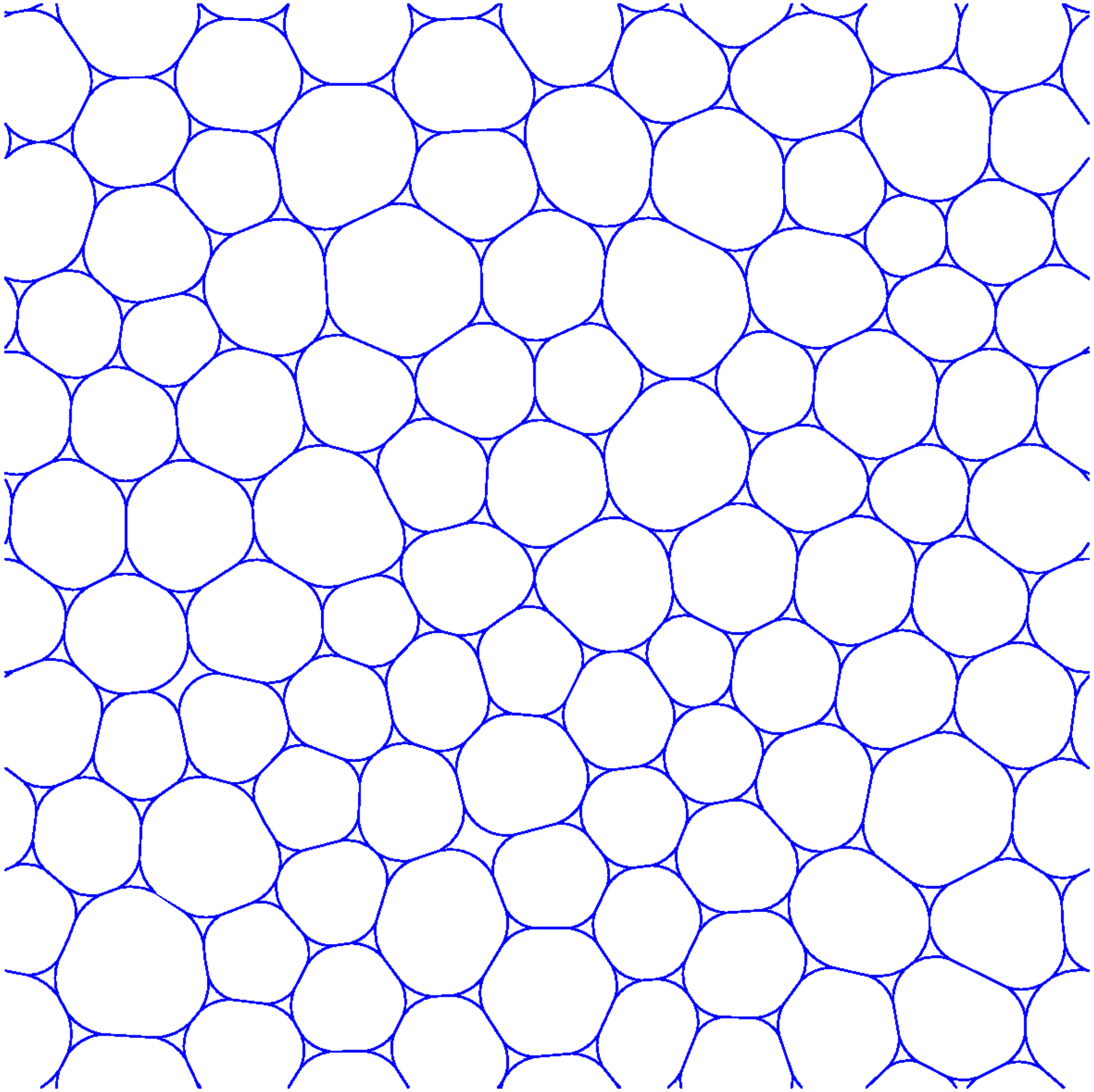


$\rho = 0.035$

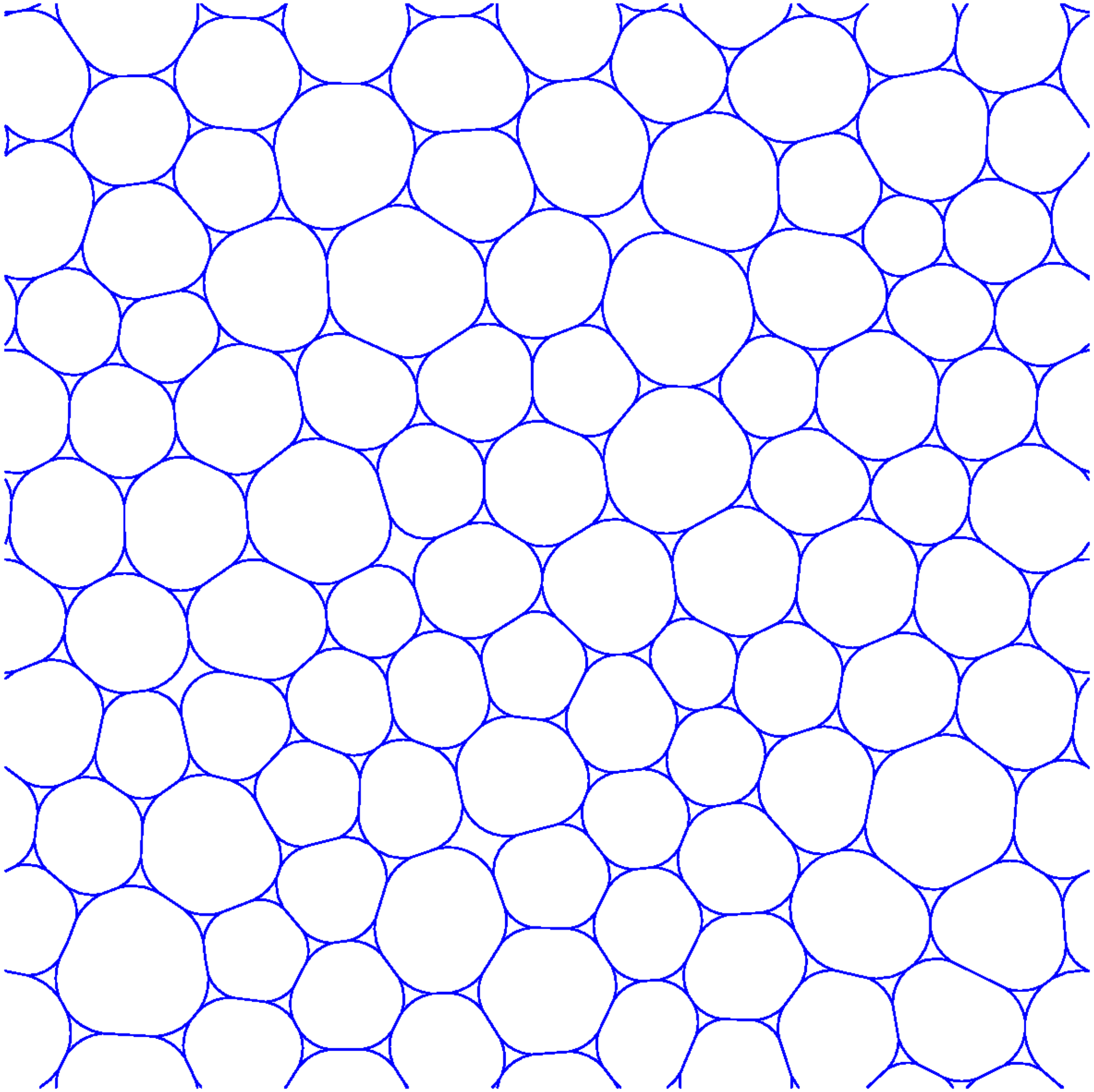


Neighbor Loss

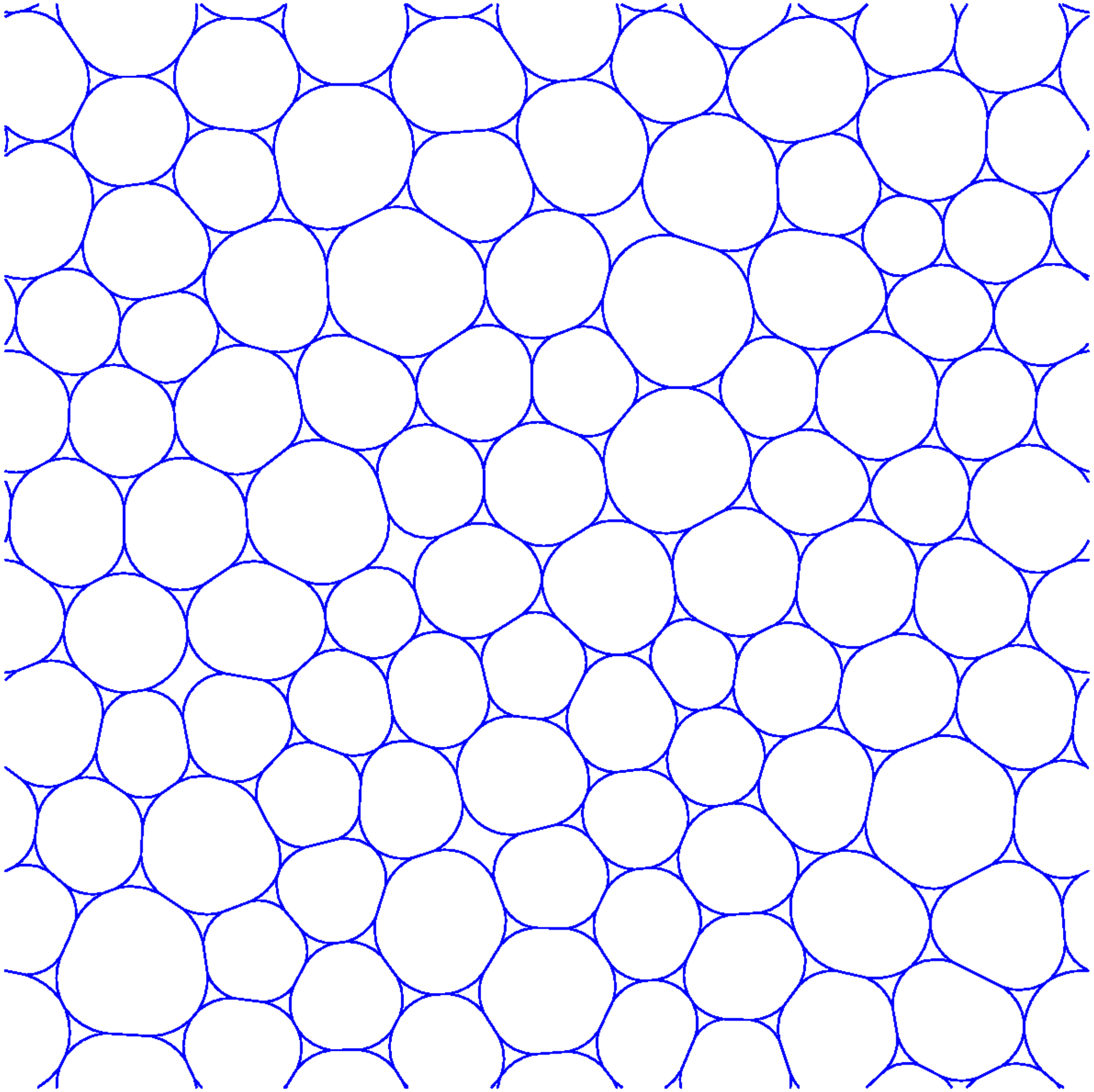
ρ = 0.04



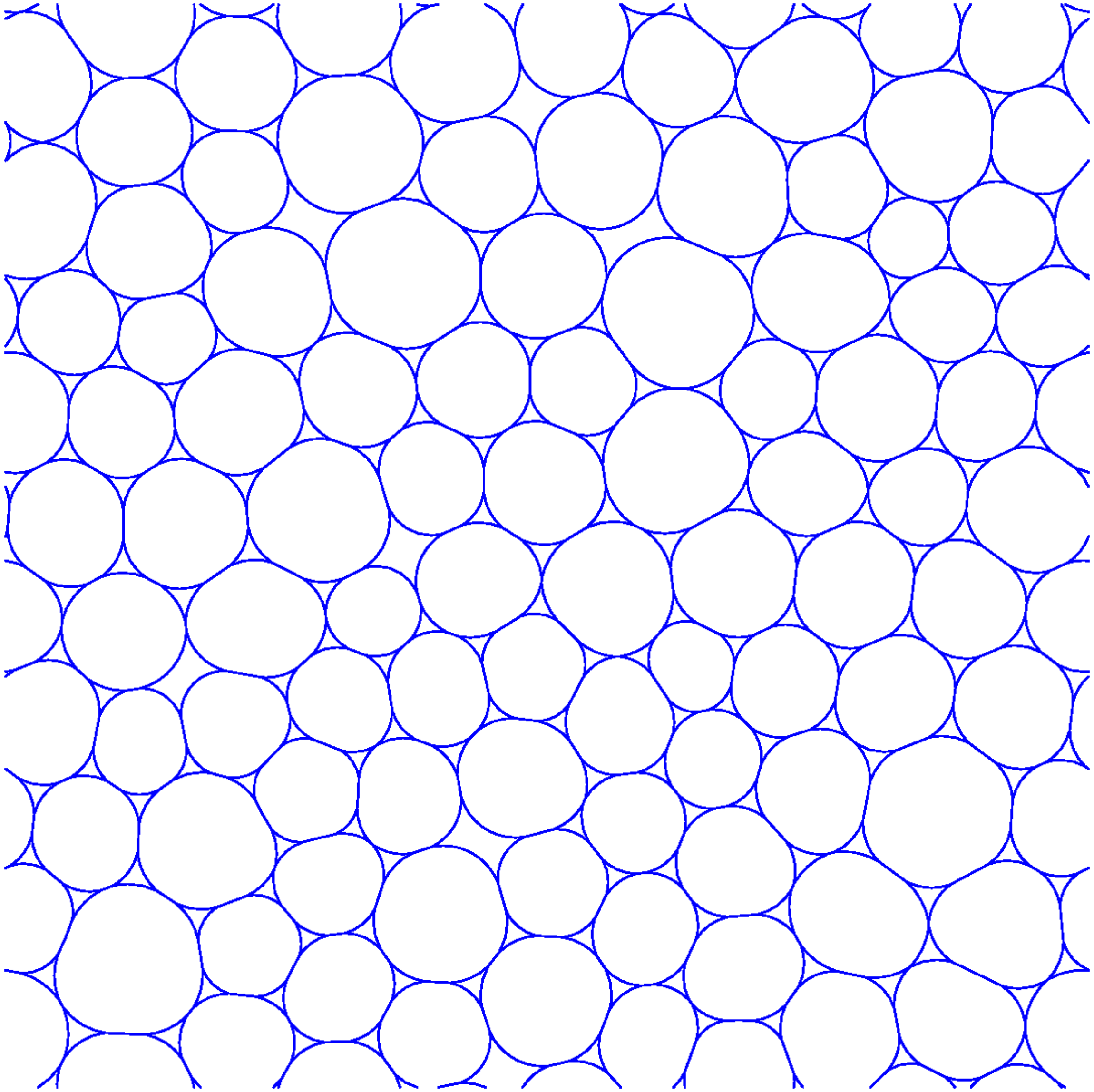
$\rho = 0.045$



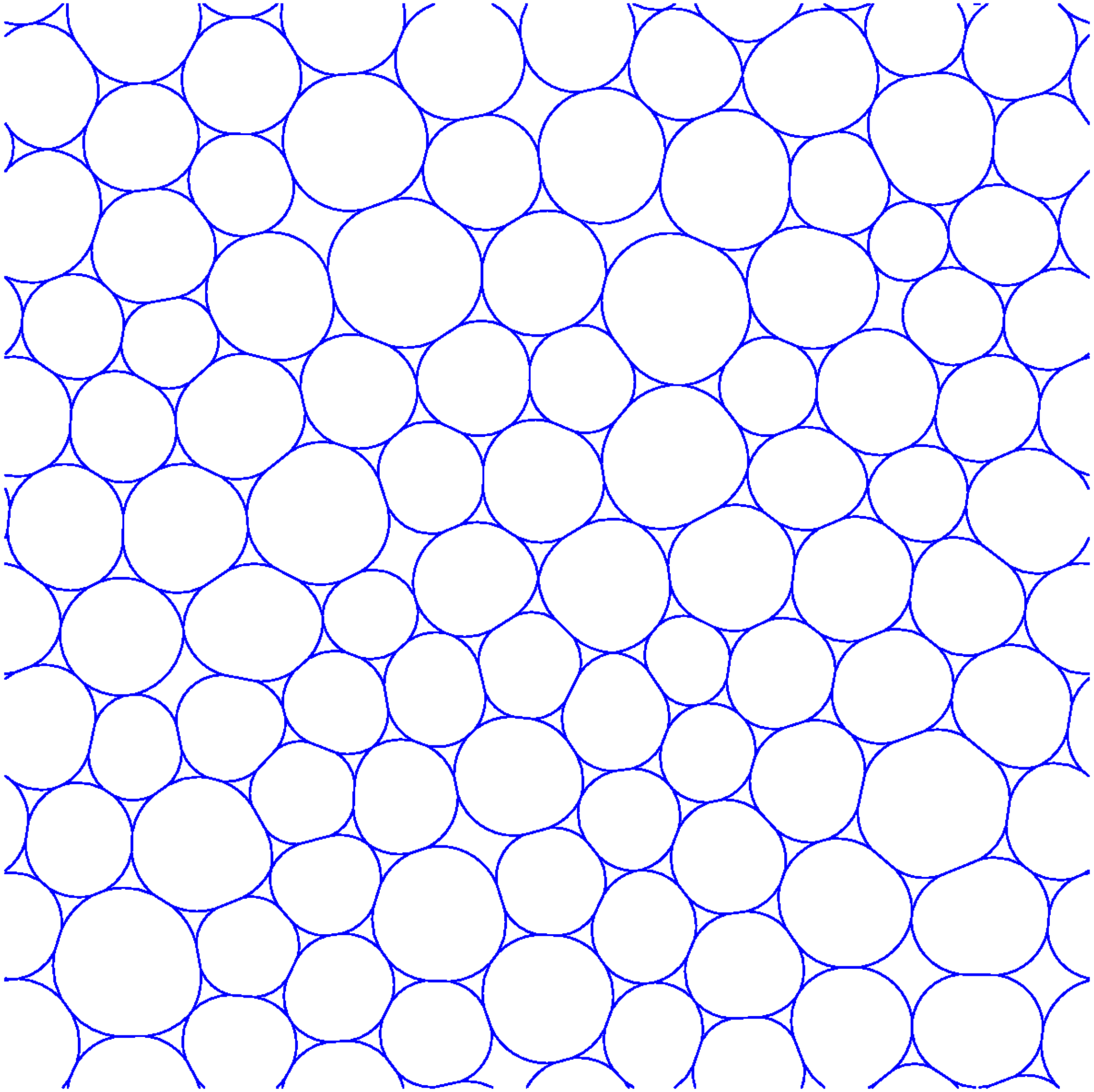
$\rho = 0.05$



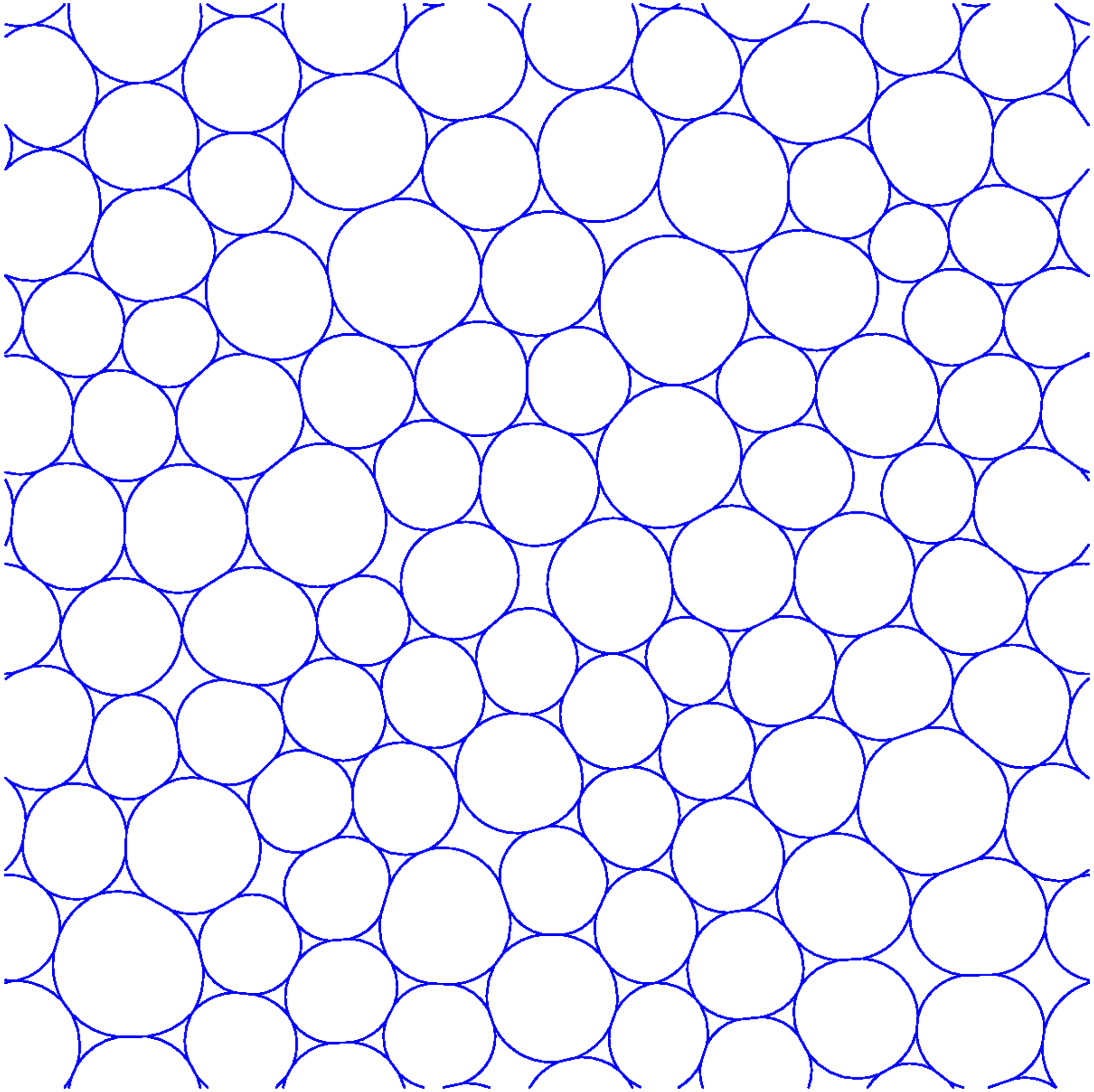
$\rho = 0.06$



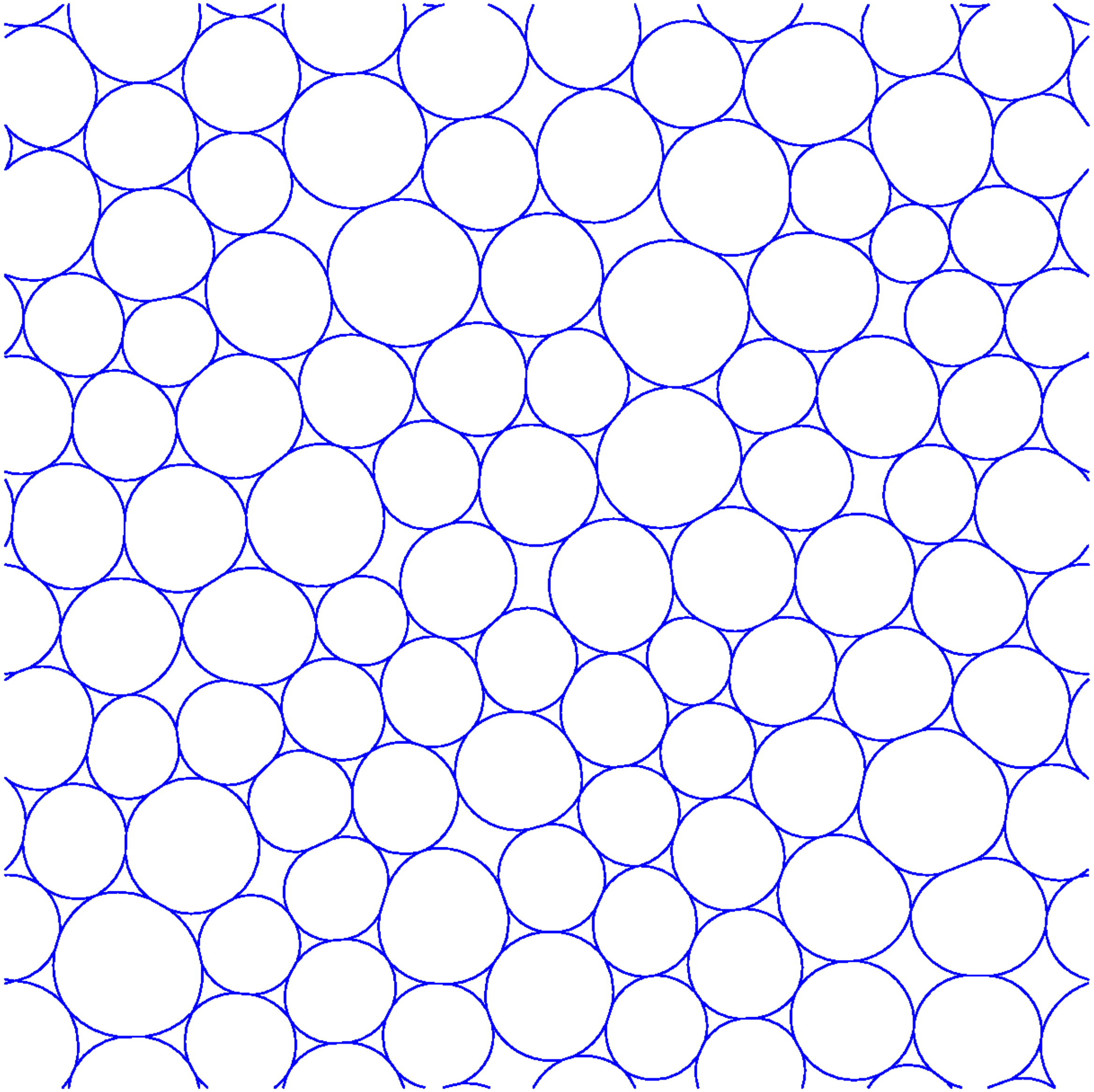
$\rho = 0.07$



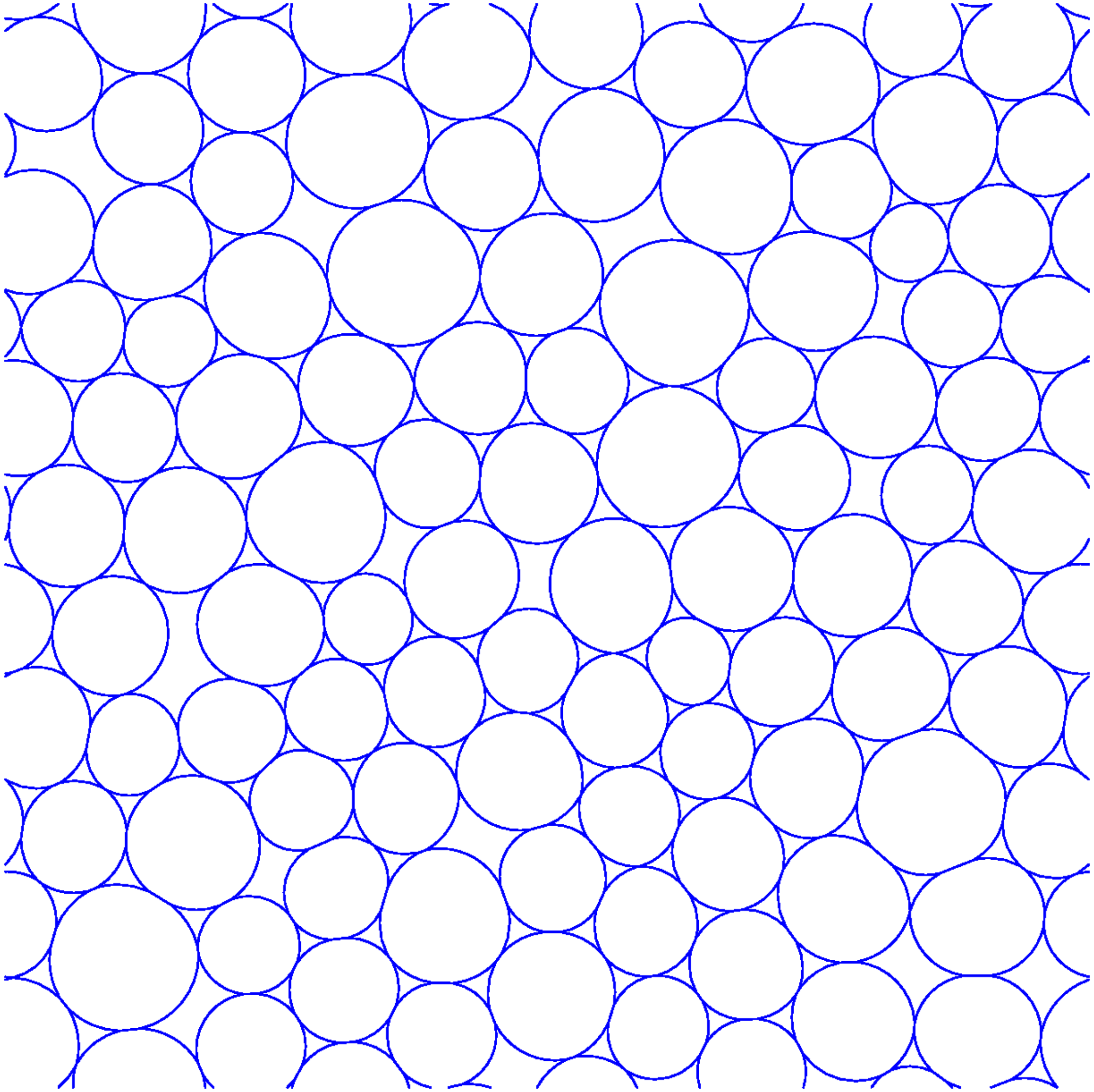
$\rho = 0.08$



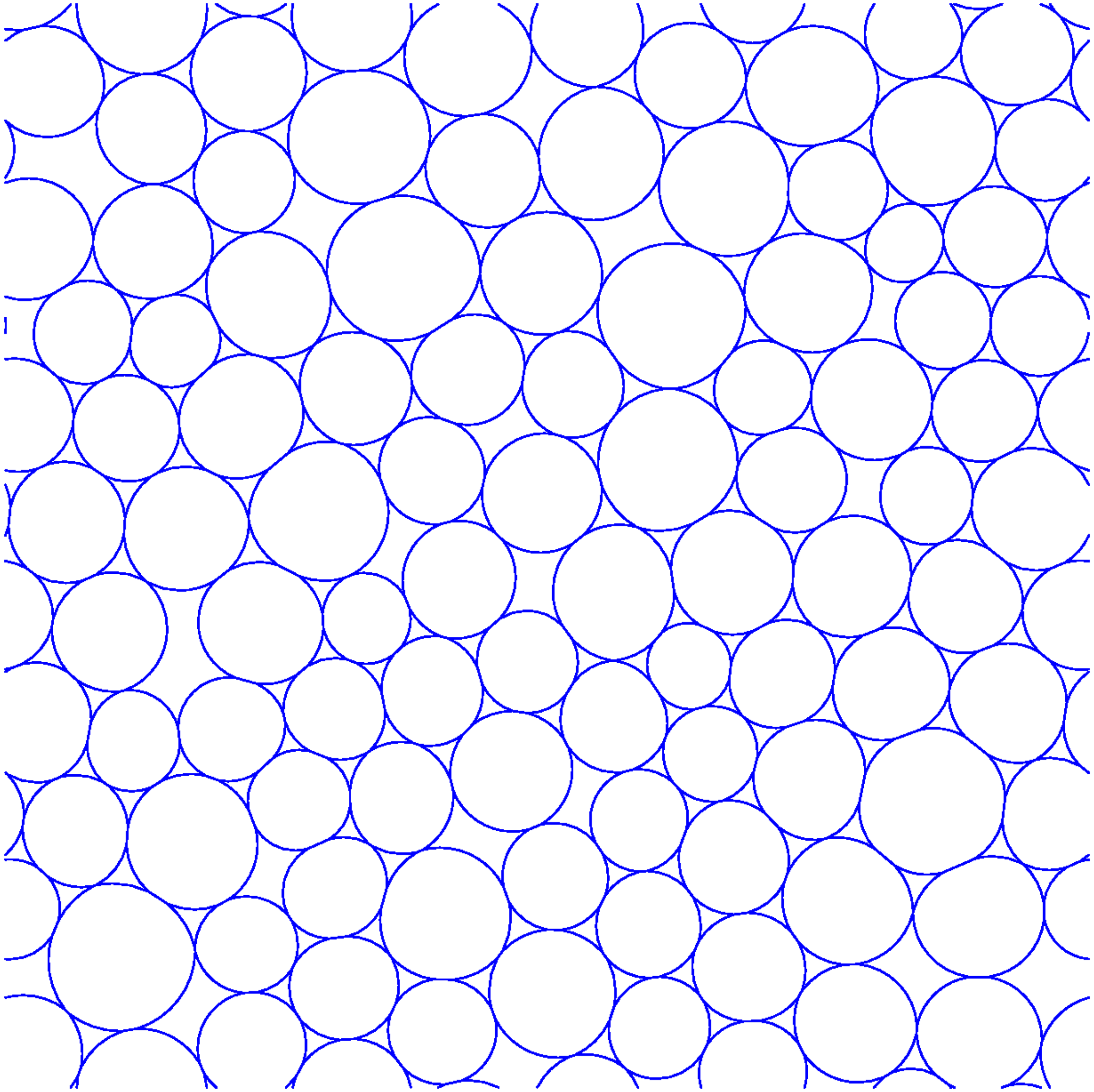
$\rho = 0.09$



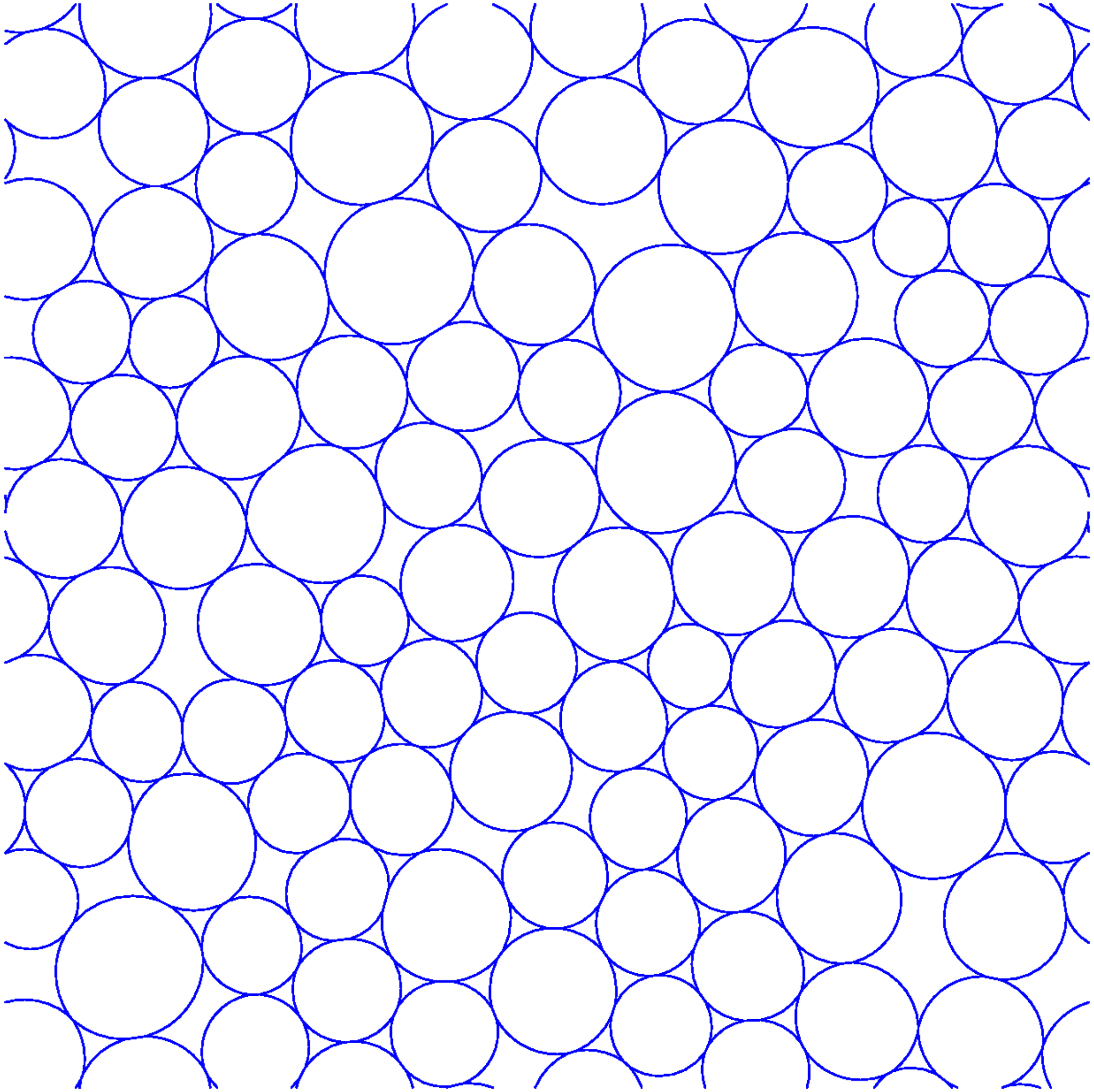
ρ = 0.10



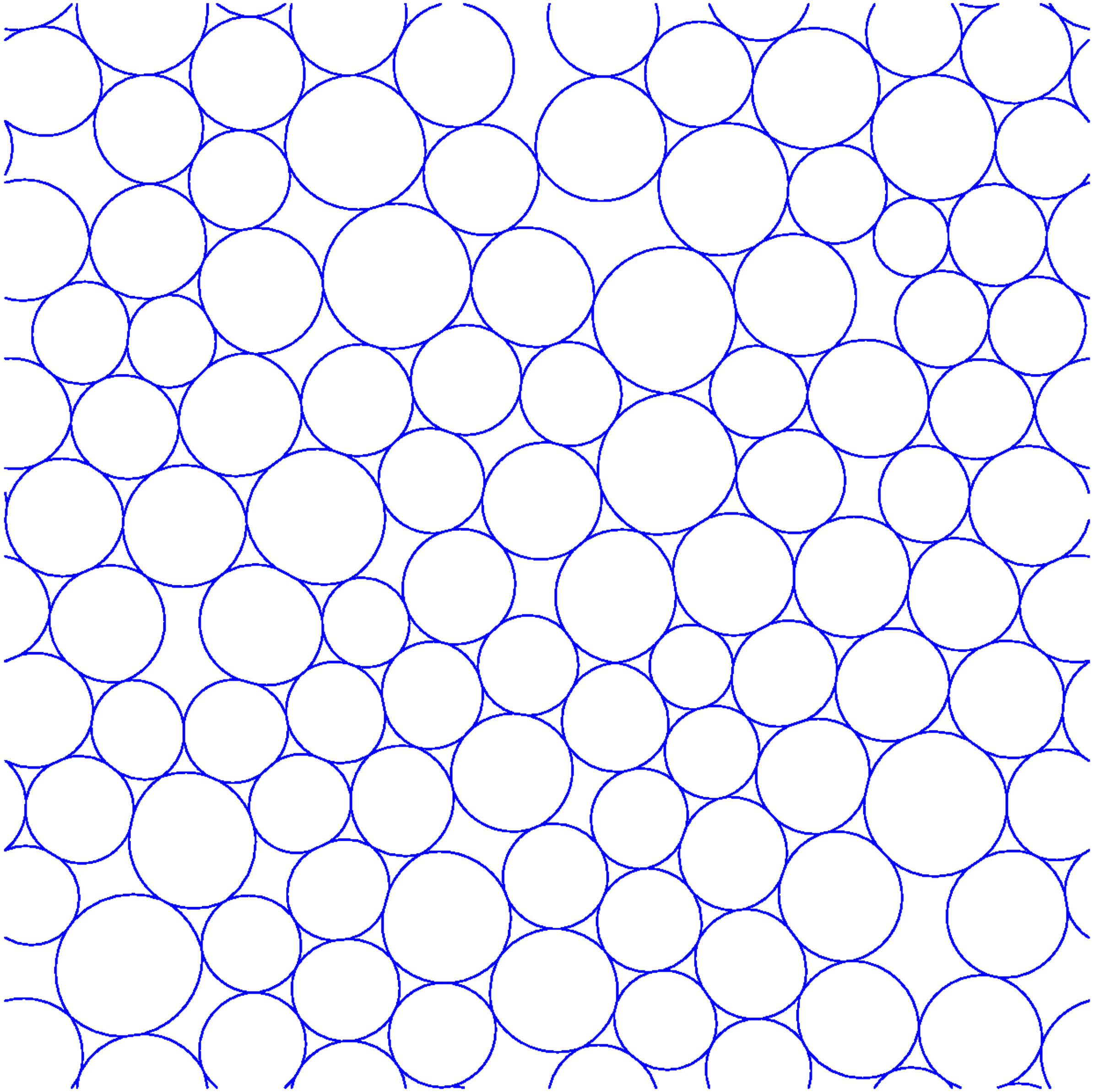
$\rho = 0.11$



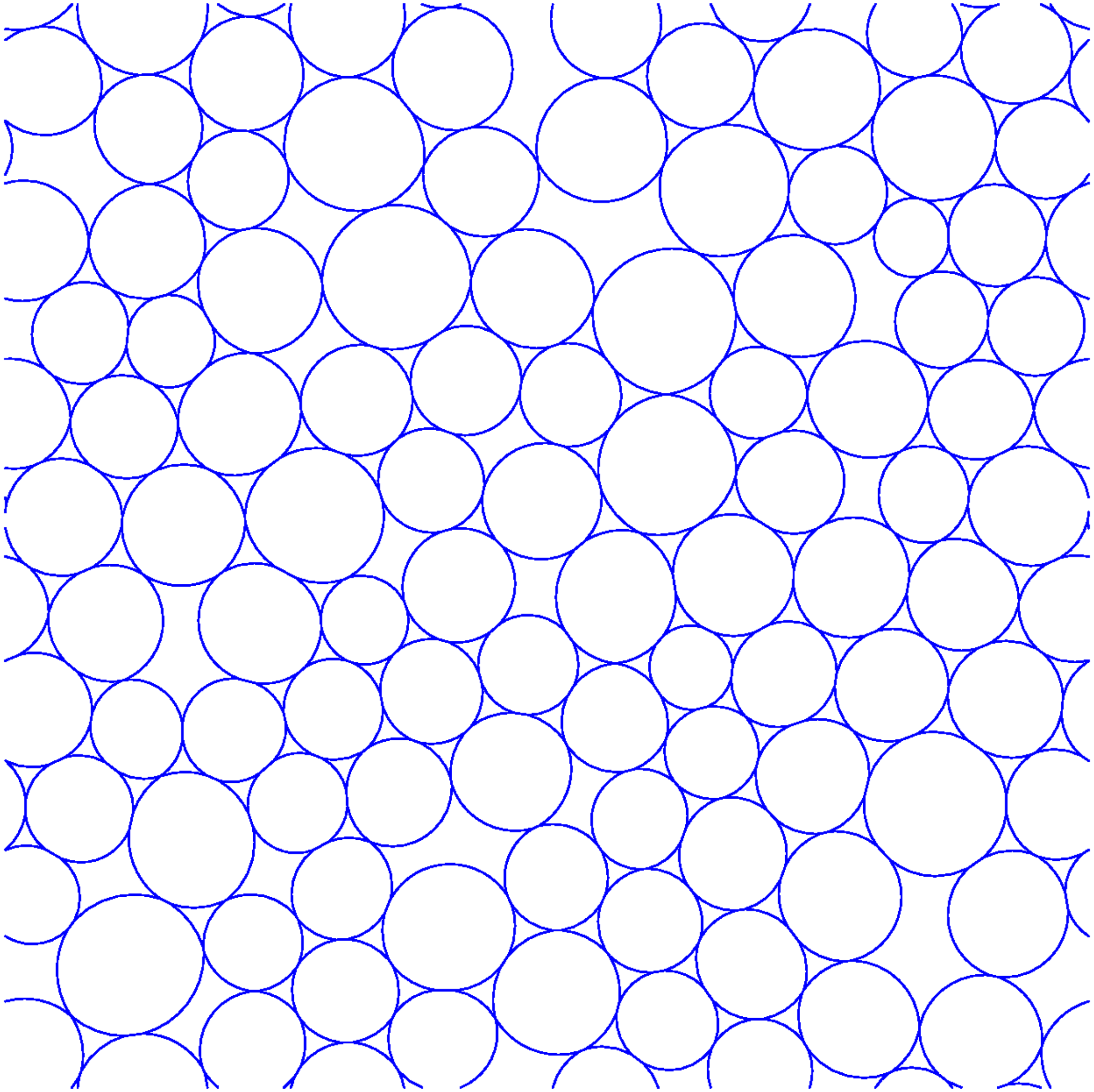
ρ = 0.12



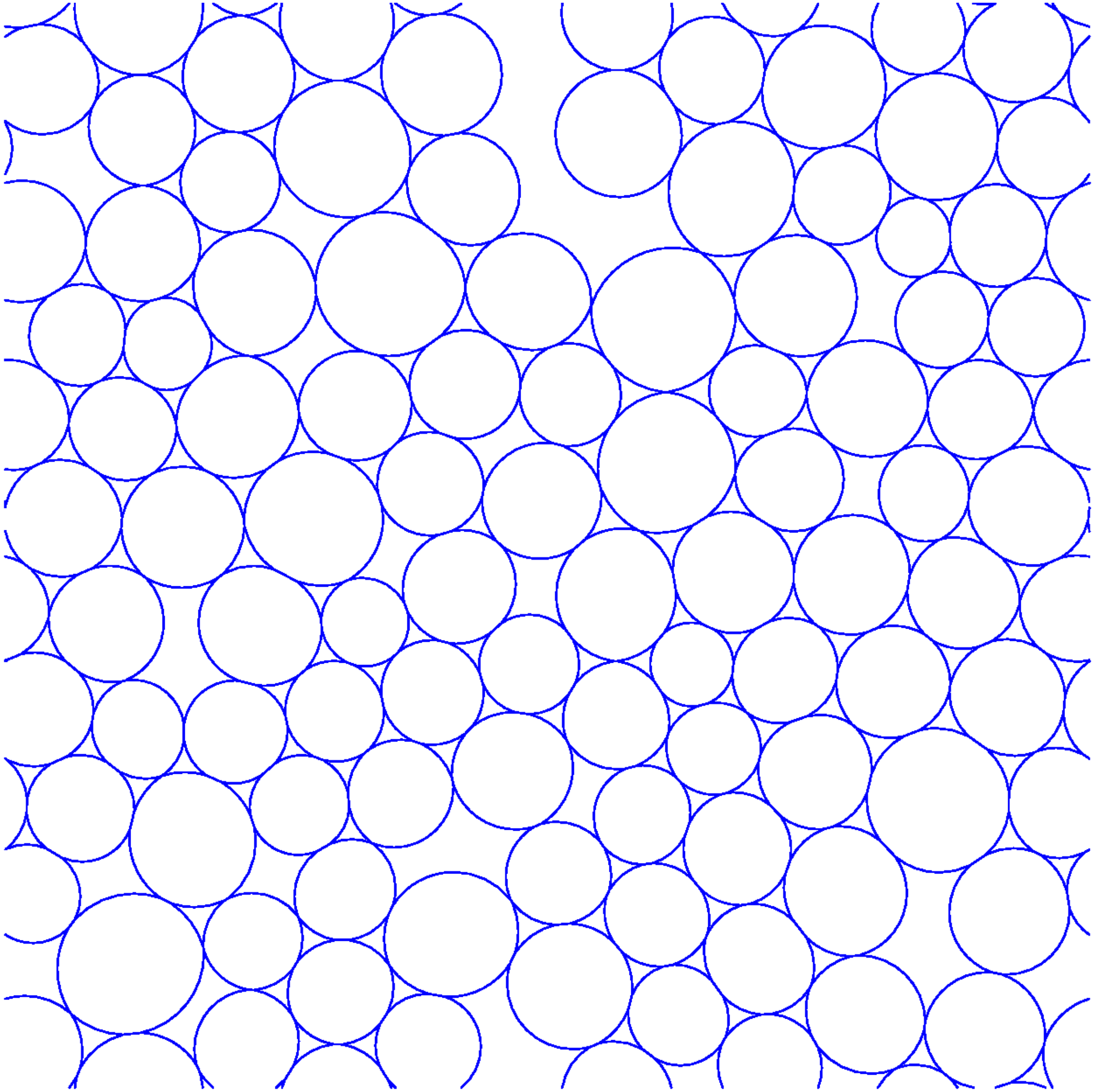
ρ = 0.13



$\rho = 0.14$

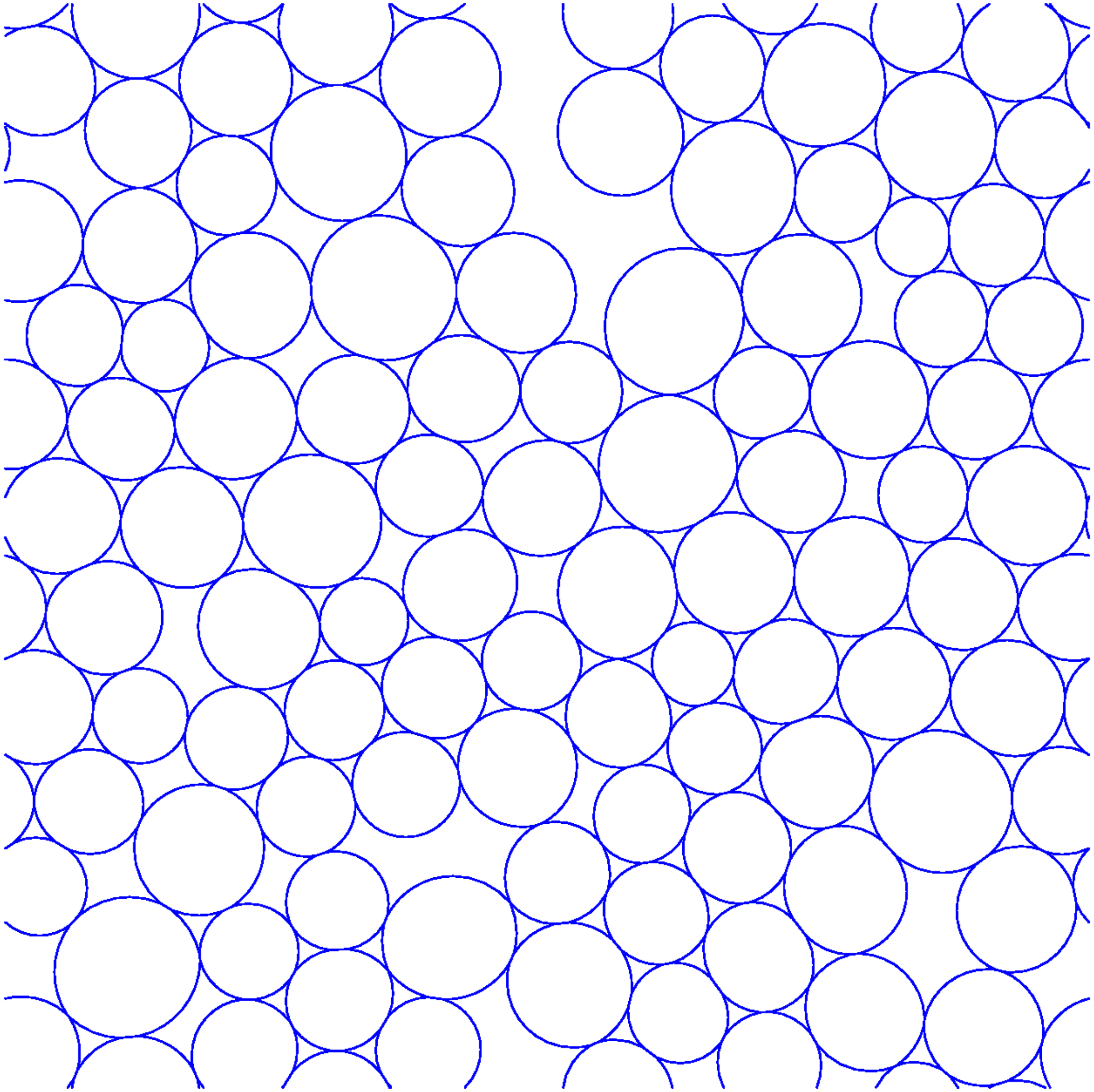


ρ = 0.145

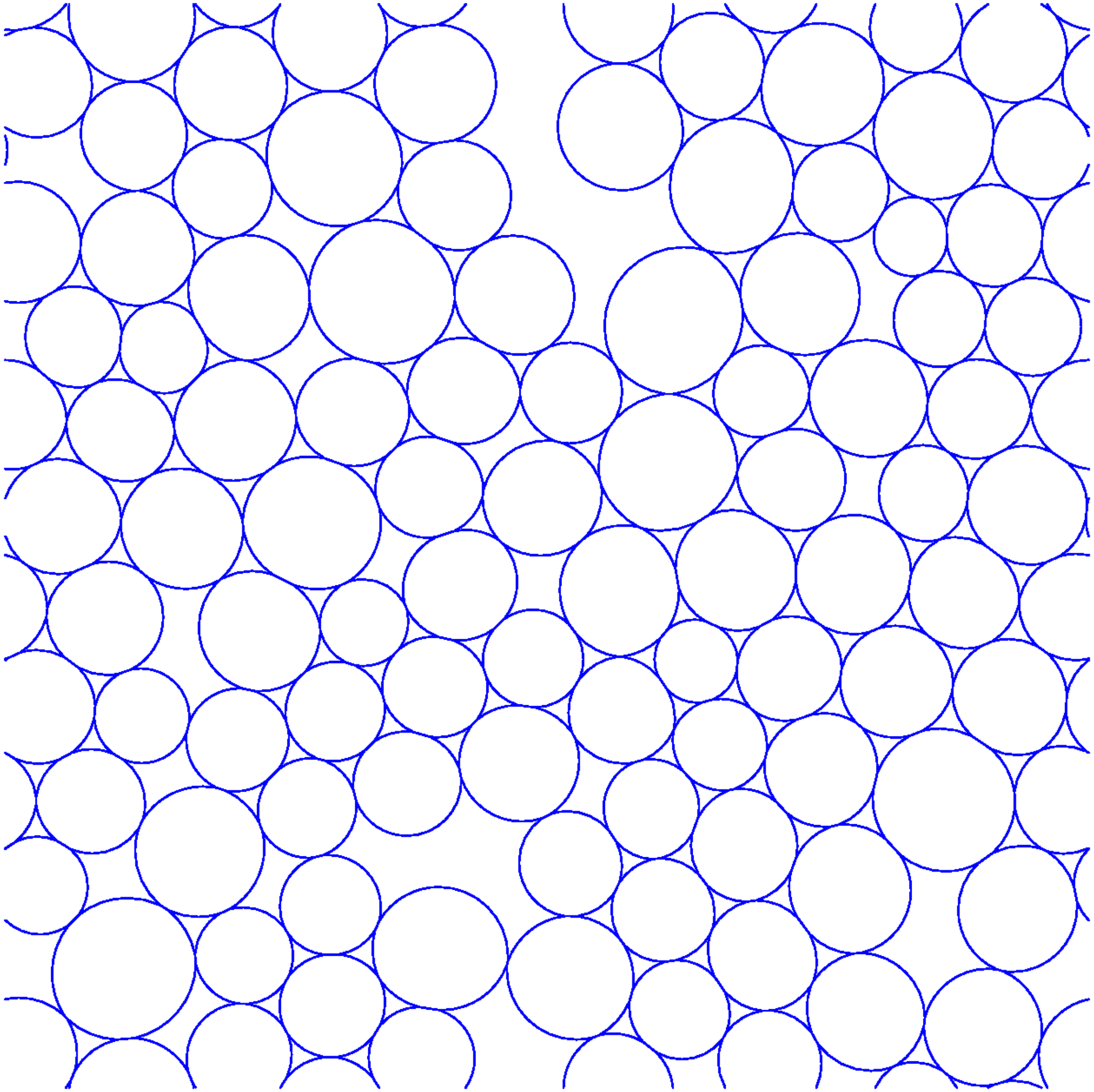


$\rho = 0.15$

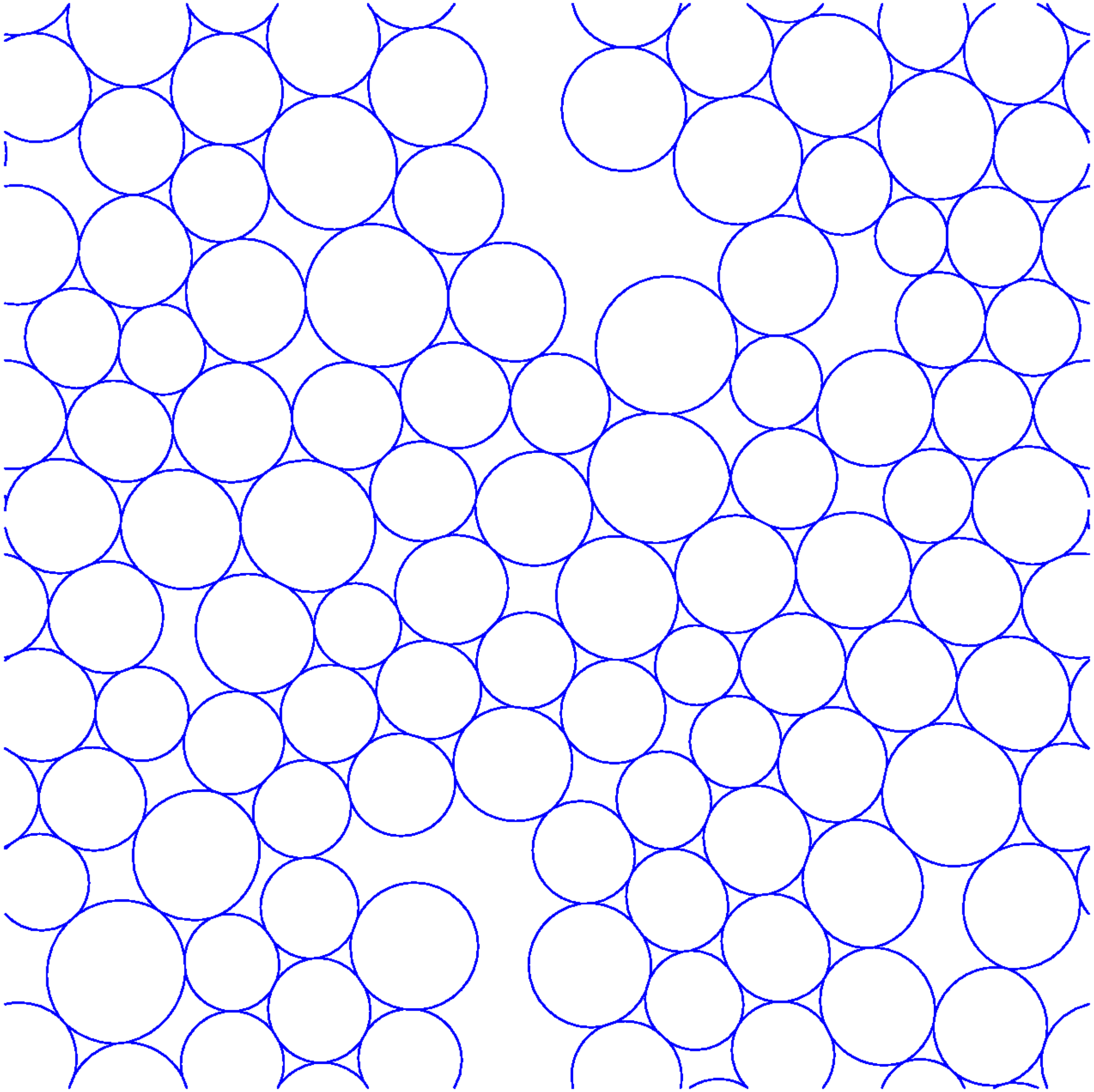
“Beyond Jamming”



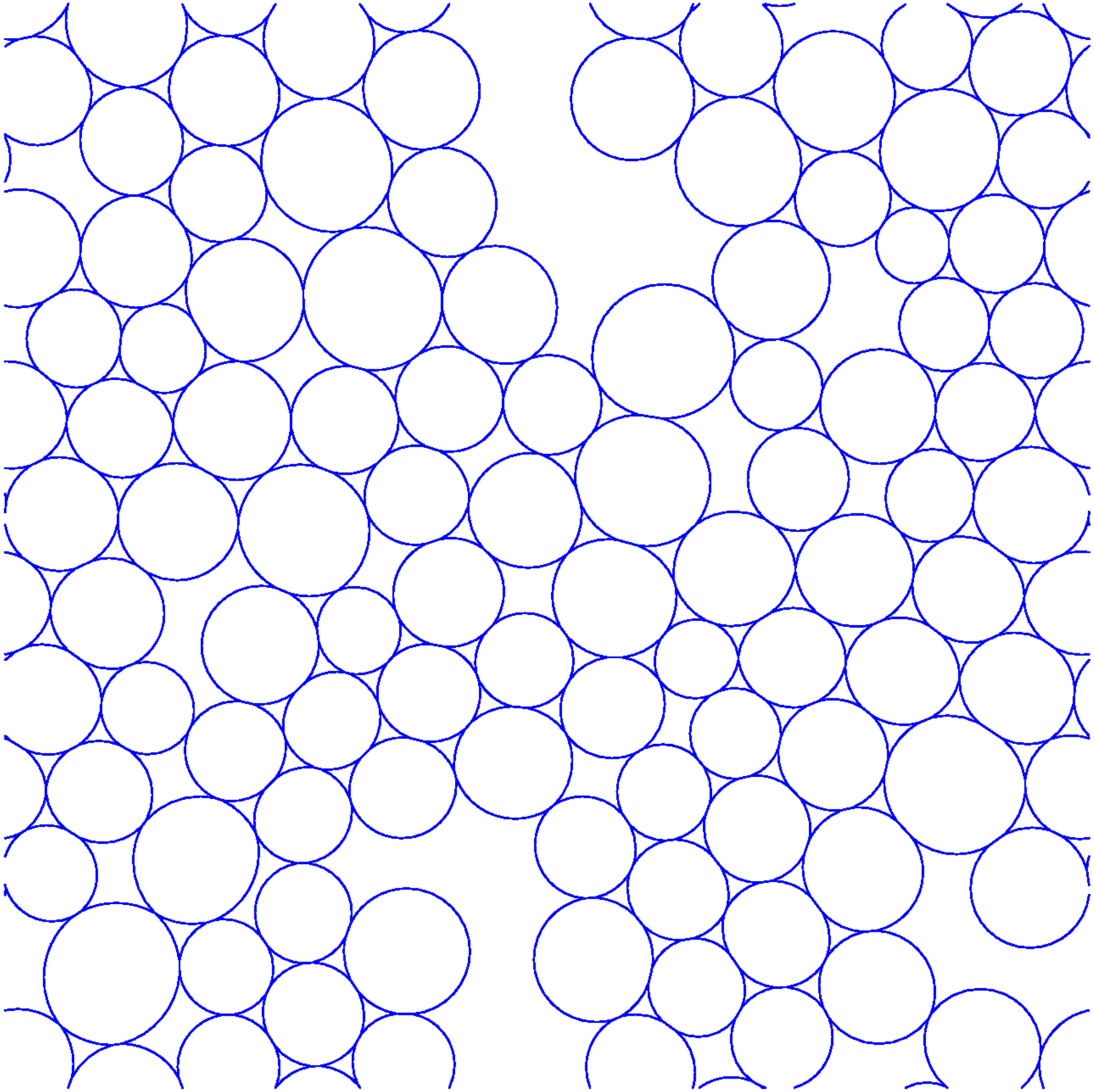
ρ = 0.155



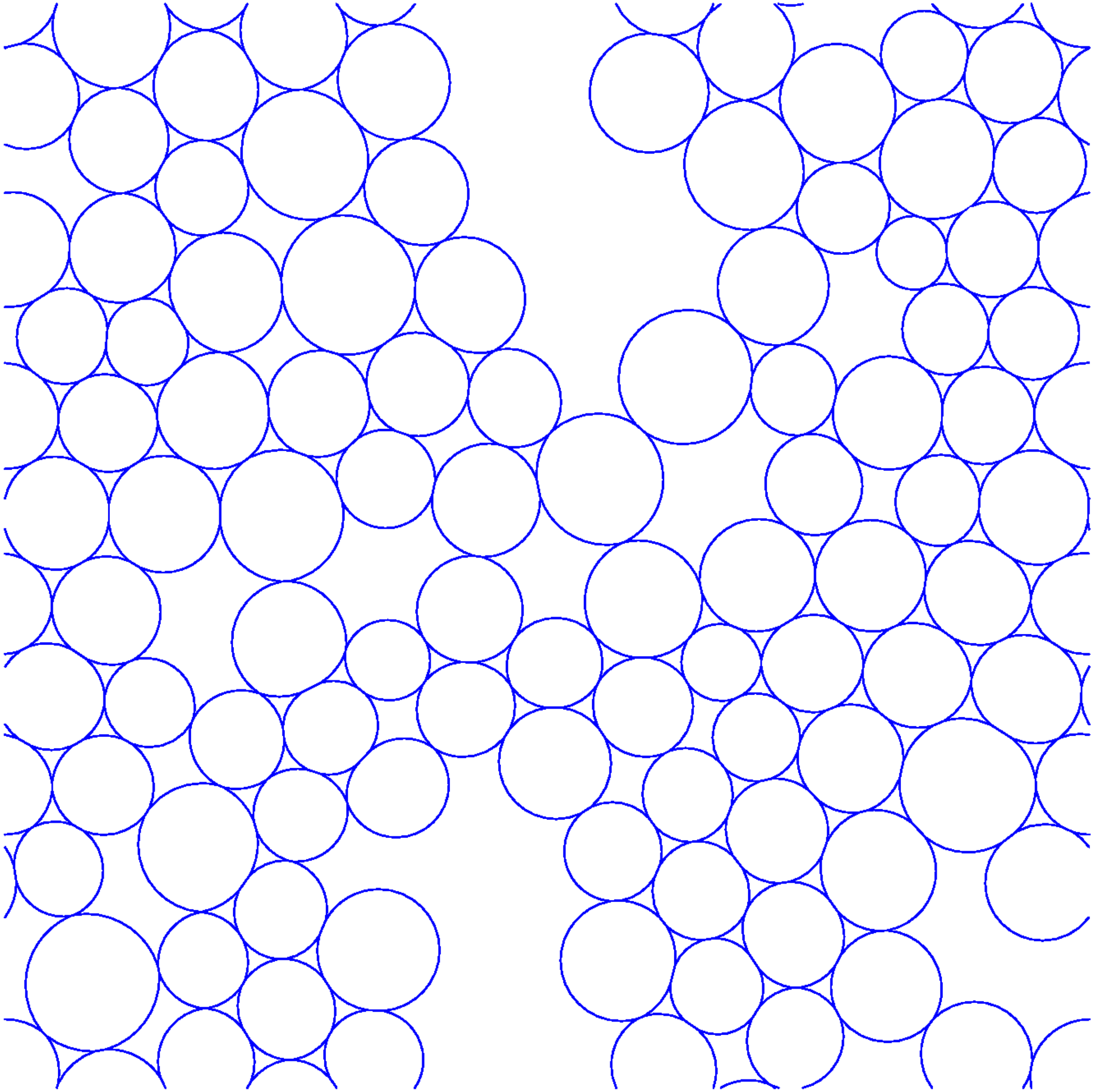
$\rho = 0.16$



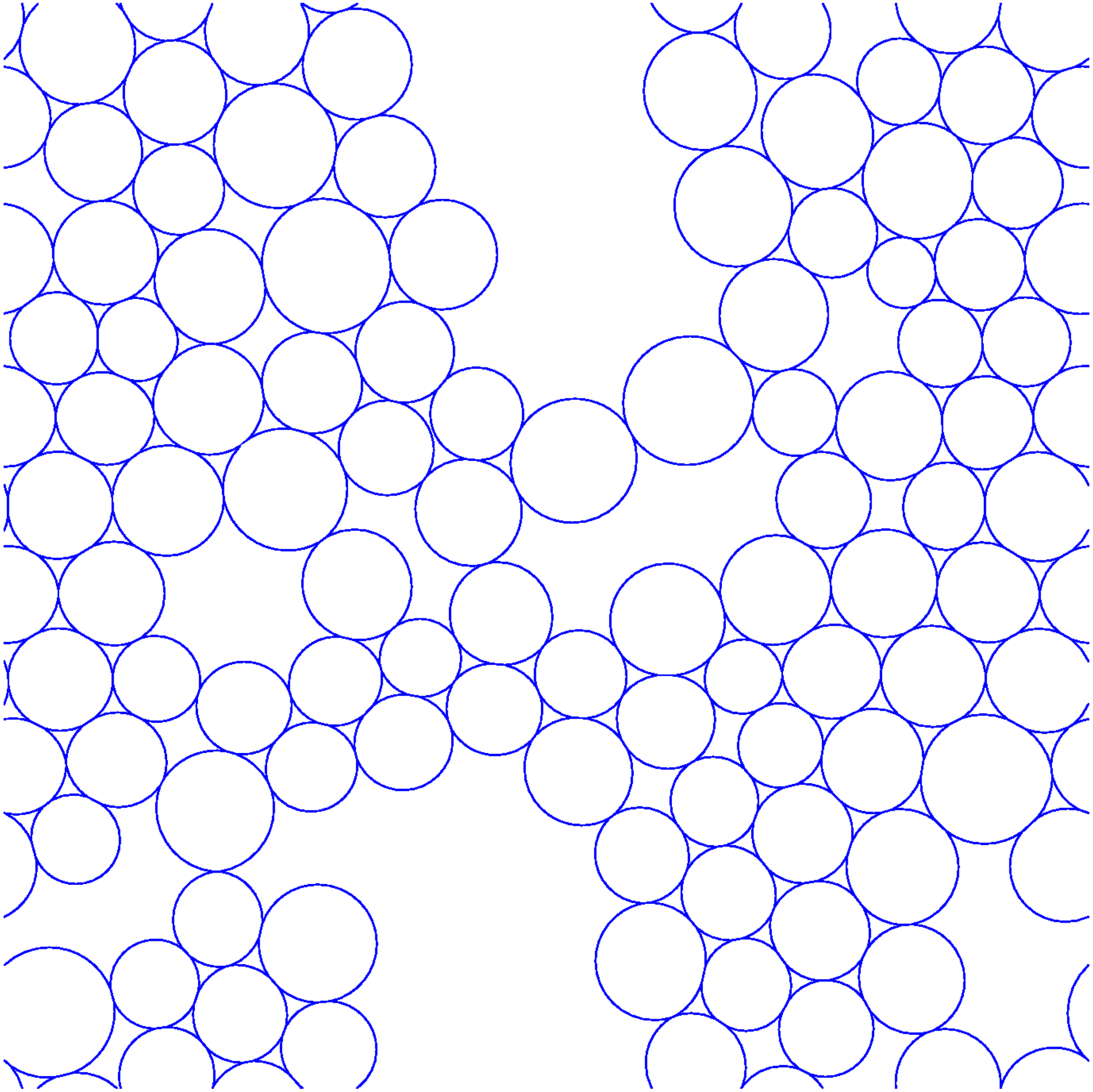
$\rho = 0.18$



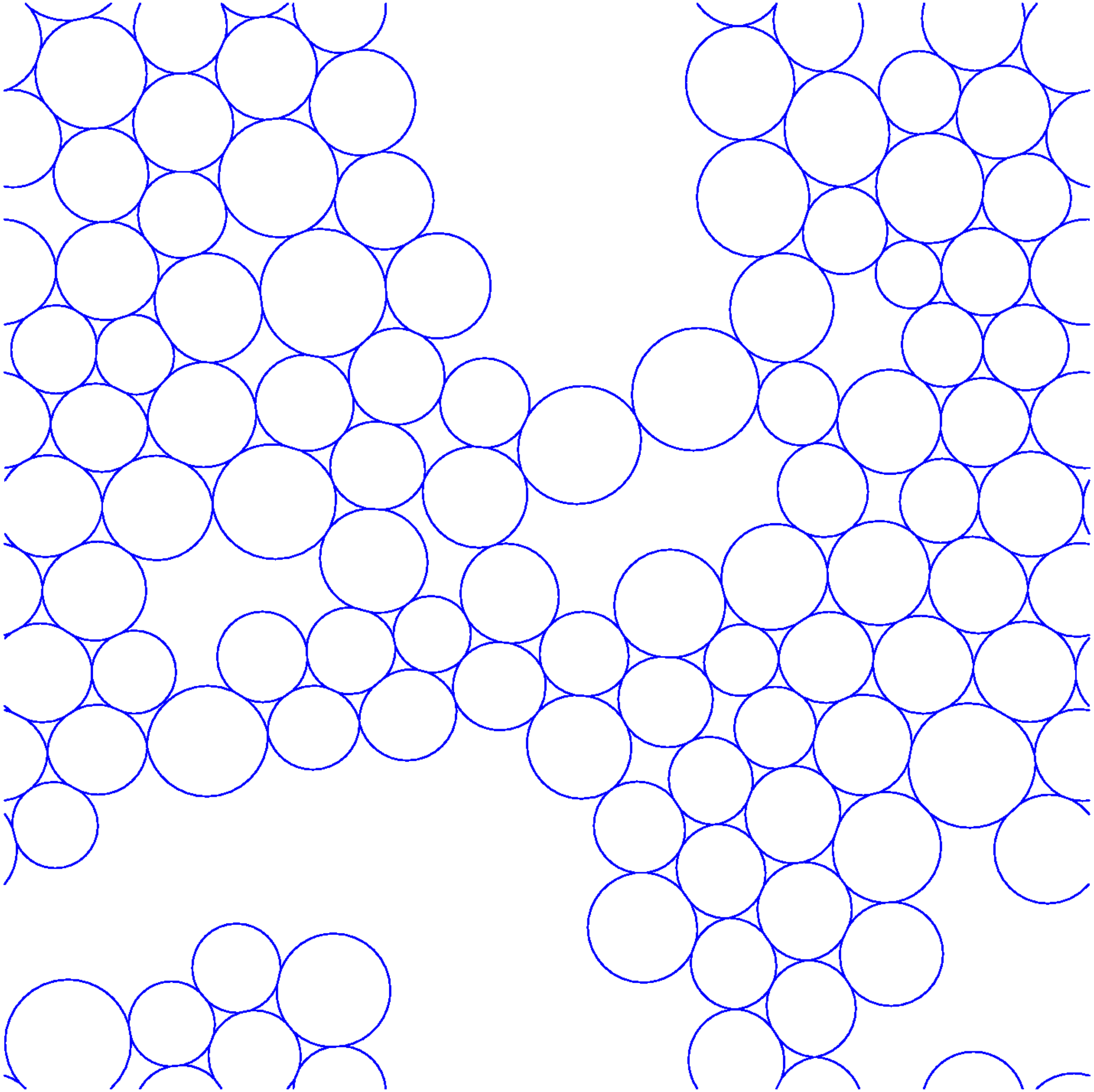
$\rho = 0.20$



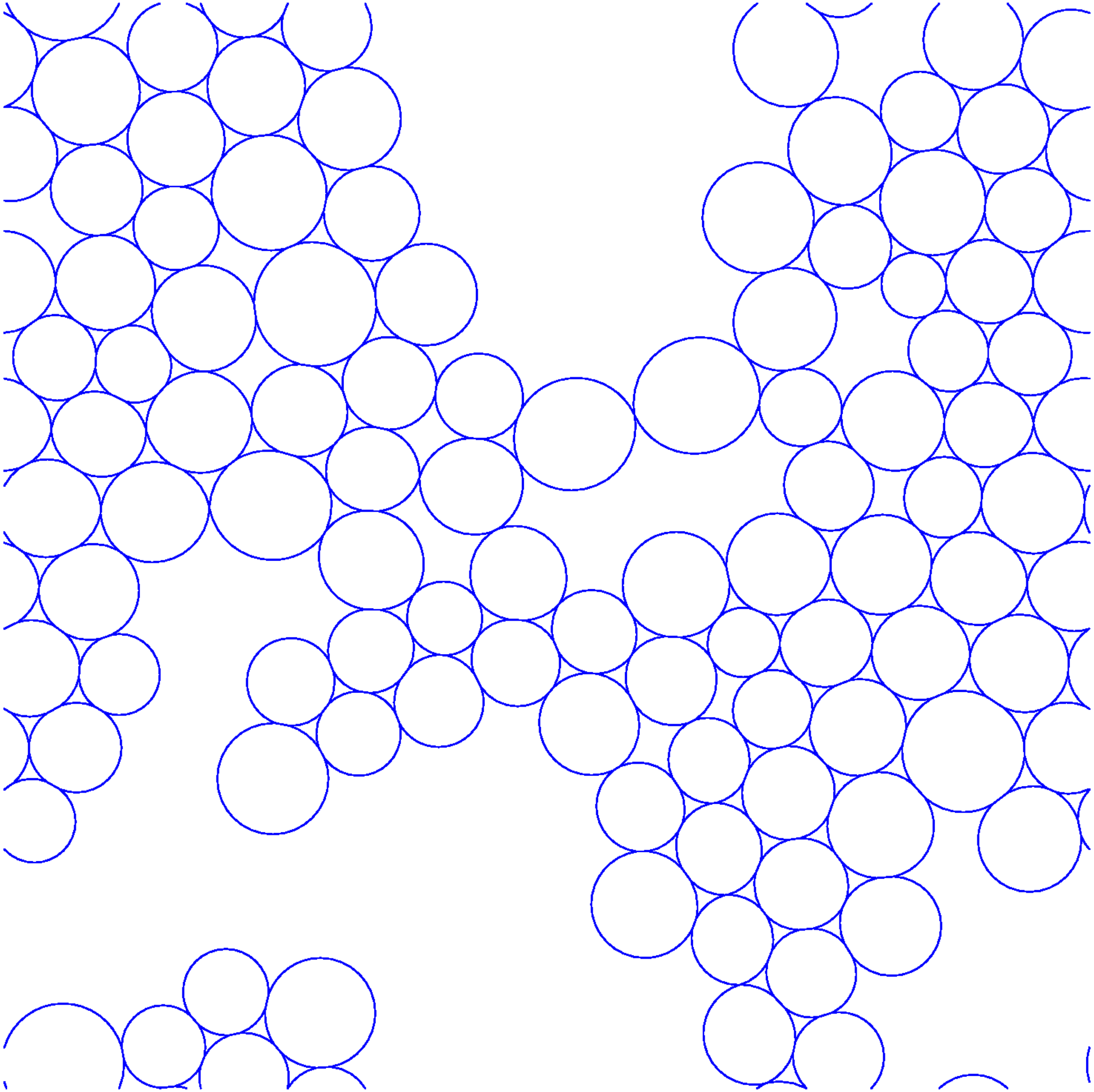
$\rho = 0.25$



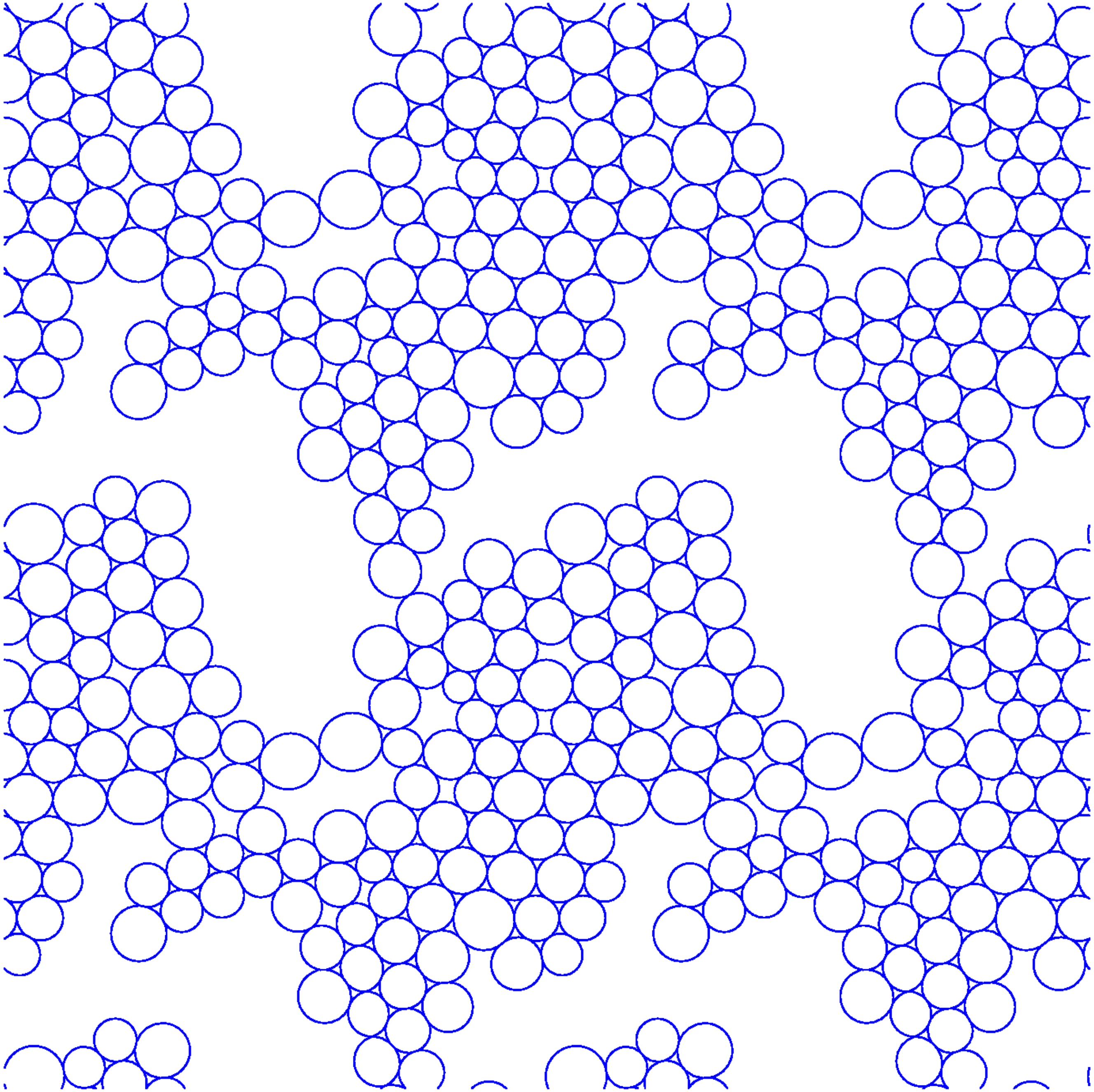
ρ = 0.30



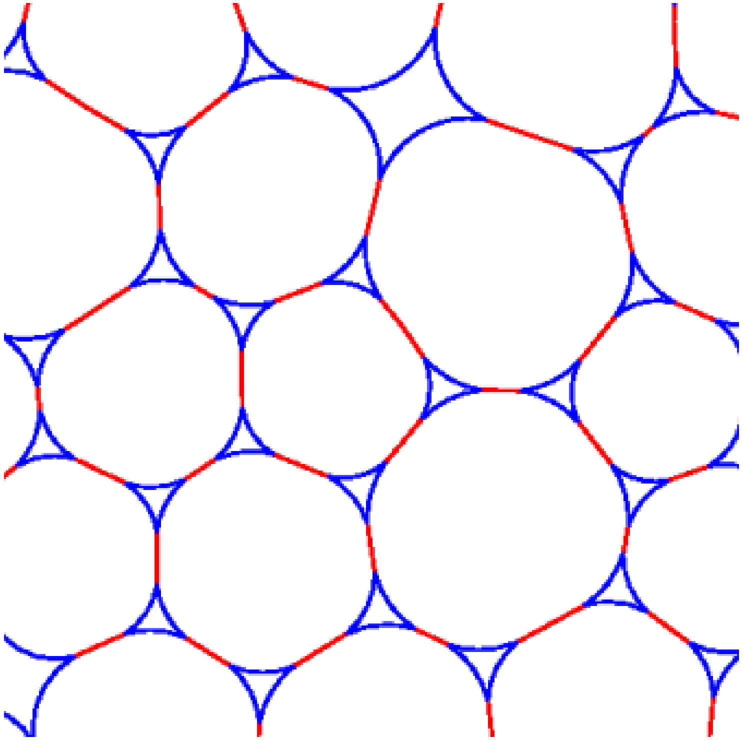
ρ = 0.35



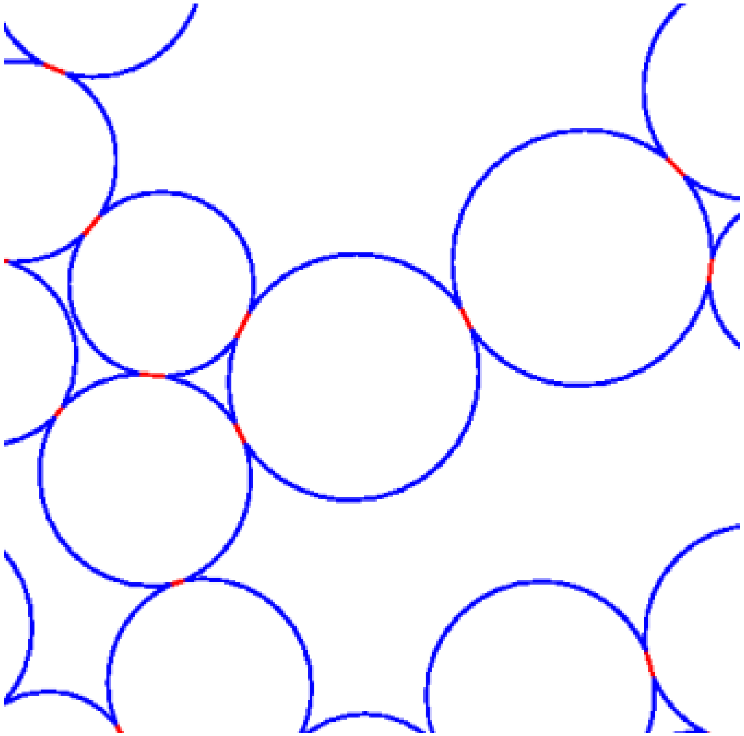
0.39



ρ = 0.39



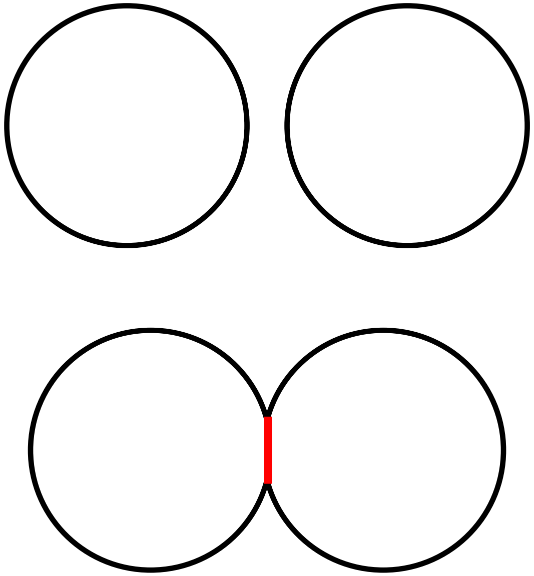
ρ = 0.05



ρ = 0.39

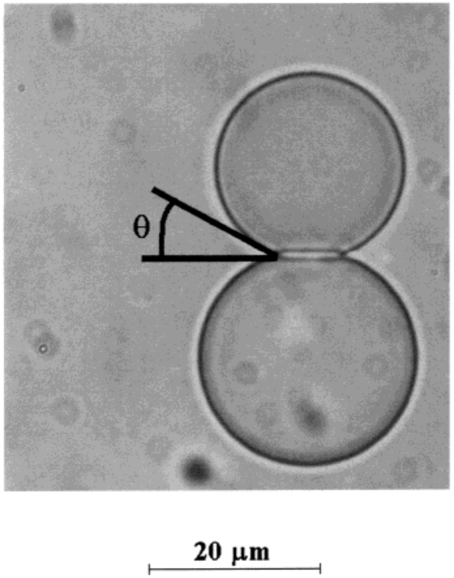
Physics of Flocculated (Adhesive, Sticky) Emulsions

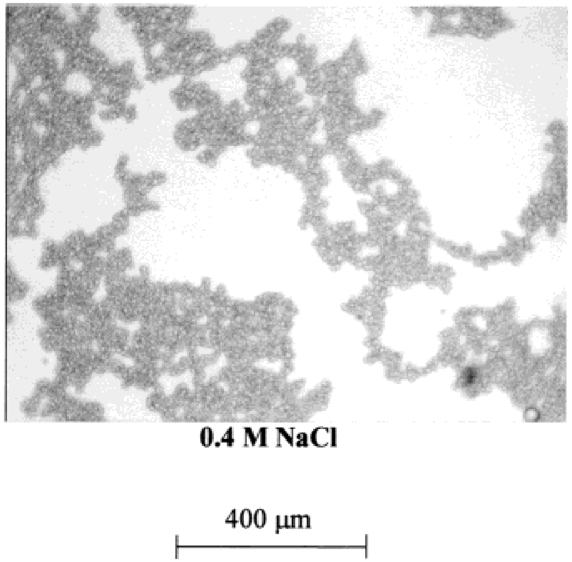
Aveyard et al. Langmuir **18**, 3487-3494, (2002)



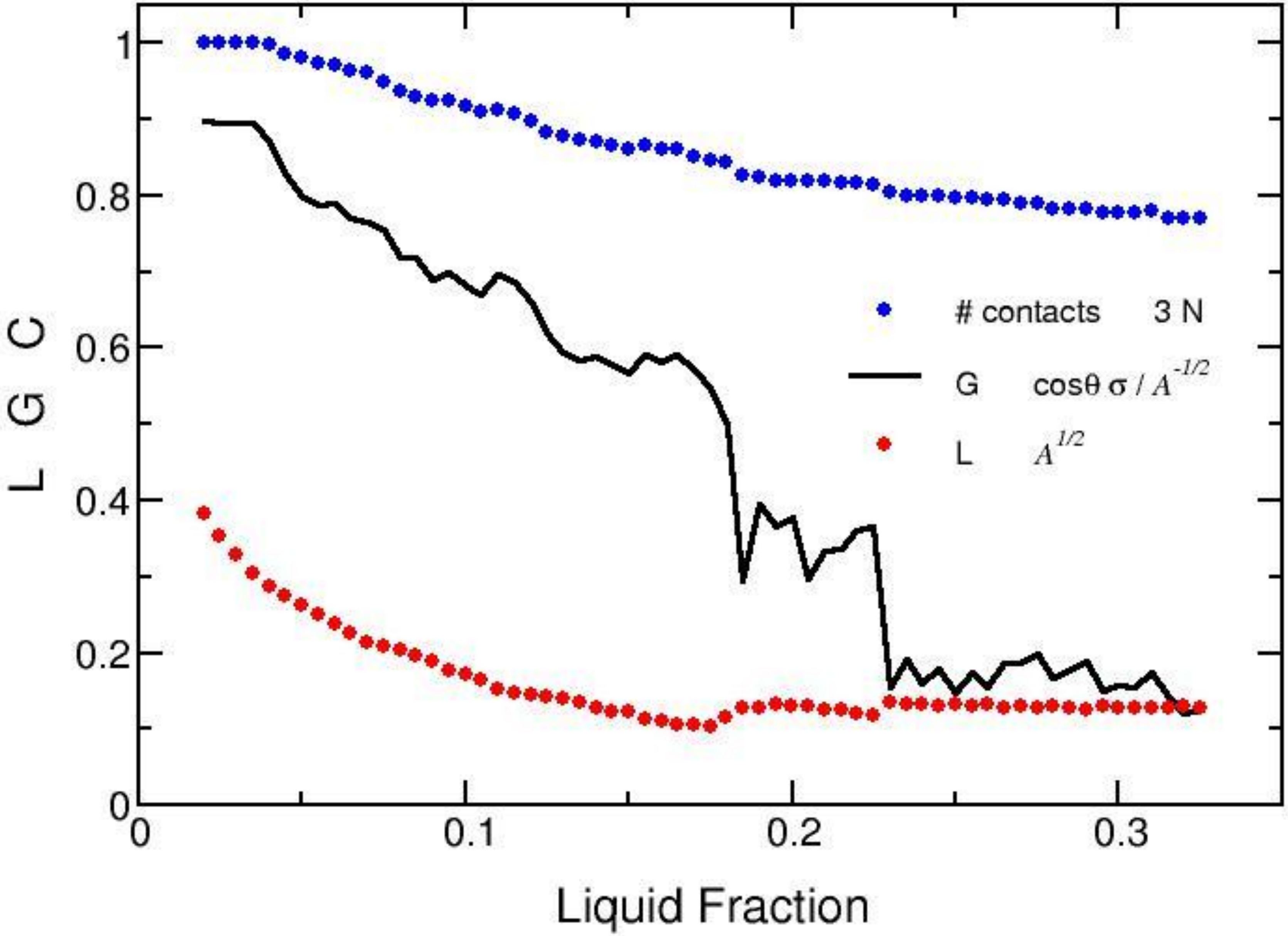
$$P = 4\pi R\left(1 + \frac{\theta^3}{6\pi} + \cdots\right)$$

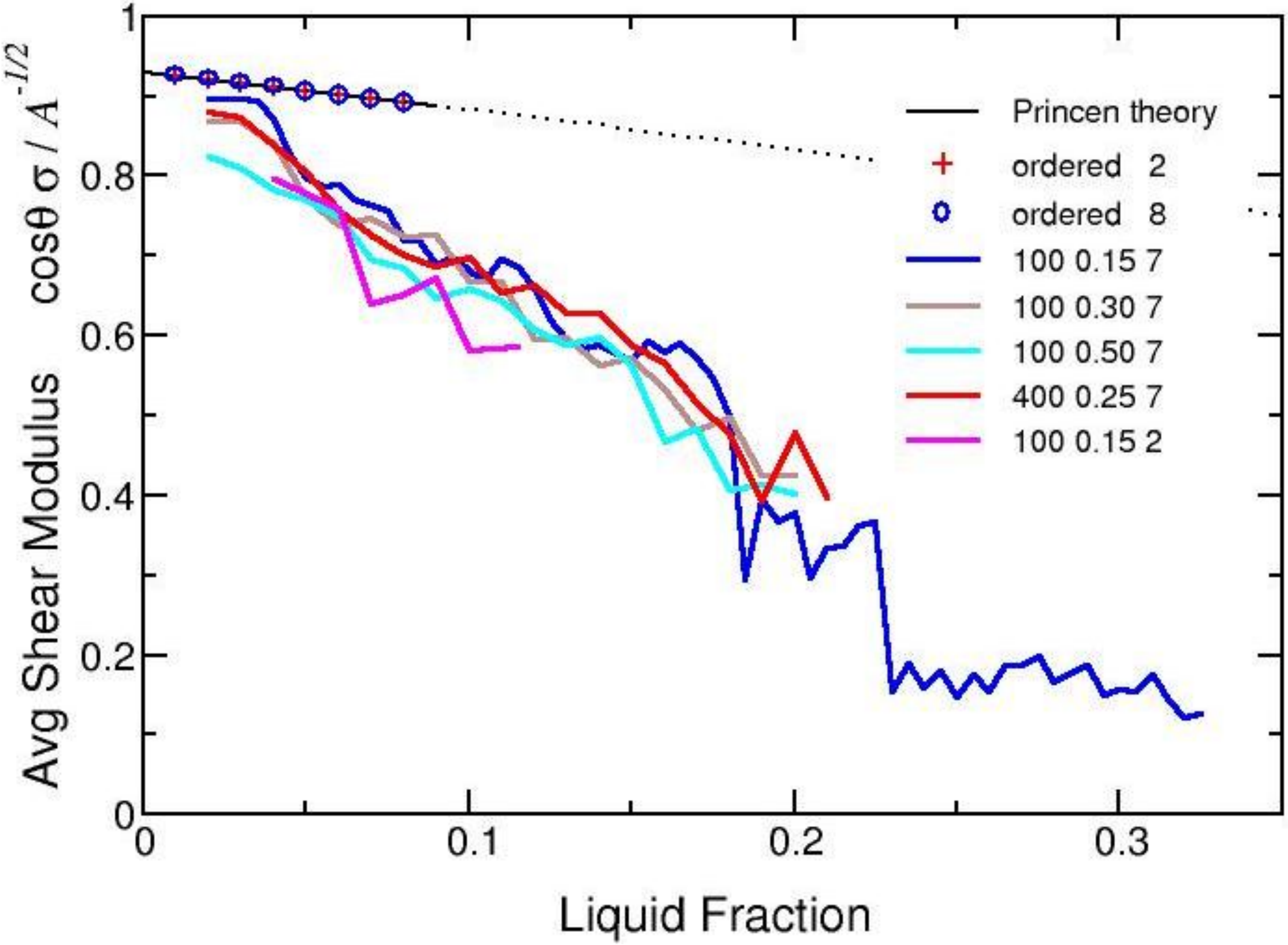
$$E = 4\pi\sigma R\left(1 - \frac{\theta^3}{3\pi} + \cdots\right)$$





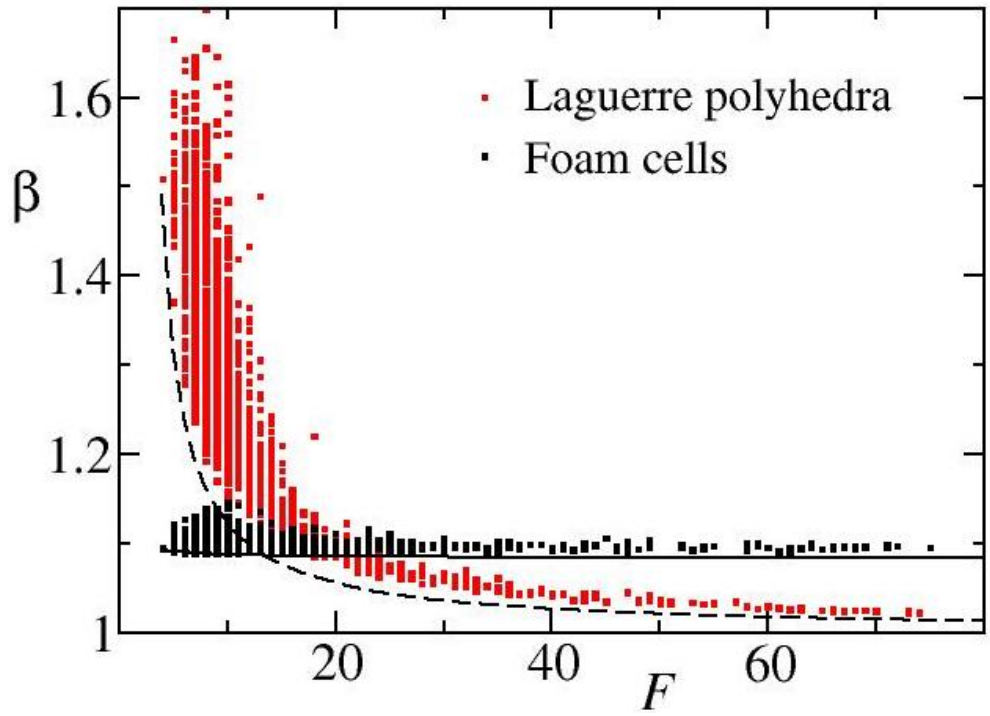
0.4 M NaCl

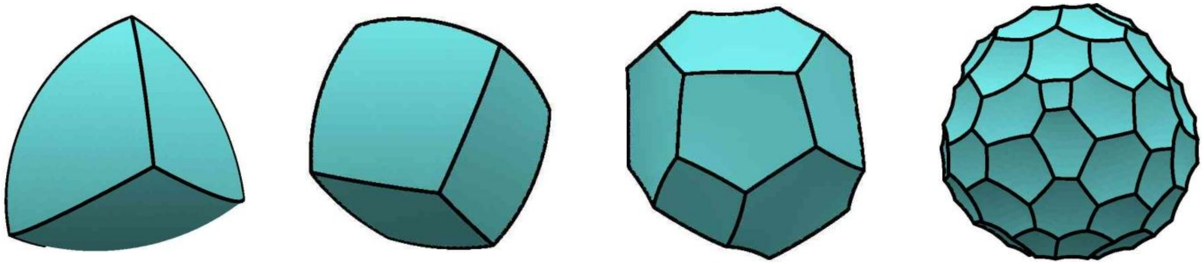

L G C
0
0.2
0.4
0.6
0.8
1
0
0.1
0.2
0.3
Liquid Fraction
contacts 3 N
G cosθ σ / $A^{-1/2}$
L $A^{1/2}$


Avg Shear Modulus cosθ σ / A^-1/2
Liquid Fraction
0
0.2
0.4
0.6
0.8
1
0
0.1
0.2
0.3
Princen theory
ordered 2
ordered 8
100 0.15 7
100 0.30 7
100 0.50 7
400 0.25 7
100 0.15 2

The surface area of a foam cell is about 10% greater than an equal-volume sphere

$$\beta = S\,(36\pi V^2)^{-1/3} = 1.101 \pm 0.006$$

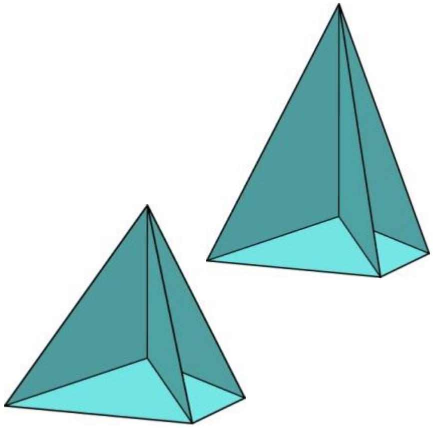




Shear Modulus: Theory

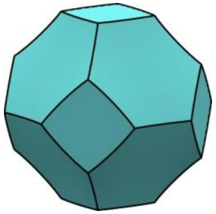
B.V. Derjaguin (1933) *Kolloid-Z*. **64**, 1.
Affine deformation of random soap films
G = 4/15 E = 0.267 E

D. Stamenovic (1991) *J. Coll. Int. Sci.* **145**, 255.
Deformation of idealized foam vertex

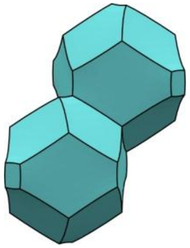


$\boxed{G = 1/6\ E = 0.167\ E}$

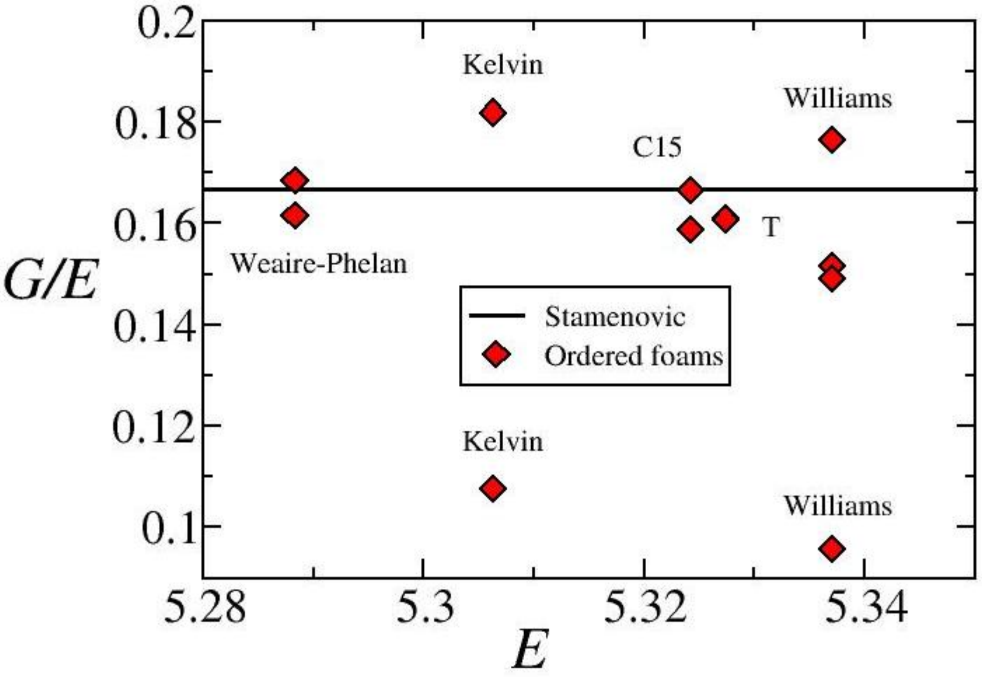
Kraynik & Reinelt (1996) *J. Coll. Int. Sci.* **181**, 511.
Surface Evolver calculations for ordered foams

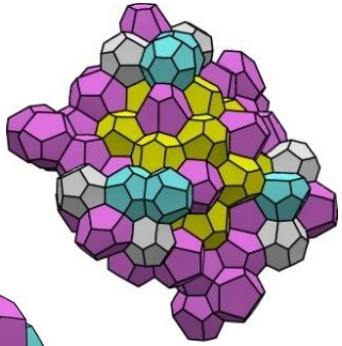


Kelvin

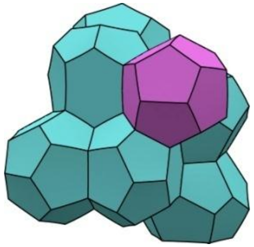


Williams

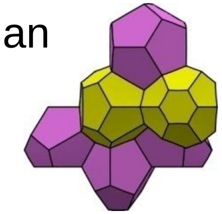




Bergman (T)



Weaire-Phelan



Friauf-Laves (C15)

Shear Modulus: Measurements for foams and emulsions

H.M. Princen & A.D. Kiss (1986) *J Coll Int Sci* **112** 427.

Static shear modulus of highly concentrated liquid-liquid emulsions

$$G \sim \sigma R_{32}^{-1} \phi^{1/3} (\phi - \phi_c)$$

$$R_{32} = \Sigma R^3 / \Sigma R^2$$

"dry" limit (ϕ=1) $\boxed{G = 0.51\, \sigma R_{32}^{-1}}$

T.G. Mason, J. Bibette & D.A. Weitz (1995) *PRL* **75** 2051.

Elasticity of compressed emulsions

$$G \sim \sigma R^{-1} \phi (\phi - \phi_c)$$

A. Saint-Jalmes & D.J. Durian (1999) *J Rheology* **43** 1411.

Vanishing elasticity of wet foams

Surface Evolver simulation of random wet foam

$\phi = 0.025$