Steady Couette flows of Elasto Visco Plastic fluids are non unique

[J. Rheol. 2012, in press]

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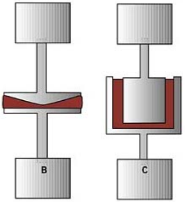
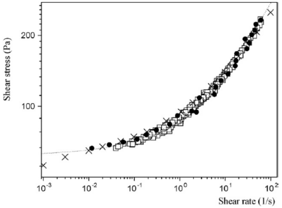
Dissipative Rheology of foams, Dublin, January, 11 2012

Herschel-Bulkley plastic dissipation

- Rheometry



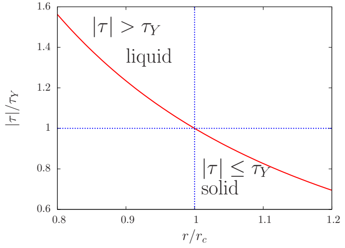


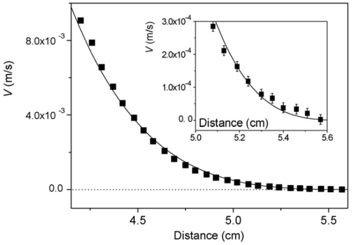
[Coussot et al. JNNFM 2009]

Herschel-Bulkley :

$$\tau(\dot{\gamma}) = \tau_Y + K\dot{\gamma}^n$$

- Prediction : localization if heterogeneous stress



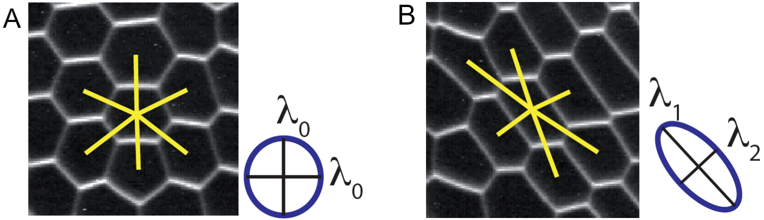


[Coussot et al. JNNFM 2009]

How to include elasticity ?

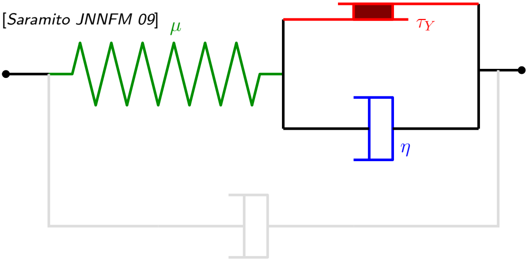
- **elastic solid under yield stress**

- **tensorial** framework because of elastic anisotropies



Elasto Visco Plastic : HB + elasticity

[*Saramito JNNFM 09*]



G : elastic modulus

τ_Y : yield stress

$\eta = K(V/L)^{n-1}$: plastic viscosity

$\lambda = \eta_2/G$: caracteristic time

$$\Longleftrightarrow$$

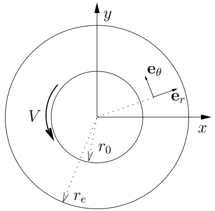
$$\frac{1}{2G}\frac{\mathcal{D}\tau}{\mathcal{D}t} + \max\left(0, \frac{|\tau|-\tau_Y}{2K|\tau|}\right)^{1/n}\tau = D(\mathbf{v})$$

$$\sigma_{tot} = -pI + \tau$$

$D(\mathbf{v}) = \frac{1}{2}\left(\nabla\mathbf{v} + \nabla\mathbf{v}^T\right)$, τ : elastic stress, $\frac{\mathcal{D}\tau}{\mathcal{D}t}$: objective derivative,
K : "consistency", n : "power-law index", $|\tau| = \left(2\tau_{xy}^2 + 0.5(\tau_{xx}-\tau_{yy})^2\right)^{1/2}$.

Cylindrical Couette

- $\mathbf{v} = v(r)\mathbf{e}_\theta \Rightarrow$ 1D resolution
- incompressible flow : $\nabla \cdot \mathbf{v} = 0$
- no external force :
  $\nabla \cdot (\tau - pI) = 0 \Rightarrow \tau_{r\theta} \propto 1/r^2$



- **EVP :** additionnal terms

$$
\begin{aligned}
\tau_{rr} &= 0,\\
\frac{1}{2G}\frac{\partial \tau_{r\theta}}{\partial t} + \max\left(0, \frac{|\tau| - \tau_Y}{2K|\tau|^n}\right)^{\frac{1}{n}} \tau_{r\theta} &= D_{r\theta},\\
\frac{1}{2G}\left(\frac{\partial \tau_{\theta\theta}}{\partial t} - 4D_{r\theta}\tau_{r\theta}\right) + \max\left(0, \frac{|\tau| - \tau_Y}{2K|\tau|^n}\right)^{\frac{1}{n}} \tau_{\theta\theta} &= 0
\end{aligned}
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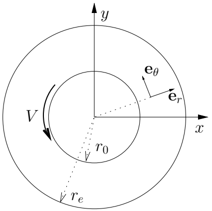

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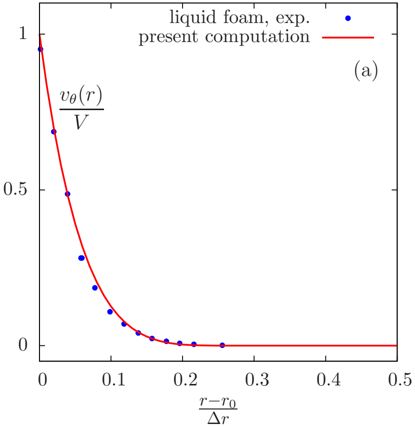
\end{aligned}
$$

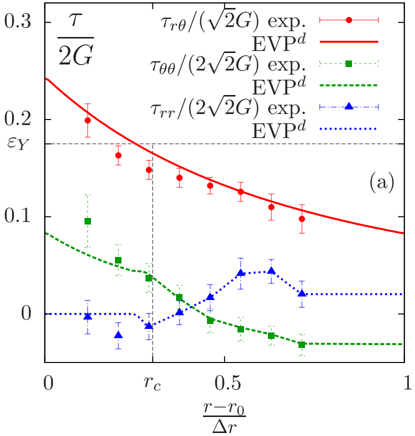
coupling between normal stress and flow

Comparison with Debrégeas et al 2001 exp.

[IC et al, JoR 2012, in press]

$$G = 10 \text{ Pa}, \ \varepsilon_Y = \tau_Y/(2G) = 0.175, \ n = 1/3, \ Bi = \frac{\tau_Y}{2\eta_2 V/L} = 10.$$



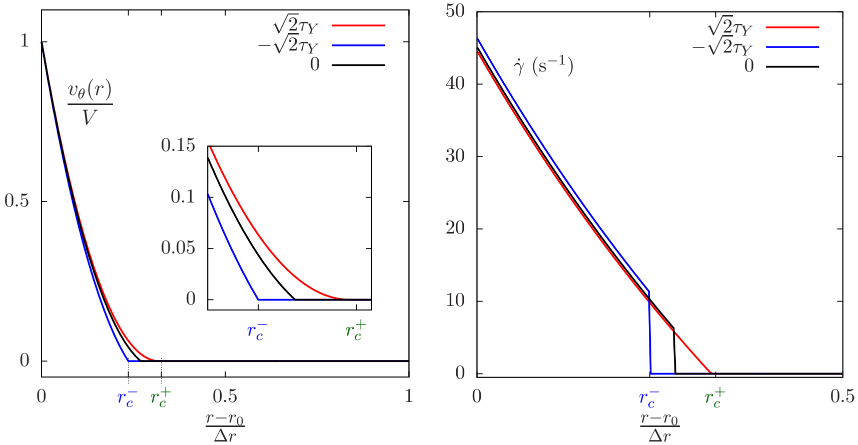


Localization + **residual stresses**

Non unique steady state !

3 initial stresses ⇒ 3 steady states :

$$\tau_{\theta\theta}(r,0) = -\sqrt{2}\tau_Y, \sqrt{2}\tau_Y, 0$$

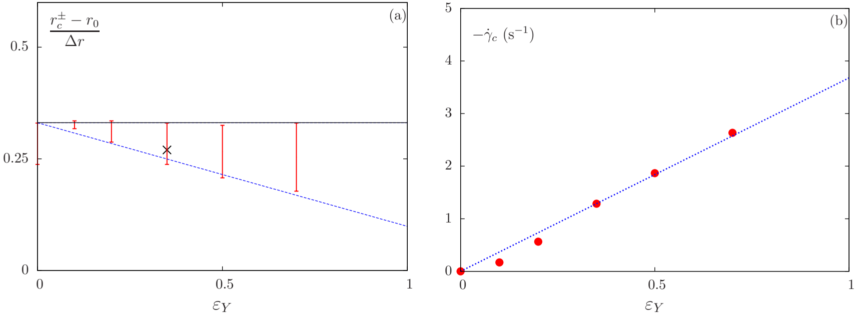


Non-smooth profiles

$\varepsilon_Y = \tau_Y/(2G) = 0.35$, $Bi = \frac{\tau_Y}{2\eta_2 V/L} = 27$, $n = 1$,

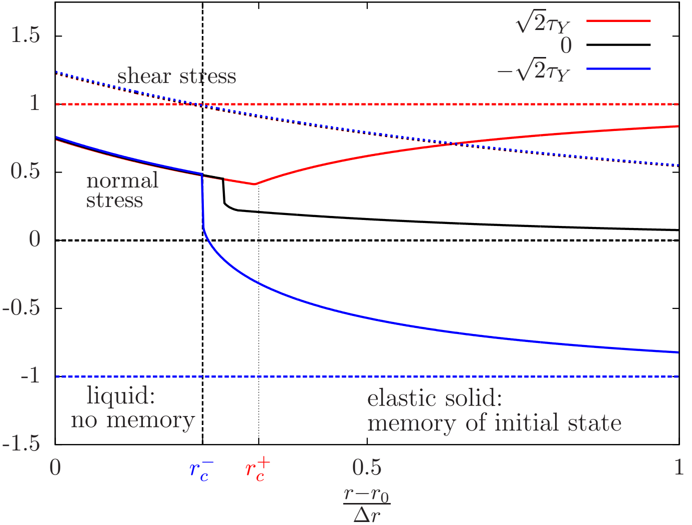
Predict localisation length ? Cheddadi et al., J. Rheol, in press, 2012

Herschel-Bulkley + correction that depend mostly on ε_Y :



Both vary according to initial conditions - trapped stresses

Memory effect of normal stresses



Conclusion

EVP tensorial effects in steady Couette :

- residual stresses
- Non-unique flow
- discontinuous velocity gradient

non-reproductibility = sensitivity to experimental details

[Coussot et al. PRL 2002] vs [Coussot Ovarlez EPJE 2010]