ph260 Theoretical Physics 2 — workshop 3

1. Solving linear higher-order ODEs.
Solve the following second and third order ODEs.

a. $\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} + \frac{\mathrm{d}y}{\mathrm{d}x} - 2y = 0; \qquad y(0) = 12; \qquad y(12) = 0.$

b. $\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} - 5\frac{\mathrm{d}y}{\mathrm{d}x} + 6y = 0; \qquad y(1) = y(-1); \qquad y(0) = 1.$

c. $\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} + 6\frac{\mathrm{d}y}{\mathrm{d}x} + 9y = 12\mathrm{e}^{-x}; \qquad y(-1) = 30; \qquad y(0) = 3.$

d. $\frac{\mathrm{d}^3 y}{\mathrm{d}x^3} - 3\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} - 9\frac{\mathrm{d}y}{\mathrm{d}x} - 5y = 0.$

e. $\frac{\mathrm{d}^2 y}{\mathrm{d}x^2} - \frac{\mathrm{d}y}{\mathrm{d}x} - 2y = 3\mathrm{e}^{2x}.$

2. Separating PDEs.
Solve the following homogeneous PDEs by Separation of Variables.

a. $\frac{\partial z}{\partial y} + 2\frac{\partial z}{\partial x} = 0.$

b. $\frac{\partial z}{\partial y} + z\frac{\partial z}{\partial x} = 0.$

c. $\frac{\partial z}{\partial y} + x\frac{\partial z}{\partial x} = 0.$

3. Modelling diffusion.
The diffusion equation in typical physical notation is the inverse of the form we have in the toolbox: In one-dimensional geometry it is: $\frac{\partial c}{\partial t} = \frac{\partial}{\partial x}\left(D\frac{\partial c}{\partial x}\right)$, where D is the diffusion coefficient, which is a constant in well-behaved systems. If this is the case, D can go before the differential: $\frac{\partial c}{\partial t} = D\frac{\partial^2 c}{\partial x^2}$, which is exactly the toolbox case. In less well-behaved systems such as those we're studying in the Materials Physics Group, $D = D(x)$ is really a function of the spatial coordinate. We don't know this dependence explicitly, but the concentration dependence of $D = D(c)$ is known from literature data, and we can guess the concentration profile $c(x)$. What differential equation do we need to solve to get $c(x,t)$? Classify the DE in terms of the properties discussed last week.

4. Factorising the characteristic polynomial.
When solving second-order ODEs with constant coefficients, we have reduced the PDE to two ODEs by factorising the characteristic polynomial. The constants in each factor were given as $k_{1,2} = \frac{b}{2} \pm \sqrt{\frac{b^2}{4} - c}$ if the polynomial is $x^2 + bx + c = (x + k_1)(x + k_2)$. Prove this.

Acknowledgement.
Examples 1 are stolen or adapted from *ML Boas; Mathematical Methods in the Physical Sciences, John Wiley, New York (USA)* ^{2}*1983.*

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