Fuzzy Diagnosis using Order of Magnitude Exaggeration

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Abstract

This paper proposes a fuzzy logic approach to allow qualitative symptoms that have been generated from order of magnitude qualitative simulation to be utilised as a fault detecting diagnostic system. The symptoms are interpreted as fuzzy rules and this allows numerical observations to be used as input to the symptoms and a ranking of faults as the output. The rule based approach results in a compact and fast (and fully testable) diagnostic system, by precompiling the results of off-line qualitative simulation and FMEA analysis. This paper outlines the technique which is being applied to automotive and aerospace electrical and fluid flow systems.

1 Introduction

This work brings together several techniques. Order of magnitude qualitative simulation provides nominal and fault behaviour which can be combined with functional interpretation [1; 2] to produce qualitative diagnostic symptoms (or rules). Fuzzy logic provides a formalism for using such Boolean rules within a real valued system, together with the ability to capture uncertainty (not probability). In this work we interpret qualitative rules using fuzzy assignment and do not incorporate fuzzy sets into the qualitative simulation itself [3] and numerous subsequent work. Qualitative order of magnitude (OM) reasoning separates qualitatively distinct behaviour and also allows qualitatively significant (extreme) effects for faults. Exaggeration reasoning provides a concept for the necessary scaling down of those effects to match real observations.

The remainder of this paper will provide background on exaggeration reasoning and OM representation, followed by a definition of the symptoms available in Section 4. Section 5 considers the fuzzy mapping of OM qualitative values and faults. Finally Section 6 provides a small example followed by conclusions.

2 Background

Exaggeration reasoning (ER) is a comparative analysis technique often employed by technicians and engineers to analyse system behaviour and malfunction and provides an alternative differential qualitative analysis [4; 5] to perform comparative analysis. Avoiding qualitative differential equations (QDE) is also a feature in the two level simulation strategy underlying our symptom generation where *global steady state* power flow analysis is combined with a higher level *local component* state based model. The global analysis we use exploits the same energy flow principles as Bond graphs [6] but unlike [7], does not generate QDEs.

The concept of ER was investigated by Weld [8]. The strategy is to qualitatively exaggerate an input perturbation, and subsequently scale the effects of a resulting simulation. To achieve this Weld utilised a qualitative magnitude space based on hyperreals. In essence this incorporates infinitesimal (but non zero) values ($negl$), and infinite (∞) into the qualitative sign $\{-, 0, +\}$based quantity space and provides exaggeration by mapping an increasing perturbation into an infinite value and a decreasing perturbation into an infinitesimal value. Thus as an example the simplest quantity space for resistance magnitude is $\{0, negl, r, \infty\}$, where r represents a resistance value (much) greater than $negl$.

It is necessary to understand that exaggeration reasoning is not guaranteed to produce a correct result if the system does not respond monotonically to a fault within the range of an exaggeration. Put another way, it must be possible for the qualitatively exaggerated effect to be 'scaled back' to represent a less severe version of the fault. For example in some (poorly designed) electrical system, a lower than normal power resistor may cause a functional failure, such as a fast running motor. We can reason that excessive speed in the motor could be caused by a low resistance fault, and faster motor the lower the errant resistance. However a qualitatively extreme case of a zero resistance causes thermal run away or a fuse blow and zero motor speed. In this case an order of magnitude representation (section 3) for the fault allows the effects to be distinguished.

For the two level simulation strategy we avoid this problem in two ways. The level one steady state resistive network analysis is a linear system. The level two analysis is based on explicit state based models, and these models will either provide a monotonic behaviour as a formula that defines a set of state changes (for example the OM time required to fill a tank based on an OM flow rate), or must provide a behaviour for any relevant (exaggerated) value. The OM simulation supports multiple levels of exaggeration, each with its own scaling function, resulting only in the requirement that the system is monotonic within each state.

The motivation for our use of ER is twofold:

- ER is suited to Failure Modes and Effect Analysis (FMEA) where the required result is a comparison between nominal and failure mode behaviour.

- The exaggeration proposed by [9] is based on infinitesimals, and we propose that it can be extended to include OM representation because the *OM assumption* (see section 3) states that each magnitude is insignificant (infinitesimal) w.r.t. a higher one.

The OM reasoning allows a finer grained behaviour to be produced than qualitative sign algebra, in particular phenomena of very different significance can be differentiated. For example in an electrical system we can separate signal voltage and/or power levels from actuator power levels by exaggerating by an order of magnitude, thus allowing qualitative behaviour prediction where faults mix these values.

The final part of exaggeration reasoning is *scaling* of the behavioural result. This is done by allowing symptoms to be generated based on the exaggerated values, to produce *exaggerated symptoms* and scale the results mapping between exaggerated qualitative symptoms and actual system observations.

Fuzzy logic (FL) provides one way to map the Boolean logical symptoms containing atoms referring to qualitative magnitudes to the precise world of system measurements. Defining a fuzzy membership function for the qualitative values allows a mapping to be produced between the accurate (correct), but imprecise qualitative values and the precise but potentially inaccurate system measurements.

The selection of the fuzzy membership for the qualitative exaggerated values allows the scaling to take place, but also allows operating regions to be identified. Considering an automotive system example, a nominal qualitative OM value for current flow of mA magnitude, may map to fuzzy set $\mu_{\mathrm{mA\_A}}$ for some LED and fuzzy set $\mu_{\mathrm{mA\_B}}$ for some relay coil (Figure 1). The mappings can be defined at the variable type, component class, or component instance in design or component instance level, perhaps becoming more specific at the lower levels (and thus improving diagnostic ability). The ranges may be based on component specification, or expected system operating ranges. In the example the LED data sheet specifies I_F = 30mA (Continuous Forward Current), I_{FP} = 100mA (Peak Forward Current), and by experiment a visible level of illumination requires 5mA. The component is usually operated at around 25mA to allow for underrating and tolerance of other components. The relay has a nominal current specification of 160mA, and is considered to be one of the low power (signalling level) devices in the system to differentiate it from the 'high power' components such as actuators and halogen lamps for example.

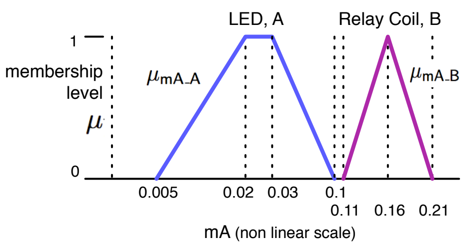

Figure 1: Two interpretations for the qualitative value mA

3 Orders of magnitude representation

The intention behind the OM representation is to separate significantly different behaviours and has been investigated for generalised reasoning by several authors [10; 11], however in this work we focus on its application to the two level modelling strategy documented in [12]. The OM circuit analysis assumption specifies that

$$nr^{\rhd a} \gg r^{\rhd (a+1)} \tag{1}$$

where $r^{\rhd a}$ is a qualitative value a orders of magniturde smaller than *r*. In practice the structure and behaviour modelling should ensure n is small enough to satisfy this. For example multiple resistors in series should not have a total numerical resistance at a higher order of magnitude than an individual resistance; and many 'short' duration events do not sum to the duration of a 'long' event. This allows significant behavioural distinctions to be included in the reasoning, for example a short circuit fault between the coil and switch of a relay does not provide enough current to operate the motor controlled by the relay.

Three order of magnitude representations (3OM) are a natural way to identify significant differences that exceed the parametric tuning of a system. Consider the significance of 'millionaire' and 'billionaire' for example. In science and engineering there are standard 3OM prefixes for most quantities, for example $\{\mathrm{m}\Omega, \Omega, \mathrm{K}\Omega, \mathrm{M}\Omega, ...\}$, or approximately $\{\mathrm{mS}, \mathrm{S}, \mathrm{H}, \mathrm{Day}, \mathrm{Year}\}$.

3.1 OM modelling

Using the OM representation and generalised Ohms law ($E = F \cdot R$) we can allocate flow in a resistive network supplied with a voltage E using Table 1. The symbol $\sqcup$ is used to specify the special case of a short circuit, since infinite flow makes little physical sense. An open circuit (lines 3,6) allows no flow. Complex circuits are reduced to a single resistance using series/parallel/star reduction. Division of OM values results as usual in index subtraction.

Effort, E	Resistance, R	Flow, F
0	0	?
0	$r^{\lhd n}$	0
0	∞	0
$u^{\lhd m}$	0	$\sqcup$
$u^{\lhd m}$	$r^{\lhd n}$	$f^{\lhd (m-n)}$
$u^{\lhd m}$	∞	0

Table 1: OM Qualitative effort flow assignment

The flow through any element of a the network can then be deduced by expanding the network using SPS technique [13; 1].

In the level two model [12] OM quantity ($Q = F \times t$) is represented as states with OM time conditional events controlling state changes. For example a set of events (tank_fill) that cause a change from the empty to full state of a storage tank of capacity size could be defined by a conditional event transition: 'if flow == $f^{\rhd n}$ tank_fill after $t^{\lhd (n+size)}$'.

3.2 Fault exaggeration and magnitude labeling

Sign based QR allows representation of extreme effects, for example that a pipe fracture leads to zero downstream flow and total loss of substance to the atmosphere. There is no possibility to represent a partial leak, since adding a resistive connection between the pipe and external atmosphere will not change the qualitative flow in the pipe. The OM representation allows a less extreme exaggeration that can provide more details. For fluid flow we define flows of the order litres per second as $f^{\rhd 3}$ since according to the SI units f^0, represents m^3/s. For example given a fluid flow system with a leak fault, and down stream resistance of $\Omega = r^{\rhd 3}$, $E = u$ and idealised pipes $\Omega = 0$ the simulation will calculate flows in table 2.

Fault	flow to Atm	downstream flow
Nominal ($\Omega = \infty$)	0	$f^{\rhd 3}$
Minor Leak ($\Omega = r^{\rhd 3}$)	$f^{\rhd 3}$	$f^{\rhd 3}$
Leak ($\Omega = r$)	f	$f^{\rhd 6}$
Major Leak ($\Omega = r^{\lhd 3}$)	$f^{\lhd 3}$	$i^{\rhd 9}$
Fracture ($\Omega = 0$)	∞	0

Table 2: Leaky pipe fault flow assignment

A Minor Leak at $\Omega = r^{\rhd 3}$ predicts a flow into the atmosphere, but does not demonstrate any qualitatively significant reduction in flow at the output. The exaggerated minor leak demonstrates the situations where a seepage causes a visible indication in the atmosphere, but no significant effect at the output (though there might be air present if this circuit was on the negative suction side of a pump since the simulation can model substance flows). Secondly, a significant leak causes the additional effect of a loss of downstream flow. The extreme Fracture case is qualitatively different again and since there is no resulting output flow, which may be a similar effect to a valve being closed.

4 Symptom Generation

Symptoms are a relation between observations and sets of component faults and can be produced automatically by comparison of nominal and failure mode simulations of the system [1]. That algorithm uses the available state space of the system encountered during production of an FMEA to extrapolate all available consistent minimal (w.r.t. observations) symptoms, using functional interpretation to extrapolate the state space based on observed correlations between structural elements and system function. The starting point for the work of this paper is a consistent set of symptoms, that have been derived by simulation. It is worth noting that the FMEA production requires a system schematic, qualitative component models (with fault descriptions), and a functional model that abstracts qualitative system behaviours into system functions. Production of the symptoms in [1] requires no further information.

A symptom is defined as a tuple $S = (E, F)$ such that E is a first order sentence referred to as a *symptom expression* (rule antecedent) that when satisfied indicates $F = \{M_1, M_2, ...\}$ as component failure modes (rule consequent). $E(s)$ evaluates E on a specific system state $s \in OBS$. In [1] E is always a simple conjunction of equivalence propositions that represent measurements. Faults may provide a fault domain $\mathcal{D}(M)$, such as a measurement over which the fault can be characterised, for example resistance for an electrical corrosion fault.

A trivial example symptom expression produced automatically from an FMEA of a simple torch might be

$$\begin{array}{rl} \mathsf{if} & \mathrm{STATE}(\mathsf{sw}) \leftrightarrow \mathsf{on} \wedge \mathsf{F}(\mathsf{lamp}) \leftrightarrow \mathsf{mA}\Delta\mathsf{A} \\ \mathsf{implicates} & \{\mathsf{switch.corroded\_contact}, \\ & \quad \mathsf{battery.terminal\_loose}\} \end{array} \quad (2)$$

In this example '$\mathsf{mA}\Delta\mathsf{A}$' is a comparative OM observation of electrical current flow magnitude 'mA' for an expected flow 'A'. 'on' is a switch state from the level two model, mapping to a qualitative switch resistance value of 0.

To enable the symptom generator to produce symptoms that can exonerate faults a symptom expression is split into two parts; E_c a Boolean *conditional*, and E_o a Boolean implicator expression. This also allows fault detection to allow evidence from nominally operating parts of a system to exonerate faults. The symptom generator may assign $E_c \equiv \mathrm{STATE}(\mathsf{sw}) \leftrightarrow \mathsf{on}$ and $E_o \equiv F(\mathsf{lamp}) \leftrightarrow \mathsf{mA}$, within a symptom $S = ((E_c, E_o), \{\mathsf{switch.corroded\_contact}, \mathsf{battery.term...}\}$ to ensure:

$$\begin{array}{l} (E_c \wedge E_o, \{\mathsf{switch.corroded\_contact}, \mathsf{battery.term...}\}) \\ (E_c \wedge \neg E_o, \{\neg\mathsf{switch.corroded\_contact}, \neg\mathsf{battery.term...}\}) \end{array} \quad (3)$$

$(E_c \wedge E_o)$ implies $\{M_1, ...\}$
$(E_c \wedge \neg E_o)$ implies $\{\neg M_1, \neg ...\}$
$\neg E_c$ rule invalid

E_c must be satisfied for the symptom to be *valid*. Any symptom where $\neg E_c = \top$ is invalid and cannot contribute any fault information. Any valid symptom that is not satisfied (i.e. $E_c \wedge \neg E_o$) can be used to exonerate the associated faults. For the example if the switch is on and the lamp is active then we can predict that the lamp is not blown, the switch contact is not dirty, and the wire to the lamp is not fractured. A valid symptom that is satisfied implicates the associated faults. A symptom that is not valid provides no information.

The observations present in symptoms are derived by comparison of nominal behaviour and the observed failure behaviour. This information is maintained for all symptom predicates and the notation $A\Delta B$ is used in following sections to indicate an observation A when B was expected. We will use this to allow the exaggerated values to be interpreted and scaled on a type, system, or component or failure mode basis in section 5.

5 Fuzzy Scaling

A fuzzy set is used to provide a concrete numerical domain for OM qualitative values. We use trapezoidal fuzzy sets with the usual parametric representation $[a, b, \alpha, \beta]$, for example V in Figure 2 is defined as $[10.8, 13.8, 0.8, 1.2]$.

Individual component faults are marked as exaggerated or not exaggerated. Some (idealised) fault models are accurately represented by the available qualitative values, for example a short circuit ($\Omega = 0$) and open circuit ($\Omega = \infty$). Exaggerated faults are used where a simple mapping of an example fault perturbation will not produce a qualitatively significant behaviour.

This work proposes fuzzy membership functions as a relatively intuitive mechanism to map numerical observations into a membership level for qualitative values used in diagnostic rules, while at the same time allowing the numerical operating ranges of the system to be specified.

Fuzzy membership functions may therefore be defined for each qualitative value associated with any specific observation, both for nominal and exaggerated values. For example if the nominal operating voltage for an automotive circuit is between 12.4V and 13.8V falling into the 3OM qualitative value $\mu_{\mathsf{u}^{\rhd 0}} = \mathsf{V}$, and an exaggerated fault $\mathsf{corroded\_terminal}$

causes a lamp output voltage to be $\mu_{\mathsf{u} \rhd 3} =$ mV, we might map mVΔV for lamp.V to a voltage range of 0-10.8V as in Figure 2. The exaggerated symptom is mapped back to

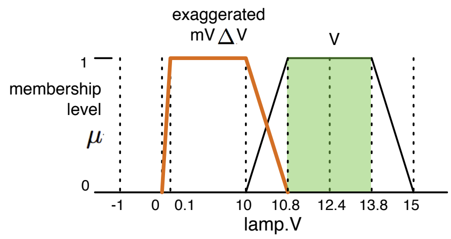

Figure 2: Automotive scaling example

the operating range of the system. The exaggerated scaling is that when the fault is mapped from an OM higher than normal resistance to merely out of the range of nominal operation, the effect is mapped from an OM lower voltage to merely out of the range of nominal voltage. This mimics the reasoning "if the contact is very badly corroded the circuit will contain a very high resistance in series with the lamp and therefore there will be almost no current flow to the lamp with almost no voltage drop across the lamp ...therefore, a lower than normal voltage across at the lamp may indicate corrosion at the switch". The chain of reasoning may be of any length, traversing more than one domain, dependent on the modelling. If required a nominal mV set could also be defined if there is nominal operation at that level to interpret.

An engineer (or some other method such as numerical simulation) can be used to identify the nominal operating ranges, and any exaggerated effects can have suitable ranges and membership functions identified for their scaled numerical effect.

We define unique membership functions μ_x and $\mu_{A\Delta x}$ within the same real order of magnitude since the former is a nominal operating range and the latter is a scaled OM exaggeration *from that range*. Multiple nominal ranges may exist, for systems with behaviour ranging over more than one qualitative OM for different operating modes (imagine a somewhat unlikely example of an oil storage tank that 'fast fills' fills and 'slow empties' through the same pipe, and where we can only measure flow rate and not direction). The exaggerated versions of each OM will fill the real valued domain space between them as in Figure 3. μ_x are shown in black (dark) and $\mu_{A\Delta x}$ are shown in orange (lighter) colours. The exaggerated flow set in Figure 3 represents both Liter/hour (L/hour) as an exaggeration of the nominal value L/s as well as the exaggeration of L/s from a nominal value of L/hour. In this example the exaggeration scaling uses a *progressive* mapping, contrasting with the *hard* mapping in Figure 2, these two approaches are discussed in section 5.3. The filled regions indicate the expected ideal range for nominal operation with a reducing membership for values considered to be outside of ideal operation, but within design tolerance. Membership of 0.5 provides neither indication or exoneration for a rule using the set and thus provide a common point for exaggerated and non exaggerated sets.

5.1 Fuzzy interpretation of faults

To allow symptoms to implicate and exonerate faults we define a domain for faults as in Figure 4. Faults have a do-

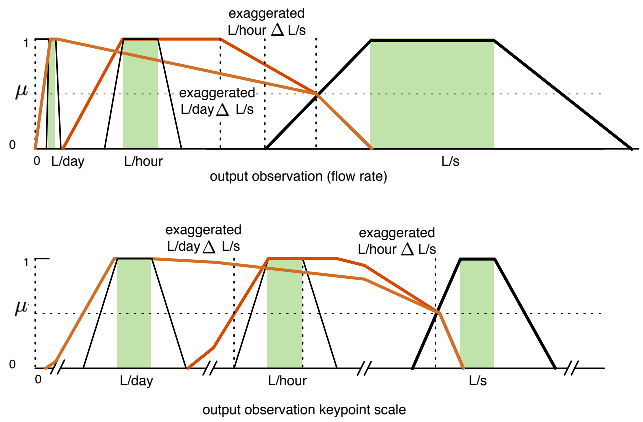

Figure 3: Default progressive exaggeration membership functions

main representing 'strength of the fault indication' or alternatively *fault rank*, where -1 is absent, and 1 is present, for fault exoneration and implication respectively. The exoneration set $\mu_{\overline{M}}$ is used to deal with valid but unsatisfied symptom effects.

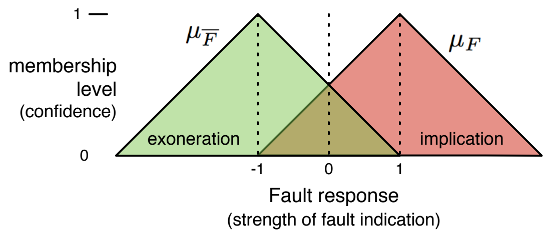

Figure 4: Binary fault fuzzy representation

The reason for this definition of the fault domain is due to the centroid defuzzification required to produce a crisp fault response (ranking) value in the fully implicated to fully exonerated range. A membership of μ_F is required to fully implicate a fault at the centroid of the set (i.e. fault response 1 on figure 4). The membership of $\mu_{\overline{F}}$ is required to fully exonerate a fault at the centroid of the set. The non truth functional nature of fuzzy implication [14] using the standard definition of $\mu_{\overline{F}}$ as $1-\mu_F$ is asymmetric on the domain $\{-2, 2\}$ required to produce a $\mu_{\overline{F}}$ centroid at -1 and is thus problematic if we truncate the set (to provide the conditional element of the symptom). The sets used in figure 4 allow a fault response membership function to be generated for any antecedent truth value.

Following standard practice for fuzzy rule interpretation, the evaluation of E_c on some set of real valued inputs values results in an antecedent truth value, $T(E_c)$, and similarly for E_o. Assuming $T(E_c) = 1$, then μ_{E_o} is a set in the fault response domain representing $T(E_o) = 1$ and $\mu_{\overline{E_o}}$ is a set representing $T(E_o) = 0$. More generally, the center of the output set is $2(T(E_o) - 1)$, and its extent is 2. The parametric definition of the output set provided by a symptom observation is therefore $\mu_{E_o} = [2(T(E_o) - 1), 2(T(E_o) - 1), 2, 2]$. Clearly, a valid symptom with $E_o = 1$ will provide no exoneration, and $E_o = 0$ will fully exonerate the fault. $E_o = 0.5$ is a fault with a truth value of 50% so it will provide a fault rank of 0 (no diagnostic information), due to equal levels of evidence both for and against the fault. Figure 4 if interpreted as the result of two symptoms ($T(E_c) = 1$) both associated with a fault, shows just such a case where we have two contradictory indications. A defuzzification (centroid method) will result in a fault response of 0 (no implication either way).

Although the support for μ_M and $\mu_{\overline{M}}$ covers the interval $[-3, 3]$, centoid (and other common) defuzzification of symptoms as described can only result in output values in the interval $[-1, 1]$, due to the symmetry of the sets and the restriction of the centers to $[-1, +1]$. In addition it is clear that exoneration is of equal 'value' to implication, although this could be changed if desired by reshaping the sets.

5.2 Conditional symptoms

The symptoms are conditional which for the Boolean case allows symptoms to be restricted in their scope. The relevance of specific observations can be narrowed, for example no output does not usually indicate a fault if the system in the off state. The Boolean semantics of $E_c = \bot$ are that the symptom cannot be used to either implicate or exonerate the fault (n/a), and conversely $E_c = \top$ says that the symptom should provide a definite fault prediction. Translating this to truth values implies intuitively that the membership level of the fault response output set should be related to validity of the rule $T(E_c)$. The min function is the most commonly used fuzzy implication method used to reshape the consequent set μ_{E_o} to produce μ_M in the fault response domain resulting in a truncation of the output set membership:

$$\mu_M = \min(T(E_c), \mu_{E_o}) \qquad (4)$$

Evaluation of $T(E_o)$ and $T(E_c)$ is a straightforward case of evaluating each predicate by mapping sensor values to a membership level for each fuzzy set in the relevant domain, and using fuzzy operators to combine these values. We utilised the common Zadeh operators $\mu_{A \,\mathsf{AND}\, B} = \min(\mu_A, \mu_B)$ $\mu_{A \,\mathsf{OR}\, B} = \max(\mu_A, \mu_B)$ to produce the antecedent truth value.

Unlike Fuzzy Associative Matrices [15] which provide only one rule for each fuzzy output set, our rule generator will typically generate multiple rules for a given fault. Intuitively the symptom with the strongest evidence prevails, so the set with the highest membership is selected for inclusion in the consequent and this is consistent with the usual Fuzzy rule output aggregation. i.e. if symptoms $S_1, S_2 \ldots S_n$ exist where $(\forall S_{1\ldots n})(M \in F(S_{1\ldots n}))$ then

$$\mu_{M'} = \max(\mu_M(S_1), \mu_M(S_2) \ldots \mu_M(S_n)) \qquad (5)$$

The final numerical fault response M' is produced by centroid calculation on $\mu_{M'}$.

5.3 Progressive fault sets

Faults may also be represented over an OM domain, resulting in a number of symptoms that indicate sets from the same fault domain (Figure 5). Multiple symptoms allow the response to act in a progressive way based on multiple OM observations.

If a single fault domain $\mathcal{D}$ has faults $\overline{M}_{\mathcal{D}}, M_A, M_B, \ldots M_m$ with predefined sets $\mu_{\overline{M}_{\mathcal{D}}}, \mu_{M_A}, \mu_{M_B}, \ldots \mu_{M_m}$ we can use the aggregation rule $\mu_{\mathcal{D}} = \mu_{M_A} \vee \mu_{M_B} \vee \ldots \mu_{M_m} \vee \mu_{\overline{M}_{\mathcal{D}}}$ (equation 5) given output sets are reshaped in accordance with the individual fault responses for $0 \le M'_A \le 1$:

$$\mu_{M'_A} = \min(M'_A, \mu_{M_A}), \text{similarly for } B, C\text{...etc} \qquad (6)$$

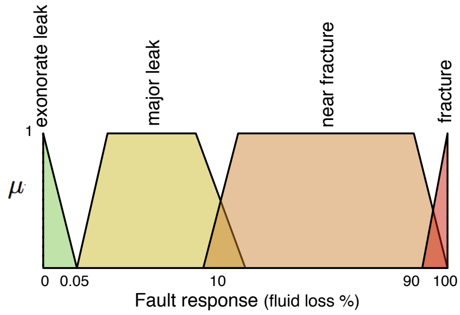

Figure 5: Faults sharing a fault domain

The exoneration set is logically defined as the negated conjunction for

$$\mu_{\overline{M}_{\mathcal{D}}} = \neg(\mu_{M_A} \vee \mu_{M_B} \vee \ldots \mu_{M_m}) \qquad (7)$$

however to avoid the problematic negation in the computation of the exoneration set, negation of the truth values is used to compute $\mu_{\overline{M}_{\mathcal{D}}}$ as follows for $-1 \le M'_A \le 0$,

$$\begin{aligned} \mu_{\overline{M}_{\mathcal{D}}} &= \mu_{\overline{M}_A} \wedge \mu_{\overline{M}_B} \wedge \ldots \mu_{\overline{M}_m} \\ \mu_{\overline{M}'_{\mathcal{D}}} &= \min(\overline{M}'_{\mathcal{D}}, \mu_{\overline{M}_{\mathcal{D}}}) \\ &\text{where,} \\ \overline{M}'_{\mathcal{D}} &= \min(-M'_A, -M'_B, \ldots) \end{aligned} \qquad (8)$$

Any fault set M_x that does not produce a result set $\mu_{M'_x}$ does not define a truth value M'_x and is excluded from $\mu_{\overline{M}_{\mathcal{D}}}$.

6 Example and Diagnosis

We are not concerned with the complexity of the simulation or size of the system in this work because the simulation and symptom generation has been proven on large multi-domain systems [16], and so for reasons of space, we use Figure 6 to illustrate the interpretation and diagnosis of a very simple fluid flow system.

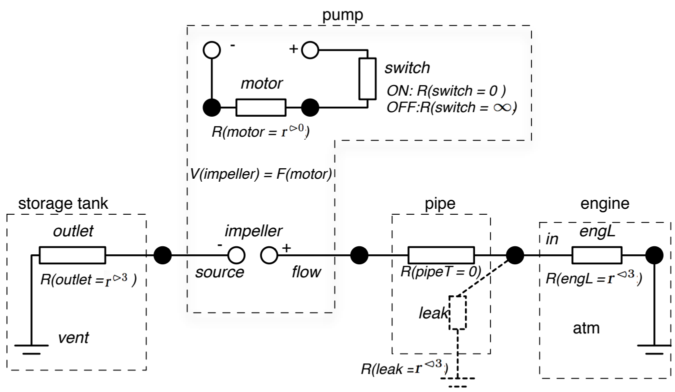

Figure 6: Fuel System

The pump links motor electrical current flow to the pump effort in a fluid flow circuit. We will assume for this example that the pump input voltage is $E(\mathsf{pump}) = u$. Normally $E(\mathsf{impeller}) = u$, but it has a fault mode 'binding' which lowers its output so for binding $E(\mathsf{impeller}) = u^{\lhd 3}$. The pipe can have a variety of leaks of different severity: minor leak $R(\mathsf{leak}) = r^{\lhd 6}$; leak $R(\mathsf{leak}) = r^{\lhd 3}$; major leak $R(\mathsf{leak}) = r^{\rhd 0}$; fracture $R(\mathsf{leak}) = \infty$.

Table 3 shows a few example results from a simulation for the state when the pump is on; in all cases the nominal flow is $F(\mathsf{engL}) = f^{\rhd 3}$.

Fault	observation	exaggerated
pipe.minor leak	$F(\mathsf{engL}) = f^{\rhd 3}$	nominal
pipe.leak	$F(\mathsf{engL}) = f^{\rhd 3}$	No
pipe.major leak	$F(\mathsf{engL}) = f^{\rhd 6}$	Yes
pipe.fracture	$F(\mathsf{engL}) = 0$	No
pipe.blockage	$F(\mathsf{engL}) = 0$	No
impeller.binding	$F(\mathsf{engL}) = f^{\rhd 6}$	Yes
$\wedge$ pipe.minor leak	$F(\mathsf{engL}) = f^{\rhd 6}$	Yes
$\wedge$ pipe.leak	$F(\mathsf{engL}) = f^{\rhd 6}$	Yes
$\wedge$ pipe.major leak	$F(\mathsf{engL}) = f^{\rhd 9}$	Yes
$\wedge$ pipe.fracture	$F(\mathsf{engL}) = 0$	Yes

Table 3: Example failure results

The symptom generator will produce symptoms such as:

$$\begin{array}{rl} \mathsf{S1: when} & V(\mathsf{pump}) \leftrightarrow u \\ \mathsf{if} & F(\mathsf{engL}) \leftrightarrow f^{\rhd 6}\Delta f^{\rhd 3} \\ \mathsf{implicates} & \{\mathsf{pipe.majorleak}, \mathsf{impeller.binding} \wedge \mathsf{pipeleak}\} \end{array} \tag{9}$$

Using the earlier symptom notation $E_c = V(\mathsf{pump}) \leftrightarrow u$ and $E_o = F(\mathsf{engL}) \leftrightarrow f^{\rhd 6}\Delta f^{\rhd 3}$ indicating a flow of $f^{\rhd 6}$ when $f^{\rhd 3}$ expected. Notice that $E_c \neq \emptyset$ because absence of $F(\mathsf{engL}) \neq f^{\rhd 6}$ does not exonerate any fault if the system is off. The switch would also provide the necessary condition in this case because only one active input voltage is ever present (for all nominal and failure modes), however it is less discriminating and therefore is not selected provided input voltage is available.

The minor leak cannot be detected and no symptom for it is available since it cannot be distinguished from nominal operation. Allowing more observations such as the tank outflow when the system is off or direct observation of the atmosphere would allow the minor leak to be detected.

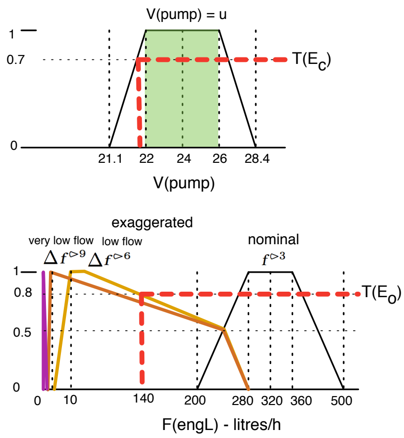

Figure 7: Scaled fuel system interpretation

The symptoms generated require the use of observations of $V(\mathsf{pump})$ and $F(\mathsf{engL})$. Figure 7 provides one interpretation of the required OM values for our system. This information must be provided by an engineer, by reference to the nominal operating values of the system and also by consideration of the exaggerated faults used. 320 is considered nominal within the litres per hour magnitude, 200 is the extreme of nominal operation and provides the center of the next exaggerated fault representing 'low flow'. $f^{\rhd 3}\Delta f^{\rhd 9}$ is double exaggerated result 'very low flow' and may be the result of a more extreme fault or multiple faults (e.g. Table 3). The 0 set extends to 0.05 to account for measurement accuracy and other factors mean that small but non zero measurements may be present when no flow is actually present.

If we observe values $V(\mathsf{pump}) = 21.8\mathsf{V}$ and $F(\mathsf{engL}) = 140\mathsf{l/h}$ then we see that for $S1$: in definition 9, $T(E_c) = \mu_u(V(\mathsf{pump})) = 0.7$ and $T(E_o) = \mu_{f^{\rhd 6}}(F(\mathsf{engL})) = 0.8$ resulting in an output set $\mu_{\mathsf{pipe.majorleak}} = 0.7[0.2, 1.4, 1.4, 1.4]$.

There are no other symptoms indicating or exonerating this fault in such a simple example so $\mu_{M'} = \mu_M(S1)$ and the fault rank is therefore directly provided by the centroid of 0.8 for the set in figure 8.

In a real system there will be many valid rules providing indications and contra indications. In our previous work aimed at assessing 'diagnosability' symptoms were validated by evaluation against qualitative results of a simulated fault. A fault ranking was then produced simply as the difference between the number of rules implicating and exonerating a fault. This simple method provided a useful indication of the validity of the symptoms at a qualitative level, however it could not work on real valued observations, and also there is no explicit significance to the number of rules generated for any specific fault. The fuzzy mapping does not simply 'count satisfied rules' but assesses their significance and combines them with respect to expected operation of the system.

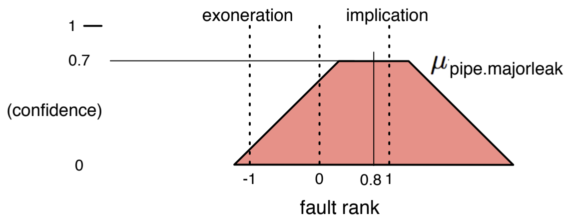

Figure 8: $\mu_{\mathsf{pipe.majorleak}}$ from $S1$

7 Conclusions

The identification of fuzzy sets provides a relatively simple method to identify the nominal operating regions of a system and by extension of these, a definition for abnormal operation. The fuzzy implication effectively interpolates the numerical operating space of the system using the control points specified by the rules and fuzzy sets, providing a very simplified numerical function for the system behaviour. We propose that since the aim is to rank faults, this simplification appears to be generally adequate, although more investigation is needed to determine the range of applicability. The comparative nature of FMEA makes it possible to provide specific interpretations for nominal and abnormal ranges of values for many systems, and is only required for the components and observations that turn out to be diagnostically interesting.

The OM reasoning allows separation of concerns and exaggeration of faults to produce qualitative significance. For example if the system in figure 6 had included a 'fast drain down valve' taking a normal flow of *f* in parallel with the engine, then the pipe.majorleak would not provide a significant effect on drain flow, but a fracture would show an exaggerated 0 flow. Using qualitative signs, the only available exaggerated fault would be fracture, and we could not differentiate the more subtle effects on systems that contain multiple OM levels. Some examples of these levels are signal level, electro-mechanical level and power train level in electrical systems; pump level and gravity level pressures in fluid flow systems; and heat exchanger versus stationary fluid conduction in thermal circuits.

The fuzzy interpretation does not deal with the *probability* of faults, or the accuracy of symptoms. Bayesian approaches [17] can rank rule based faults, however the difficulty is in determining the probabilities. Even when this can be done, the fuzzy value membership of observations appears to provide a more direct interpretation of the uncertainty based on actual system state, although statistical information could be used to differentiate similar ranked faults.

Although this paper includes a very simple example, we have applied the method to symptoms generated for a complex aircraft fuel system[18] and with a suitable fuzzy set mapping the results provide a progressive ranking of faults as the behaviour of the system moves away from expected operating parameters. That system was a topologically complex set of valves and pumps, with many stable steady states and clearly identifiable operating regions. Further work will be aimed at characterising the effects of set selection on the ranking characteristics of the approach and on the limitations of the OM exaggerations.

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